УДК 621.382

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THE STABILITY OF THE TWO-CHANNEL DISTRIBUTED-PARAMETERS WITH THE LOSS CONTROL SYSTEM

In this paper, we demonstrate the analysis of the stability of the two-channel control systems containing the optical fibre links as a through-coupling.. The analysis of the stability of systems the indicial equation of which can be reduced to the form where its roots are located in the defined sector of the left half plane of the complex variable s was conducted on the basis of the method of the plane of parameters.

Приведено аналіз стабільності двоканальних керуючих систем, які містять волоконно-оптичні ланки в якості наскрізних зв'язків. На основі методу матриці параметрів проведено аналіз стабільності систем визначальне рівняння яких може бути зменшена до форми в якій його корені розміщені в певному місці лівої половини площини комплексних змінних.

1. Introduction

Among the multi-channel automatic control systems one can find the systems consisting of two [1,3] or even three [2] identical channels. There are also situations when the system operates in the high EMI environment that affects the coupling between channels. Therefore, the application of optical fibre links in order to eliminate the influence of EMI is necessary.

The method of the analysis of the stability of single channel linear systems containing optical fibre links designed to operate in high EMI environments is presented in this paper[5].

This paper presents the analysis of the stability of two-channel control systems containing the optical fibre links as a through-coupling (Fig. 1).

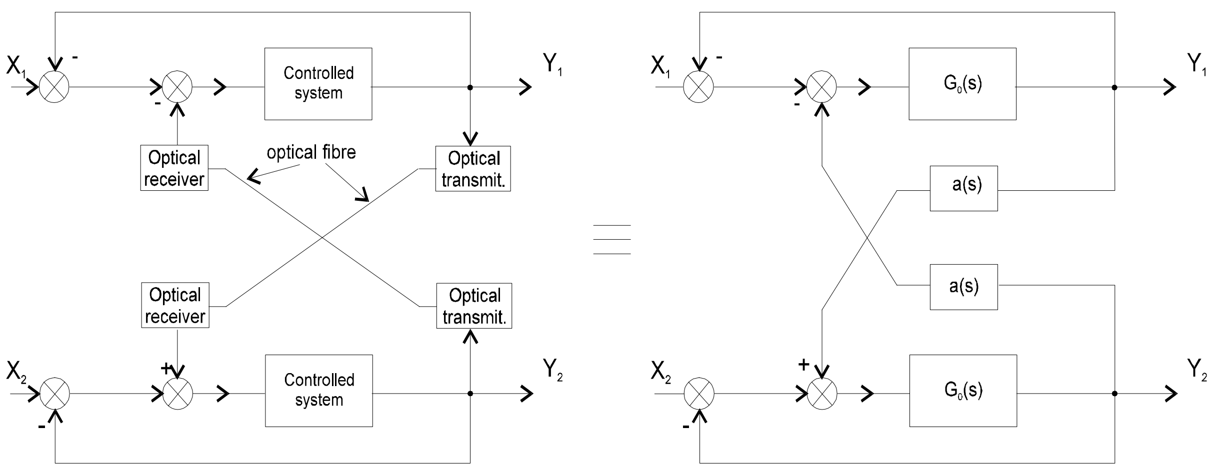

Fig. 1. A block diagram of an automatic control system containing the optical fibre links. a) Physical layout, b) Diagram showing the transmittances of individual components

2. The Analysis of the stability of two-channel systems optical with the fibre through-coupling

We assume that two identical linear objects of lumped constants (Fig. 1) have the analytical transfer function $G_0(s)$ in the right half plane and can be represented in the form of a fraction

$$G_0(s) = \frac{A(s)}{B(s)} \tag{1}$$

when

$$A(s) = \sum_{\nu=0}^{n} a_\nu s^\nu \qquad (a_\nu, b_\nu \in R, a_n \neq 0) \tag{2}$$
$$B(s) = \sum_{\nu=0}^{n} b_\nu s^\nu$$

The a(s) denotes the transmittance of a through-coupling, of which the transfer function is determined as:

$$a(s) = \exp\left(\frac{1}{2}\gamma(s)l\right) \tag{3}$$

where

$$\gamma(s) = \sqrt{s^2c + sd + h} \tag{4}$$

l, c, d, h – parameters of the distributed-parameters components.
The complex operator transmittance [5] of a closed system G(s) has the following form:

$$G(s) = \frac{B(s)\left(e^{\frac{1}{2}\nu l} + j\right)}{[A(s)+B(s)]e^{\frac{1}{2}\nu l} + jB(s)} \tag{5}$$

The stability of the above system depends on the position of zeros in the characteristic equation:

$$M(s) = [A(s) + B(s)]e^{\frac{1}{2}\gamma(s)l} + jB(s) \tag{6}$$

Let us transform the expression:

$$N(s) = s^2c + sd + h\,, \tag{7}$$

in which, in order to eliminate the free term h in expression, we introduce a new complex variable:

$$p=s+k.$$

The expression (7) takes the following form:

$$N(s) = N(p-k) = c'p^2 + d'p + h' \tag{8}$$

where:

$$c' = c\,,$$
$$d' = d - 2ck\,,$$
$$h' = ck^2 - dk + h\,.$$

By equating h' to zero we can determine the parameter k which, in general, is a complex number of the form:

$k = \sigma \pm j\delta$

For further investigation we assumed $k = \sigma + j\delta$ for which the $k = \sigma - j\delta$ calculations are analogous.

Now, let us transform the expression $\frac{1}{2}\gamma l$:

$$\frac{1}{2}\gamma l = \frac{1}{2} l\sqrt{s^2 c + sd + h} = \frac{1}{2} l\sqrt{p^2 c + pd'} = \frac{1}{2}\tau\sqrt{p^2 + p\frac{d'}{c}} = \varphi(p), \tag{9}$$

where:

$$\tau = l\sqrt{c}.$$

The expression (9) can be represented as:

$$\varphi(p) = \frac{1}{2}\tau p\sqrt{1 + \frac{d'}{pc}}. \tag{10}$$

By the series expansion of $\sqrt{1 + \frac{d'}{pc}}$ we obtain:

$$\varphi(p) = \frac{1}{2}\tau p\left[1 + \frac{1}{2}\frac{d'}{c}\frac{1}{p} - \frac{1}{2\cdot 4}\left(\frac{d'}{c}\right)^2 \frac{1}{p^2} + \frac{1\cdot 3}{2\cdot 4\cdot 6}\left(\frac{d'}{c}\right)^3 \frac{1}{p^3} - \dots\right] \tag{11}$$

This series is convergent for $\left|\frac{d'}{c}\frac{1}{p}\right| \le 1$.

Substituting $p = s + k$ for (11) we obtain

$$\varphi(p) = \frac{1}{2}\tau(s+k)\left[1 + \frac{1}{2}\frac{d'}{c}\frac{1}{s+k} - \frac{1}{2\cdot 4}\left(\frac{d'}{c}\right)^2 \frac{1}{(s+k)^2} + \right.$$
$$\left. + \frac{1\cdot 3}{2\cdot 4\cdot 6}\left(\frac{d'}{c}\right)^3 \frac{1}{(s+k)^3} - \dots\right], \tag{12}$$

and the condition of convergence of the series (12) acquires the following form:

$$|s + k| \ge \left|\frac{d'}{c}\right|.$$

The expression (10) can also be represented in another form:

$$\varphi(p) = \frac{1}{2}\tau\sqrt{\frac{d'}{c}p}\sqrt{1 + \frac{c}{d'}p} \tag{13}$$

By the series expansion of $\sqrt{1 + \frac{c}{d'}p}$ we obtain:

$$\varphi(p) = \frac{1}{2}\tau\sqrt{\frac{d'}{c}p}\left[1 + \frac{1}{2}\frac{c}{d'}p - \frac{1}{2\cdot 4}\left(\frac{c}{d'}\right)^2 p^2 + \frac{1\cdot 3}{2\cdot 4\cdot 6}\left(\frac{c}{d'}\right)^3 p^3 - \dots\right] \tag{14}$$

This series is convergent for $\left|\frac{c}{d'}p\right| \le 1$. Substituting $p = s + k$ into (14) we have:

$$\varphi(p)=\frac{1}{2}\tau\sqrt{\frac{d'}{c}(s+k)}\left[1+\frac{1}{2}\frac{c}{d'}(s+k)-\frac{1}{2\cdot 4}\left(\frac{c}{d'}\right)^2(s+k)^2+\frac{1\cdot 3}{2\cdot 4\cdot 6}\left(\frac{c}{d'}\right)^3(s+k)^3-\ldots\right] \quad (15)$$

and the condition of convergence is: $|s+k|\le\left|\frac{c}{d'}\right|$.

The series (12) and (15) can be limited to the first two terms of expansion. For $|s+k|>\left|\frac{d'}{c}\right|$ and assuming that $d'=d-2ck$, $\varphi(s)$ is as follows:

$$\varphi(s)=\frac{1}{2}\tau\left(s+k+\frac{1}{2}\frac{d'}{c}\right)=0.5\tau(s+m) \quad (16)$$

where:

$$m=\frac{1}{2}\frac{d}{c}.$$

Considering (16) and (9), equation (4) can be represented as:

$$M(s)=[A(s)+B(s)]e^{0,5\tau(s+m)}+j\,B(s) \quad (17)$$

or, considering (2)

$$M(s)=\sum_{\nu=0}^{n}\left[(a_\nu+b_\nu)e^{0.5m\tau}e^{0.5\tau s}+j\,b_\nu\right]s^\nu \quad (18)$$

when, in equation (18) the real terms a_ν and b_ν are the linear functions of the two parameters α and β:

$$\begin{cases}a_\nu=a_{\nu1}\alpha+a_{\nu2}\beta+a_{\nu3}\\ b_\nu=b_{\nu1}\alpha+b_{\nu2}\beta+b_{\nu3}\end{cases} \quad (19)$$

The function:

$$s=-|w|e^{j\frac{2}{\pi}(\arg w-\pi)\arccos\xi}-\eta, \qquad \eta>0 \quad (20)$$

for $\arg w\in\left\langle\frac{\pi}{2},\frac{3}{2}\pi\right\rangle$ and $\arg(s+\eta)\in\langle\pi-\arccos\xi,\pi+\arccos\xi\rangle$ maps the left half plane of the plane of the complex variable „w" into the region Ω on the plane of the complex variable „s" which is limited by two half lines l_1 and l_2 (fig. 2a) of the following equations:

$$\begin{cases}s=\omega_n e^{j\arccos\xi}-\eta & \text{for } \omega_n\in(-\infty,0)\\ s=-\omega_n e^{j\arccos\xi}-\eta & \text{for } \omega_n\in\langle 0,\infty)\end{cases} \quad (21)$$

where ω_n is the pulsation of the undamped proper vibration and ξ is the relative damping coefficient – and the arc of an infinite radius.

For the sake of uniformity of the formulae we introduce the following variable:

$$z=\begin{cases}-\xi & \text{for} \quad \omega_n\in(-\infty,0)\\ \xi & \text{for} \quad \omega_n\in\langle 0,\infty)\end{cases} \quad (22)$$

Then

$$\arccos\xi=\begin{cases}\pi-\arccos z & \text{for} \quad \omega_n\in(-\infty,0)\\ \arccos z & \text{for} \quad \omega_n\in\langle 0,\infty)\end{cases}$$

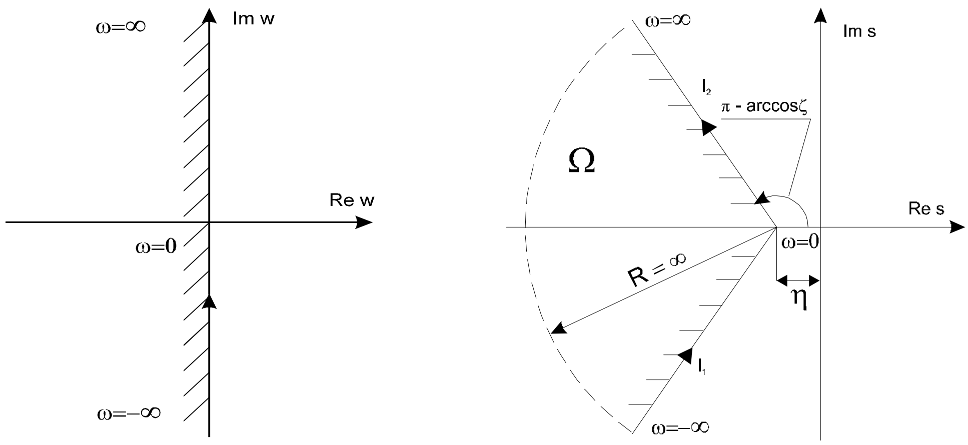

Fig. 2. Mapping of the left half plane of the complex variable „w" into the region Ω of the complex variable „s"

and (21) can be rewritten as:

$$s = -\omega_n e^{-j\arccos z} - \eta \quad \text{for} \quad \omega_n \in (-\infty, \infty), \tag{23}$$

while:

$$\operatorname{sign} z = \operatorname{sign} \omega_n . \tag{24}$$

Considering that

$$\cos(\arccos z) = z$$

and

$$\sin(\arccos z) = \sqrt{1 - z^2} ,$$

the relationship (23) can be rewritten as:

$$s = -\omega_n \left(z - j\sqrt{1 - z^2} \right) - \eta \quad \text{for} \quad \omega_n \in (-\infty, \infty). \tag{25}$$

The need for zeros of the function M(s) to be located within the given region Ω, i.e. the determination of the degree of stability η and the relative damping coefficient ξ, is equivalent to the need for zeros of the function

$$\widetilde{M}(w) = M\left(-|w| e^{j\frac{2}{\pi}(\arg w - \pi)\arccos\xi} - \eta \right) \tag{26}$$

to be located within left half plane of the complex variable „w". The mapping described maps the zeros of function $\widetilde{M}(s)$ into the zeros of the function $\widetilde{M}(w)$ in such a way that the real zeros correspond to the real zeros and complex conjugate pairs correspond also to complex conjugate pairs.

By mapping, using function (26), the left half plane of the plane of the complex variable „w", an area on the plane of parameters α and β can be found in which the stability of the system described can be guaranteed. In order to do the, the following equation has to be solved:

$$\widetilde{M}(j \cdot \omega_n) = 0$$

that is

$$M(s) = 0 . \tag{27}$$

In order to determine the dependence:

$$M(s) = \sum_{\nu=0}^{n}\left[\left(a_\nu + b_\nu\right)e^{(m+0,5s)\tau} + j b_\nu\right]s^\nu \tag{28}$$

according to (23)we calculate s^ν :

$$\begin{aligned} s^\nu &= \left(-\omega_n e^{-j\arccos z} - \eta\right)^\nu = \sum_{\chi=0}^{\nu}(-1)^\nu \binom{\nu}{\chi}\eta^{\nu-\chi}\omega_n{}^\chi e^{-j\chi\arccos z} \\ &= \sum_{\chi=0}^{\nu} H_{\nu,\chi}\omega_n{}^\chi\left[T_\chi(z) - jU_\chi(z)\right], \end{aligned} \tag{29}$$

where

$$H_{\nu,\chi} = (-1)^\nu \binom{\nu}{\chi}\eta^{\nu-\chi}$$

and functions:

$$T_\chi(z) = \cos(\chi\arccos z),$$
$$U_\chi(z) = \sin(\chi\arccos z),$$

are Tshebyshev terms of the first and second kind.

Next substituting dependence (25) for $e^{0,5\tau(s+m)}$ and separating the real part from the imaginary part, we get:

$$e^{0,5\tau(s+m)} = e^{0,5m\tau}e^{0,5\left[-\omega_n\left(z-j\sqrt{1-z^2}\right)-\eta\right]} = I(\omega_n) + jK(\omega_n)$$

where:

$$\begin{aligned} J(\omega_n) &= e^{-0.5\tau[(\omega_n z+\eta)-m]}\cos 0.5\tau\left(\omega_n\sqrt{1-z^2}\right) \\ K(\omega_n) &= e^{-0.5\tau[(\omega_n z+\eta)-m]}\sin 0.5\tau\left(\omega_n\sqrt{1-z^2}\right). \end{aligned} \tag{30}$$

Considering expressions (29) and (30) in relation to (28) we obtain the following equation:

$$\sum_{\nu=0}^{n}\sum_{\chi=0}^{\nu} H_{\nu,\chi}\omega_n{}^\chi\left[T_\chi(z) - jU_\chi(z)\right]\left[a_\nu\left(J(\omega_n) + jK(\omega_n)\right) + b_\nu\right] = 0 \tag{31}$$

According to (27) the real and the imaginary part of equation (31) are simultaneously equal to zero, i.e.:

$$\begin{aligned} &\sum_{\nu,\chi} H_{\nu,\chi}\omega_n{}^\chi\left[a_\nu K(\omega_n)U_\chi(z) + \left(a_\nu J(\omega_n) + b_\nu\right)T_\chi(z)\right] = 0 \\ &\sum_{\nu,\chi} H_{\nu,\chi}\omega_n{}^\chi\left[a_\nu K(\omega_n)T_\chi(z) - \left(a_\nu J(\omega_n) + b_\nu\right)U_\chi(z)\right] = 0 \end{aligned} \tag{32}$$

Using the dependence (19), equation (32) can be expressed as follows:

$$\begin{cases} A_1(\omega_n)\alpha + A_2(\omega_n)\beta + A_3(\omega_n) = 0 \\ B_1(\omega_n)\alpha + B_2(\omega_n)\beta + B_3(\omega_n) = 0, \end{cases} \tag{33}$$

where:

$$\begin{cases} A_i(\omega_n) = \sum\limits_{\nu,\chi} H_{\nu,\chi}\omega_n{}^{\chi}\left[a_{\nu i}K(\omega_n)U_\chi(z) + \left(a_{\nu i}J(\omega_n) + b_{i\nu}\right)T_\chi(z)\right] = 0 \\ B_i(\omega_n) = \sum\limits_{\nu,\chi} H_{\nu,\chi}\omega_n{}^{\chi}\left[a_{\nu i}K(\omega_n)T_\chi(z) - \left(a_{\nu i}J(\omega_n) + b_{\nu i}\right)U_\chi(z)\right] = 0 \end{cases} \tag{34}$$

$(i = 1,2,3)$.

We have obtained two equations containing two unknown values α and β. By resolving the system of equations (34) in respect to α and β, assuming that the main determinant is $\Delta_0(\omega_n) \neq 0$, we have:

$$\begin{cases} \alpha = \dfrac{\Delta_1(\omega_n)}{\Delta_0(\omega_n)} \\ \beta = \dfrac{\Delta_2(\omega_n)}{\Delta_0(\omega_n)} \end{cases} \tag{35}$$

where the determinants Δ_j $(j = 0,1,2)$ are defined as follows:

$$\begin{cases} \Delta_0(\omega_n) = A_1(\omega_n)B_2(\omega_n) - B_1(\omega_n)A_2(\omega_n) \\ \Delta_1(\omega_n) = -A_3(\omega_n)B_2(\omega_n) + B_3(\omega_n)A_2(\omega_n) \\ \Delta_2(\omega_n) = -A_1(\omega_n)B_3(\omega_n) + B_1(\omega_n)A_3(\omega_n) \end{cases} \tag{36}$$

For $|s+k| < \left|\dfrac{d'}{c}\right|$ we will use the following expression of $\varphi(s)$:

$$\varphi(s) = 0.5\tau\sqrt{2(m-k)(s+k)}\left[1 + \frac{1}{4(m-k)}(s+k)\right], \tag{37}$$

where:

$$m = \frac{1}{2}\frac{d}{c}.$$

Assuming (37), the equation (18) will take the following form:

$$M(s) = \sum_{\nu=0}^{n}\left((a_\nu + b_\nu)e^{0.5\tau\sqrt{2(m-k)(s+k)}\left[1+\frac{1}{4(m-k)}(s+k)\right]} + j b_\nu\right)s^\nu, \tag{38}$$

when a_ν and b_ν are defined by the dependence (19) and s^ν by (29).

Substitution (25) for $e^{\varphi(s)}$ and considering $k = \sigma + j\delta$ and then separating the real part from the imaginary part we obtain:

$$e^{0.5\tau\sqrt{2(m-k)(s+k)}\left[1+\frac{1}{4(m-k)}(s+k)\right]} = e^{N_1(\omega_n)+N_2(\omega_n)} = \tilde{J}(\omega_n) + j\tilde{K}(\omega_n), \tag{39}$$

when:

$$\begin{cases} \widetilde{J}(\omega_n) = e^{N_1(\omega_n)} \cos N_2(\omega_n) \\ \widetilde{K}(\omega_n) = e^{N_1(\omega_n)} \sin N_2(\omega_n), \end{cases} \tag{40}$$

where:

$$N_1(\omega_n) = 0.5\tau\left[u + \frac{1}{4}\frac{1}{(m-\sigma)^2+\delta^2}(ur - vq)\right]$$

$$N_2(\omega_n) = 0.5\tau\left[v + \frac{1}{4}\frac{1}{(m-\sigma)^2+\delta^2}(uq - vr)\right]$$

and

$$r = -\omega_n z(m-\sigma) - \delta\omega_n\sqrt{1-z^2} + m\sigma - m\eta + \sigma\eta - \sigma^2 - \delta^2,$$

$$q = \omega_n\sqrt{1-z^2}(m-\sigma) - \omega_n z\delta + \delta m - \delta\eta,$$

$$u = \pm\sqrt{x + \sqrt{x^2+y^2}},$$

$$v = \pm\frac{y}{2\sqrt{x+\sqrt{x^2+y^2}}},$$

$$x = -\omega_n z(m-\sigma) + \delta\omega_n\sqrt{1-z^2} + m\sigma - m\eta + \sigma\eta - \sigma^2 - \delta^2,$$

$$y = \omega_n\sqrt{1-z^2}(m-\sigma) + \omega_n z\delta + \delta m - \delta\eta - 2\sigma\delta.$$

Substituting (29) and (40) for (38) we obtain:

$$\sum_{\nu=0}^{n}\sum_{\chi=0}^{\gamma} H_{\nu,\chi}\omega_n{}^{\chi}\left[T_\chi(z) - jU_\chi(z)\right]\left[a_\nu\left(\widetilde{J}(\omega_n) + j\widetilde{K}(\omega_n)\right) + b_\nu\right] = 0 \tag{41}$$

According to (27) the real and the imaginary part of equation (41) are simultaneously equal to zero, i.e.:

$$\begin{cases} \sum\limits_{\nu\chi} H_{\nu\chi}\omega_n{}^{\chi}\left[a_\nu\widetilde{K}(\omega_n)U_\chi(z) + \left(a_\nu\widetilde{J}(\omega_n) + b_\nu\right)T_\chi(z)\right] = 0 \\ \sum\limits_{\nu\chi} H_{\nu\chi}\omega_n{}^{\chi}\left[a_\nu\widetilde{K}(\omega_n)T_\chi(z) - \left(a_\nu\widetilde{J}(\omega_n) + b_\nu\right)U_\chi(z)\right] = 0 \end{cases} \tag{42}$$

Assuming the dependence (19), equation (42) can be expressed as follows:

$$\begin{cases} \widetilde{A}_1(\omega_n)\alpha + \widetilde{A}_2(\omega_n)\beta + \widetilde{A}_3(\omega_n) = 0 \\ \widetilde{B}_1(\omega_n)\alpha + \widetilde{B}_2(\omega_n)\beta + \widetilde{B}_3(\omega_n) = 0 \end{cases} \tag{43}$$

where:

$$\begin{cases} \widetilde{A}_i(\omega_n) = \sum\limits_{\nu,\chi} H_{\nu,\chi}\omega_n{}^{\chi}\left[a_{\nu i}\widetilde{K}(\omega_n)U_\chi(z) + \left(a_{\nu i}\widetilde{J}(\omega_n) + b_{i\nu}\right)T_\chi(z)\right] = 0 \\ \widetilde{B}_i(\omega_n) = \sum\limits_{\nu,\chi} H_{\nu,\chi}\omega_n{}^{\chi}\left[a_{\nu i}\widetilde{K}(\omega_n)T_\chi(z) - \left(a_{\nu i}\widetilde{J}(\omega_n) + b_{\nu i}\right)U_\chi(z)\right] = 0 \end{cases} \tag{44}$$

$(i = 1,2,3)$.

By resolving the system of equations (43) in respect to α and β, assuming that the main determinant is $\widetilde{\Delta}_0(\omega_n) \neq 0$, we have:

$$\begin{cases} \alpha = \dfrac{\widetilde{\Delta}_1(\omega_n)}{\widetilde{\Delta}_0(\omega_n)} \\ \beta = \dfrac{\widetilde{\Delta}_2(\omega_n)}{\widetilde{\Delta}_0(\omega_n)} \end{cases} \tag{45}$$

where determinants $\widetilde{\Delta}_k$ (k=0,1,2) are defined as follows:

$$\begin{cases} \widetilde{\Delta}_0(\omega_n) = \widetilde{A}_1(\omega_n)\widetilde{B}_2(\omega_n) - \widetilde{B}_1(\omega_n)\widetilde{A}_2(\omega_n) \\ \widetilde{\Delta}_1(\omega_n) = -\widetilde{A}_3(\omega_n)\widetilde{B}_2(\omega_n) + \widetilde{B}_3(\omega_n)\widetilde{A}_2(\omega_n) \\ \widetilde{\Delta}_2(\omega_n) = -\widetilde{A}_1(\omega_n)\widetilde{B}_3(\omega_n) + \widetilde{B}_1(\omega_n)\widetilde{A}_3(\omega_n) \end{cases} \tag{46}$$

Dependencies (35) and (45) represent parametric equations of the curve Γ (the limit of the division region D) on the plane of parameters α and β. This curve is an image of the imaginary axis on the plane of the complex variable „w".

We will show that functions (35) are even. Indeed, from equations (30) and assuming that (24), we will obtain:

$$J(-\omega_n) = J(\omega_n), \qquad K(-\omega_n) = -K(\omega_n). \tag{47}$$

Considering the following properties of the Tchebyshev polynomials:

$$T_\chi(-z) = (-1)^\chi T_\chi(z), \quad U_\chi(-z) = (-1)^{\chi-1} U_\chi(z),$$

from the dependence (34) we obtain:

$$\begin{aligned} &A_i(-\omega_n) = A_i(\omega_n), \\ &B_i(-\omega_n) = B_i(\omega_n), \\ &(i = 1,2,3\ldots), \end{aligned}$$

and from the relation(36) we have:

$$\begin{aligned} &\Delta_k(-\omega_n) = -\Delta_k(\omega_n) \\ &(k = 0,\ 1,\ 2,\ \ldots) \end{aligned}$$

so finally, according to (45) we obtain:

$$\begin{aligned} \alpha(-\omega_n) &= \alpha(\omega_n) \\ \beta(-\omega_n) &= \beta(\omega_n) \end{aligned} \tag{48}$$

By carrying out the same calculations as above it can be shown that functions (45) are also even. The curve Γ is being circulated twice then, once when $\omega_n \in (-\infty, 0)$ and in the opposite direction when $\omega_n \in \rangle 0, \infty)$. The area of stability on the α-β plane is limited by the curve Γ and certain singular lines (when $\Delta_0(\omega_n) = 0$ and $\widetilde{\Delta}_0(\omega_n) = 0$). The positioning of the stability region in relation to the border can be determined by, for example, Neymark criterion [8].

3. Conclusions

The method of the analysis of the stability of two-channel control systems containing the optical fibre links as a through-coupling, applied in this work consists of the choice of such a function (22), which would map the left half plane of the plane of the complex variable „w" into a given region on the plane of the complex variable „s". As a result, the question of testing the roots positioning in the given region on the plane of the complex variable „s" was reduced to a well-known question of the testing of the roots positioning in the left half plane of the complex variable „w". Following that, the left half plane of the complex variable „w" was mapped by (35) and (45) into stability region on the plane of parameters α and β.

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УДК 621.372

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ДОСЛІДЖЕННЯ ФОРМИ ПОЛІМЕРНИХ ПЛОСКООПУКЛИХ МІКРОЛІНЗ

Подано наочний метод оцінки форми мікролінз, виготовлених полімеризацією лежачої краплини фоточутливої композиції. Розраховано відхилення виготовлених зразків від заданої кривизни.

In this paper the method for estimation of shape of microlenses formed by polymerization of sessile drop of photosensitive composition is considered. Curvature deviation of formed samples from specific curvature is calculated.

Скляні і полімерні мікролінзи широко застосовують як фокусуючі пристрої для підвищення ефективності вводу випромінювання між окремими елементами волоконно-оптичних систем [1].