Modeling of Fermentation Processes under Limited by Amplitude Errors of a Technological Process

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Abstract - **Current study is devoted to identification of organic waste treatment process model in case of limited by amplitude measerement errors using interval methods.**

Keywords - **interval analysis, identification, organic waste fermentation, biogas, Monod model.**

I. INTRODUCTION

Organic wastes may be used as a source of renewable energy – biogas (methane), which is a product of anaerobic microbiological fermentation. Moreover, fermented substrate has less content of solid compounds and could be used as an organic fertilizer in crops growing.

Mathematical modeling of this process is studied in a sequence of works [1]. However, despite universality, application of these models is limited due to the fact that their identification is very difficult problem and could be solved with necessary accuracy only in the laboratory. Therefore these models need to be refined to be able to simulate industrial processes under uncertainty due to methodical errors and limited by amplitude measurement errors.

An effective technique to take uncertain conditions into account is interval data analysis. Using interval analysis we constructed dynamic interval models which are able to find tolerance corridors for trajectories of state variables. Methods identifying these models in an accurate way with low computational complexity are developed in [2]. These methods provide ability to solve optimal control problems under uncertainty of industrial fermentation processes.

II. STATEMENT OF PROBLEM

Let $X(t)$ be mass of microorganisms, $S(t)$ – mass of organic waste, $P_1(t)$ – methane mass, $P_2(t)$ – mass of fermented substrate (organic fertilizer), $P_{1,\max}$ – maximal methane mass, $P_{2,\max}$ – maximal mass of fermented substrate, A_1 - $A_8 \geq 0$ – unknown coefficients.

According to [1] the state of an isothermal waste treatment process would be described by a Cauchy problem for Monod system of ordinary differential equations in discrete form:

$$\begin{cases} X_{k+1} = X_k + h\left(A_1 \dfrac{S_k}{A_2 + S_k} - A_3\right) X_k, \\ S_{k+1} = S_k - hA_4 \dfrac{S_k X_k}{A_2 + S_k}, \\ P_{1,k+1} = P_{1,k} + hA_5 \dfrac{S_k X_k}{A_2 + S_k}\left(1 - \dfrac{P_{1,k}}{P_{1,\max}}\right), \\ P_{2,k+1} = P_{2,k} + hA_6 \dfrac{S_k X_k}{A_2 + S_k}\left(1 - \dfrac{P_{2,k}}{P_{2,\max}}\right). \end{cases} \quad (1)$$

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With initial state $X(0) = X_0 > 0$, $S(0) = S_0 > 0$, $P_1(0) = P_2(0) = 0$. Waste mass trajectory $S(t)$ depends on many inputs, which in an autonomous model are taken in constants A_1 - $A_8 \geq 0$ into account.

Let's denote state variables as $\hat{x}_{1,k} = X_k$, $\hat{x}_{2,k} = S_k$, $\hat{x}_{3,k} = P_{1,k}$, $\hat{x}_{4,k} = P_{2,k}$. Observations under process will be described using following equations

$$\vec{y}_{k+1} = C \cdot \vec{x}_{k+1} + \vec{e}_{k+1}, \quad k = 0,...,N-1. \quad (2)$$

where $\vec{x}_{k+1} = (x_{1,k+1}; x_{2,k+1}; x_{3,k+1}; x_{4,k+1})$ – a vector of system state variables (masses of microorganisms, waste, methane and fermented substrate) at $k+1$-th discrete time moment; $\vec{y}_{k+1} = (y_{1,k+1}; y_{2,k+1}; y_{3,k+1}; y_{4,k+1})$ – a vector of measured „outputs" of a system; C – a square (4×4) matrix, which sets up an observation channel; $\vec{e}_{k+1} = \left(e_{1,k+1}; e_{2,k+1}; e_{3,k+1}; e_{4,k+1}\right)^T$ – a vector of random errors, limited by amplitude in which

$$\left|e_{1,k+1}\right| = ... = \left|e_{4,k+1}\right| \leq \Delta_{k+1}, \quad \Delta_{k+1} > 0 \quad \forall\ k = 0,...,N-1, \quad (3)$$

where Δ_{k+1} – an error amplitude at $k+1$-th discrete time moment.

Taking the observation channel equations (2) and limits (3) of errors amplitude $\vec{e}_{k+1}$ into account, we obtain interval relationships:

$$\vec{y}_{k+1} - \Delta_{k+1} \cdot \vec{I} \leq C \cdot \vec{x}_{k+1} \leq \vec{y}_{k+1} + \Delta_{k+1} \cdot \vec{I}, \quad k = 0,...N-1, \quad (4)$$

where $\vec{I}$ is a unity vector.

Let's denote that, not all of the state variables are directly observable in this process. However they would be indirectly estimated using measurements of other physical meters. So we will consider structure of the observation channel as a square, nonsingular matrix *C*, i.e. number of output variables equals to number of state parameters, *m*=*n*.

Let $C^{-1} = \left\{c^*_{ij},\ i = 1,...,m;\ j = 1,...,m\right\}$ be an inverse matrix to C matrix. In this case *C* matrix is nonsingular. Therefore a transformation for estimation of state parameters at *k*-th time moment could be obtained from the system of interval equations (4) as

$$\begin{cases} \min\limits_{\Delta_{k+1} \in \{-\Delta_{k+1}; +\Delta_{k+1}\}} \left\{c^*_{i1} \cdot (y_{1k+1} - \Delta_{k+1}) + ... + c^*_{im} \cdot (y_{mk+1} - \Delta_{k+1})\right\} \leq x_{ik+1} \leq \\ \leq \max\limits_{\Delta_{k+1} \in \{-\Delta_{k+1}; +\Delta_{k+1}\}} \left\{c^*_{i1} \cdot (y_{1k+1} + \Delta_{k+1}) + ... + c^*_{im} \cdot (y_{mk+1} + \Delta_{k+1})\right\}, \end{cases}$$

$$i = 1,\ldots,m, \ k = 0,...,N-1. \quad (5)$$

Let's denote:

$$y^-_{ik+1} = y_{ik+1} - \Delta_{k+1}; \quad y^+_{ik+1} = y_{ik+1} + \Delta_{k+1};$$

$$z^-_{ik+1} = \min\limits_{\Delta_{k+1} \in \{-\Delta_{k+1}; +\Delta_{k+1}\}} \left\{c^*_{i1} \cdot (y_{1k+1} - \Delta_{k+1}) + ... + c^*_{im} \cdot (y_{mk+1} - \Delta_{k+1})\right\}; \quad (6)$$

$$z^{+}_{ik+1}=\max_{\Delta_{k+1}\in\{-\Delta_{k+1};+\Delta_{k+1}\}}\left\{c^{*}_{i1}\cdot(y_{1k+1}+\Delta_{k+1})+...+c^{*}_{im}\cdot(y_{mk+1}+\Delta_{k+1})\right\}. \quad (7)$$

Conditions for tolerance corridors of state variable trajectories would be obtained as:

$$\begin{cases}[\hat{x}_{1,k+1}]=[\hat{x}^{-}_{1,k+1};\hat{x}^{+}_{1,k+1}]\subseteq[z_{1,k+1}]=[z^{-}_{1,k+1};z^{+}_{1,k+1}],\\ [\hat{x}_{2,k+1}]=[\hat{x}^{-}_{2,k+1};\hat{x}^{+}_{2,k+1}]\subseteq[z_{2,k+1}]=[z^{-}_{2,k+1};z^{+}_{2,k+1}],\\ [\hat{x}_{3,k+1}]=[\hat{x}^{-}_{3,k+1};\hat{x}^{+}_{3,k+1}]\subseteq[z_{3,k+1}]=[z^{-}_{3,k+1};z^{+}_{3,k+1}],\\ [\hat{x}_{4,k+1}]=[\hat{x}^{-}_{4,k+1};\hat{x}^{+}_{4,k+1}]\subseteq[z_{4,k+1}]=[z^{-}_{4,k+1};z^{+}_{4,k+1}],\end{cases}$$

where $k=0,...N-1$ – time discretes;

$$\begin{cases}[\hat{x}^{-}_{1,k+1};\hat{x}^{+}_{1,k+1}]=\left(1+h\left(A_1\dfrac{[\hat{x}^{-}_{2,k};\hat{x}^{+}_{2,k}]}{A_2+[\hat{x}^{-}_{2,k};\hat{x}^{+}_{2,k}]}-A_3\right)\right)[\hat{x}^{-}_{1,k};\hat{x}^{+}_{1,k}],\\ [\hat{x}^{-}_{2,k+1};\hat{x}^{+}_{2,k+1}]=[\hat{x}^{-}_{2,k};\hat{x}^{+}_{2,k}]-hA_4\dfrac{[\hat{x}^{-}_{1,k};\hat{x}^{+}_{1,k}][\hat{x}^{-}_{2,k};\hat{x}^{+}_{2,k}]}{A_2+[\hat{x}^{-}_{2,k};\hat{x}^{+}_{2,k}]},\\ [\hat{x}^{-}_{3,k+1};\hat{x}^{+}_{3,k+1}]=[\hat{x}^{-}_{3,k};\hat{x}^{+}_{3,k}]+hA_5\dfrac{[\hat{x}^{-}_{1,k};\hat{x}^{+}_{1,k}][\hat{x}^{-}_{2,k};\hat{x}^{+}_{2,k}]}{A_2+[\hat{x}^{-}_{2,k};\hat{x}^{+}_{2,k}]}\left(1-\dfrac{[\hat{x}^{-}_{3,k};\hat{x}^{+}_{3,k}]}{x_{3,\max}}\right),\\ [\hat{x}^{-}_{4,k+1};\hat{x}^{+}_{4,k+1}]=[\hat{x}^{-}_{4,k};\hat{x}^{+}_{4,k}]+hA_6\dfrac{[\hat{x}^{-}_{1,k};\hat{x}^{+}_{1,k}][\hat{x}^{-}_{2,k};\hat{x}^{+}_{2,k}]}{A_2+[\hat{x}^{-}_{2,k};\hat{x}^{+}_{2,k}]}\left(1-\dfrac{[\hat{x}^{-}_{4,k};\hat{x}^{+}_{4,k}]}{x_{4,\max}}\right).\end{cases}$$

Then system (5) will be the following

$$\begin{cases}z^{-}_{1,k+1}\le\left(1+h\left(A_1\dfrac{[\hat{x}^{-}_{2,k};\hat{x}^{+}_{2,k}]}{A_2+[\hat{x}^{-}_{2,k};\hat{x}^{+}_{2,k}]}-A_3\right)\right)[\hat{x}^{-}_{1,k};\hat{x}^{+}_{1,k}]\le z^{+}_{1,k+1},\\ z^{-}_{2,k+1}\le[\hat{x}^{-}_{2,k};\hat{x}^{+}_{2,k}]-hA_4\dfrac{[\hat{x}^{-}_{1,k};\hat{x}^{+}_{1,k}][\hat{x}^{-}_{2,k};\hat{x}^{+}_{2,k}]}{A_2+[\hat{x}^{-}_{2,k};\hat{x}^{+}_{2,k}]}\le z^{+}_{2,k+1},\\ z^{-}_{3,k+1}\le[\hat{x}^{-}_{3,k};\hat{x}^{+}_{3,k}]+hA_5\dfrac{[\hat{x}^{-}_{1,k};\hat{x}^{+}_{1,k}][\hat{x}^{-}_{2,k};\hat{x}^{+}_{2,k}]}{A_2+[\hat{x}^{-}_{2,k};\hat{x}^{+}_{2,k}]}\left(1-\dfrac{[\hat{x}^{-}_{3,k};\hat{x}^{+}_{3,k}]}{x_{3,\max}}\right)\le z^{+}_{3,k+1},\\ z^{-}_{4,k+1}\le[\hat{x}^{-}_{4,k};\hat{x}^{+}_{4,k}]+hA_6\dfrac{[\hat{x}^{-}_{1,k};\hat{x}^{+}_{1,k}][\hat{x}^{-}_{2,k};\hat{x}^{+}_{2,k}]}{A_2+[\hat{x}^{-}_{2,k};\hat{x}^{+}_{2,k}]}\left(1-\dfrac{[\hat{x}^{-}_{4,k};\hat{x}^{+}_{4,k}]}{x_{4,\max}}\right)\le z^{+}_{4,k+1},\end{cases}$$

$$k=0,...N-1, \quad (9)$$

The resulting system is an interval system of nonlinear algebraic equations (ISNAE). Solution of this system is a vector of model parameters $\vec{A}=(A_1\quad A_2\quad A_3\quad A_4\quad A_5\quad A_6)$ for organic waste treatment.

An algorithm solving this system, peculiarities of its application are shown and its convergence is proven in the full-text paper.

III. CONCLUSIONS

A problem of modeling organic waste treatment processes in case of limited by amplitude errors in measurements of state parameters in industrial process is studied and obtained following new scientific results:

1. For the first time, during identification of Monod systems describing organic waste fermentations, criterion of a given corridor prediction based on these models is used, thereby avoiding complex optimization problems of parametric identification from multiextremum functions using algorithms with low convergence;

2. To explain this problem we proposed to use a well-known interval method solving ISNAE, which provides high convergence and insignificant computational complexity.

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