Method of Universal Algebra Construction for Image Processing

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Abstract – **New universal algebra and method for its construction is described. Operations of addition and multiplication by real scalar are defined for interval (-M, M), where M>0. New algebra generalizes known algebraic structures of Jourlin-Pinoli and Pătraşcu. The using of triangular norms and the laws of human visual system (HVS) is the base of proposed approach.**

Keywords – **logarithmic image processing, abstract linear algebra, vector space, image enhancement, HVS.**

I. INTRODUCTION

One of the important tasks in image processing is their quality improvement. Two paths can be taken for its solution: the first one is the contrast enhancement in the spatial domain, the second one is the enhancement in the frequency domain. In both cases traditional operations of addition, multiplication and subtraction are used for realizing these different algorithms. Another approach consist in using of logarithmic image processing model founded by Jourlin and Pinoly in 1985 [1] and developed by Pătraşcu in 2001 [2] (similar approach for image quality evaluation through its contrast was proposed by Nesteruk in 1980 [3], developed by Vorobel and his scientific school in 1992-2009 [4]). All these methods are based on different algebras. We propose new general approach for universal algebra construction, which is also based on the human visual perception laws. As the base of theoretical foundation for construction of algebra we will prove theorem, which will be presented in the next section.

II. BASIC THEOREM

Theorem. Let $(\mathbf{R};+,\cdot)$ be the field of the real numbers, whose elements we shall call scalars, $\mathbf{E}=(-\mathbf{M},\mathbf{M})$, where $\mathbf{M}>0$, be a set of real numbers and $\mathbf{G}=(\mathbf{E};\langle+\rangle)$ be an Abelian additive group with addition operation $\forall\,\mathbf{u},\mathbf{v}\in\mathbf{E}$

$$\mathbf{u}\langle+\rangle\mathbf{v}\overset{\text{def}}{=}\begin{cases}\mathbf{sign}\,(\mathbf{u}+\mathbf{v})\cdot\mathbf{g}^{-1}(\mathbf{g}\,(|\,\mathbf{u}\,|\,/\,\mathbf{M})+\\ +\,\mathbf{g}\,(|\,\mathbf{v}\,|\,/\,\mathbf{M})),\ \text{if}\ \ \mathbf{sign}\,(\mathbf{u})=\mathbf{sign}\,(\mathbf{v}),\\ \mathbf{sign}\,(\mathbf{u}+\mathbf{v})\cdot\mathbf{q}^{-1}(|\,\mathbf{q}\,(|\,\mathbf{u}\,|\,/\,\mathbf{M})-\\ -\,\mathbf{q}\,(|\,\mathbf{v}\,|\,/\,\mathbf{M})\,|),\ \text{if}\ \ \mathbf{sign}\,(\mathbf{u})=-\mathbf{sign}\,(\mathbf{v}),\end{cases}\qquad(1)$$

where

$$\mathbf{sign}(\mathbf{x})=\begin{cases}-1, & \text{if}\ \mathbf{x}<0,\\ 0, & \text{if}\ \mathbf{x}=0,\\ 1, & \text{if}\ \mathbf{x}>0,\end{cases}$$

$\mathbf{g}$, $\mathbf{q}$ are bijective functions and inner additive generators of the strict triangular **t**-conorm (**s**-norm) **S** and **t**-norm **T** suitably. Then $\forall\,\alpha\in\mathbf{R}$ and $\forall\,\mathbf{u}\in\mathbf{E}$ a mapping operation $\mathbf{R}\langle\times\rangle\mathbf{E}\to\mathbf{E}:(\alpha,\mathbf{u})\to\alpha\langle\times\rangle\mathbf{u}$, which is describe as

$$\alpha\langle\times\rangle\mathbf{u}\overset{\text{def}}{=}\mathbf{sign}\,(\alpha\cdot\mathbf{u})\cdot\mathbf{g}^{-1}(\alpha\cdot\mathbf{g}(|\,\mathbf{u}\,|\,/\,\mathbf{M}))\,,\qquad(2)$$

forms a scalar by vector multiplication, set $\mathbf{E}$ becomes vector real space over the field $\mathbf{R}$ with isomorphism $\varphi(\mathbf{u})=\mathbf{sign}(\mathbf{u})\cdot\mathbf{M}\cdot\mathbf{g}(|\,\mathbf{u}\,|\,/\,\mathbf{M})$. Such assigned operations $\langle+\rangle$ (1) and $\langle\times\rangle$ (2) with the set $\mathbf{E}$ create universal algebra $(\mathbf{E};\langle+\rangle,\langle\times\rangle)$.

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III. SOME RESULTS

Using this theorem we construct various algebras, based on the additive generator functions of logarithmic type. As an example of usage the generator function $\mathbf{g}$, that generalizes properties of human perception of light, a consequence of the described above theorem are those operations of addition and multiplication by a scalar:

$$\mathbf{u}\langle+\rangle\,\mathbf{v}=\mathbf{sign}\,(\mathbf{u}+\mathbf{v})\cdot\frac{|\mathbf{u}+\mathbf{v}|+(1-\mathbf{k})\cdot(\mathbf{p}-2)\cdot\mathbf{u}\cdot\mathbf{v}\,/\,\mathbf{M}}{1+(\mathbf{p}-1)\cdot\mathbf{u}\cdot\mathbf{v}\,/\,\mathbf{M}^2+\mathbf{k}\cdot(\mathbf{p}-2)\cdot\mathbf{min}(|\mathbf{u}|,|\mathbf{v}|)\,/\,\mathbf{M}},$$

$$\alpha\langle\times\rangle\mathbf{u}=\mathbf{sign}\,(\alpha\cdot\mathbf{u})\cdot\mathbf{M}\cdot\frac{(\mathbf{M}+(\mathbf{p}-1)\cdot|\mathbf{u}|)^{|\alpha|}-(\mathbf{M}-|\mathbf{u}|)^{|\alpha|}}{(\mathbf{M}+(\mathbf{p}-1)\cdot|\mathbf{u}|)^{|\alpha|}+(\mathbf{p}-1)\cdot(\mathbf{M}-|\mathbf{u}|)^{|\alpha|}},$$

for $\mathbf{p}>0$ with $\mathbf{k}=0$ for $\mathbf{sign}(\mathbf{u})=\mathbf{sign}(\mathbf{v})$ and $\mathbf{k}=1$ for $\mathbf{sign}(\mathbf{u})=-\mathbf{sign}(\mathbf{v})$.

These operations allow to realize the traditional image processing algorithms dependent on the laws of human perception of light.

IV. CONCLUSION

The presented theorem is the base of constructing algebras, for which mathematical operations are dependent on the properties of the simulated processes and objects.

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