Hotelling's Statistic Recurent Formation

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Abstract – **The synthesis of adaptive filter recurrent forming Hotelling's statistics is conducted.**

Keywords – **adaptive filter, recurrent formation, Hotelling's statistics, interference conditions.**

INTRODUCTION

One of the main requirement to radiolocation system is providing stability of quality characteristics of signal detection in priori unknown interference conditions.

Herewith the main parameter which stability it is necessary to provide during solving detection problem is probability of false alarm. A possible approach to solving problem of stabilization probability of false alarm is using the decision statistics which are invariant to involving interferences. It is known [1], that in Gaussian interference conditions Hotelling's statistics has this property of invariance.

One of the ways of building signal processing systems in the conditions of unknown interferences supposes using statistics, which are invariant to unknown parameters of interferences. In a number of sources for solving a problem of finding unknown determined signal on the Gaussian interference background with zero average and unknown covariance matrix it is suggested to use Hotelling's statistic

$$Z_n = X_n^* \hat{B}_n^{-1} X_n \qquad (1)$$

where X_n – n - dimensional vector of complex envelope's samples of input signal; $\hat{B}_n^{-1}$ – the maximum credibility estimation of matrix inverse to covariance interference's matrix B_n ,which is uniform, most powerful and invariant to B_n verifying criterion of non zero average hypothesis of multidimensional normal combination.

In this article is solving the problem of design the recurrent formation algorithm of Hotelling's statistic (1) in conditions of stationary interferences and processing the time samples with identical interelement time interval T_n. For solving this synthesis problem we use adaptive Bayesian approach [3] which involves the synthesis of adaptive algorithm with known parameters of interferences. Then we replace unknown characteristic with their consistent estimates, particularly maximum likelihood estimate.

Considering matrix B_n hermiticity and positive definite, using decomposition of B_n^{-1} in triangle multipliers with unit diagonal $B_n^{-1} = W_n^{*T} D_n W_n$ [4], statistics (1) can be represented in next way:

$$Z_n = \left(W_n X_n\right)^{*T} D_n \left(W_n X_n\right), \qquad (2)$$

where

$$W_n = \begin{bmatrix} 1 & 0 & \dots & 0 \\ w_{21} & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ w_{n1} & w_{n2} & \dots & 1 \end{bmatrix} = \begin{bmatrix} P_1^{*T} & 0 & \dots & 0 \\ & P_2^{*T} & \dots & 0 \\ & \vdots & & \\ & P_n^{*T} & & \end{bmatrix}$$

– lower triangular matrix of decorrelative processing[5];

$$P_i^{*T} = \left[w_{i1}\ \ w_{i2}\ \ \dots\ \ w_{ii-1}\ \ 1\right];\ D_n = diag\left\{d_{ii}\right\};$$

$d_{ii} > 0$, $i = 1 \dots n$;

$*$, T – sign of complex conjugation and transposition respectively.

Providing W_n and B_n in block form and using Treatment algorithm for block matrices we can show [6], that $P_i = \left(B_i^{-1} e_i\right) / \left(B_i^{-1}\right)_{ii}$ is normalized to $\left(B_i^{-1}\right)_{ii}$ last column B_i^{-1} and $d_{ii} = \left(B_i^{-1}\right)_{ii}$; where $e_i^T = [0\ 0\ \dots\ 0\ 1]$.

From the (2) and last relations that

$$Z_n = \sum_{i=1}^{n} \left(B_i^{-1}\right)_{ii} \left|S_i\right|^2; \qquad (3)$$

where $S_i = P_i^{*T} X_i$.

To simplify forming S_i we use the structure Toeplitz property of covariance matrix B_n from the stationary time samples with identical interelement time intervals T_n.

It is known [4] that in this case vectors P_i satisfy:

$$P_i = \begin{bmatrix} 0 \\ - \\ P_{i-1} \end{bmatrix} - \mu_i^* \begin{bmatrix} \tilde{P}_{i-1}^* \\ - \\ 0 \end{bmatrix}; \qquad (4)$$

where $\mu_i^* = -\left(B_i^{-1}\right)_{1i} / \left(B_i^{-1}\right)_{ii}$, $\sim$ – sign labeling the elements of a vector reversed.

Basing on (4) we can show that there are next ration:

$$\begin{aligned} Z_i(t) &= Z_{i,2}(t) - \mu_i Z_{i,1}(t) \\ Z_{i,2}(t) &= Z_{i-1}(t) = Z_{i-1,2}(t) - \mu_{i-1} Z_{i-1,1}(t) \\ Z_{i,1}(t) &= Z_{i-1,1}(t - T_n) - \mu_{i-1}^* Z_{i-1,2}(t - T_n) \end{aligned} \qquad (5)$$

Ratio analysis (3), (5) show, that in a case of Toeplitz structure of covariance matrix B_n, statistic's (1) formation is provided by means of recurrent processing input picks. It can be shown [6] that coefficients μ_i and $\left(B_i^{-1}\right)_{ii}$ which are necessary to organize the recurrent processing, satisfy next equations

$$\mu_i = r_{Z_{i,1} Z_{i,2}} \qquad (6)$$

$$\left(B_i^{-1}\right)_{ii} = 1 / \sigma_{Z_i}^2 \qquad (7)$$

where $r_{Z_{i,1} Z_{i,2}}$ – correlation coefficient of $Z_{i,1}$ and $Z_{i,2}$ processes; $\sigma_{Z_i}^2$ – process Z_i dispersion.

According to (3), (5)-(7) and the Bayesian approach to overcome the parametric a priori uncertainty is composed structure scheme of adaptive filter, which realizes recurrent formation of statistics (1) in a background of interferences with unknown covariance properties. (Fig. 1, where 1 – delay element T_n, 2 – measuring the correlation coefficient; 3 – complex conjugation block; 4 – multiplier; 5 –subtraction block; 6 – measuring dispersion; 7 – divider; 8 – accumulator. The dashed line circled part of the scheme, the connection which increases the filter order by one).

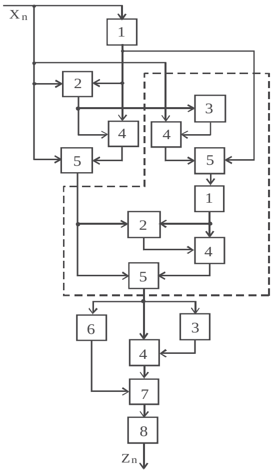

Fig. 1. Functional scheme of filter forming Hotelling's statistics

A characteristic feature of the synthetic algo-rhythm is its convergence with an arbitrary amount of training samples used for estimation of unknown coefficients of correlation.

This is due to the positive definiteness constructed on the basis of filter weights equivalent to a sample matrix $\hat{B}_n^{-1} = \hat{W}^{*T}{}_n \hat{D}_n \hat{W}_n$, an arbitrary $m \geq 1$.

In this case the formation of statistics (1) using treatment maximum likelihood estimator of the unknown covariance matrix possible only when $m \geq n$, as in the opposite case, the evaluation is confluent.

CONCLUSION

Thus, the use of a priori information about the structure of sample covariance matrices of stationary nterferences with the same interelementi time intervals can not only simplify the implementation, but also to improve the qualitative characteristics of detector uses a recurrent formation of solving statistics.

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