

Calculations Of Transfer Functions Of The Defense Control Systems

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Abstract - the convenient method for programming of calculation of transfer functions of the defense control systems has been proposed.

I PART I

The calculation of transfer functions of the defense control systems (DCS) which are the discrete systems of high order, requires much labour intensiveness, that's why it is naturally necessary to make the process of calculation automatic. The known algorithms of solving this problem are directed [1, 2] to working out of individual programs and mathematical abilities of modern ECMs and are used only in a measure.

II. THE BASIC PART

Let's examine open loop DCS, consisting of consecutive connected ideal impulse element, predictor, zero transistor [3] and continuous parts with transfer function by means of proper irreducible fraction:

$$\begin{aligned} W(s) &= B(s) / A(s) = \\ &= \left(\sum_{k=1}^q B_k s^{q-k} \right) / \left(s^q + \sum_{k=1}^q A_k s^{q-k} \right) = \\ &= B(s) / \left[(s - s_1)^{r_1-1} \prod_{k=2}^l (s - s_k)^{r_k} \right] \end{aligned} \quad (1)$$

where S - a complex variable; A_k, B_k - material factors

$$\left(\sum_{k=1}^q B_k^2 \neq 0 \right); q, l, r_k - \text{natural numbers}$$

$(l \leq q, \sum_{k=1}^l r_k = q + 1); S_1=0, S_2, \dots, S_l$ - not equal strips of fraction (1).

Transfer function of the considered DCS

$$W(z, \sigma) = \frac{z-1}{z} \sum_{n=0}^{\infty} h(n, \sigma) z^{-n}, \quad (2)$$

when

$$h(n, \sigma) = \left| L^{-1} \{ W(s) / s \} \right|_t = (n + \sigma) T_0 \quad (3)$$

- the function [3] corresponding transitive function of LF is solved; L^{-1} - a symbol of inverse Laplas transformation; z - a complex variable; T_0 - the period of discreteness; σ - parameter of displacement.

Let's enter numbers

$$z_k = \exp S_k T_0 \quad (k=1, 2, \dots, l). \quad (4)$$

Also we shall be limited by the most frequent practice case, when $z_1 z_2 \neq \dots \neq z_l$. Then

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$$\sum_{n=0}^{\infty} h(n, \sigma) z^{-n} = z b(z, \sigma) / [(z-1)c(z)]. \quad (5)$$

Here

$$b(z, \sigma) = \sum_{k=0}^q b_k(\sigma) z^{q-k}; \quad (6)$$

$$c(z) = (z-1)^{r_1-1} \prod_{k=2}^l (z - z_k)^{r_k} = z^q + \sum_{k=1}^q c_k z^{q-k}; \quad (7)$$

$b_k(\sigma), c_k$ - real coefficients and polynoms (6) and (7) are mutually simple [4]. From parities (2) and (5) follows, that

$$W(z, \sigma) = b(z, \sigma) / c(z) \quad (8)$$

Apparently, at the accepted assumptions the problem of calculation of transfer function (2) is reduced to the problem of definition of two finite-dimensional vectors:

$$b(\sigma) = \left\| b_0(\sigma) b_1(\sigma) \dots b_q(\sigma) \right\|', \quad (9)$$

$$c = \left\| c_1 c_2 \dots c_q \right\|' \quad (10)$$

(a stroke - a sign of transposition), made of the multinomial coefficients (6) and (7).

Expression (7) shows, that at known poles of fraction (1) calculation the component of a vector (10) is reduced to easier algorithmized problem of finding of factors of a multinomial in its norms (4) [5].

The greatest difficulties are caused usually with calculation of factors of numerator (6) of fractions (8). Let

$$\nabla_{c_r} = c_k - c_{k-1} \quad (11)$$

($k=1, 2, \dots, q+1; c_0=1, c_{q+1}=0$). Then from the formula (5) follows, that

$$h(n, \sigma) + \sum_{k=1}^{q+1} \nabla_{c_k} h(n-k, \sigma) = \sum_{k=0}^q b_k(\sigma) \delta_{kn} \quad (12)$$

($n=0, 1, \dots$)

Where $\delta_{kn}=1$ at $n=k$ and $\delta_{kn}=0$ at $n \neq k$. From here

$$b(\sigma) = C h_1(\sigma) \quad (13)$$

Where

$$h_1(\sigma) = \left\| h(0, \sigma) h(1, \sigma) \dots h(q, \sigma) \right\|' \quad (14)$$

- a vector from values of trellised function (3) at $n \in [0, q]$, and

$$c = \left\| \nabla_{c_{i-k}} \right\|_0^q$$

- a triangular matrix (q+1)-th order [6] from among (11) (i - number of a line, k - number of a column $\nabla_{c_0} = 1, \nabla_{c_{i-k}} = 0$ at $k > i$).

Expression (13) is easy for realizing on the computer by means of the standard program of multiplication of a matrix on a vector. It reduces a problem of calculation of a vector (9) and more simple and well studied to a problem of definition of a vector (14). Let

$$W(s)/s = \sum_{k=1}^l \sum_{i=1}^{r_k} N_{ki} / (s - s_k)^i,$$

where N_{ki} - factors of decomposition [7].

After simple transformations we shall receive, that

$$h_1(\sigma) = M(\sigma)N. \quad (15)$$

Here

$$N = \left\| N_{11} N_{12} \dots N_{1r_1} N_{21} \dots N_{2r_2} \dots N_{l_1} N_{l_2} \dots N_{lr_l} \right\|, \quad (16)$$

$$M(\sigma) = \left\| M_1(\sigma) M_2(\sigma) \dots M_l(\sigma) \right\| \quad (17)$$

-square matrix of the (q+1) -th order, consisting of rectangular cells

$$M_m(\sigma) = \left\| \frac{T_0^k}{k!} (i + \sigma)^k z_m^{i+\sigma} \right\|$$

($i=0, 1, \dots, q; k=0, 1, \dots, r_m-1$) the size (q+1) x r_m ($m=1, 2, \dots, l$). Elements of a matrix (17) are defined on the computer by combining of standard programs of elementary functions calculation.

From parities (13) and (15) follows, that

$$b(\sigma) = CM(\sigma)N \quad (18)$$

The vector functioning here (16) can be calculated on the computer by means of a technique offered in the work [8].

Expression (18) together with the method of calculation of a vector mentioned above (10) form simple and effective algorithm of calculation of transfer functions of DCS which basic stages are realized by means of the standard programs which are a part of a software of all the computer.

III. CONCLUSIONS

The technique of calculation of transfer functions of DCS offered in the present work is easily generalized on systems with delay and multivariate systems

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