The Irreducible Represent for Analysis of Telecommunication Networks

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Abstract – **Using of irreducible repress is proposed for the analysis of telecommunication networks. The formulas decompositions of second, third and fourth ranks tensors are given on parts of irreducible.**

Keywords - **the tensor, the irreducible represent**, **K-track routing.**

I. INTRODUCTION

The methodology that successfully combines the notion of the graph and queuing system based on the tensor analysis. This methodology of the calculation of the telecommunication network allow in combining processes and structures in which these processes occur. This network structure determines the coordinate system, and any change in the topology converts the coordinate system [1-3]. At the moment tensor analysis of the networks confined to uniform traffic type. The functioning of modern telecommunication networks that integrate different types of the traffic to increase their functionality require the qualitatively different approach.

The aim of the paper is decision the problem of analysis of the telecommunication networks by the irreducible representation.

II. APPLICATION OF THE IRREDUCIBLE REPRESENT FOR ANALYSIS OF TELECOMMUNICATION NETWORKS

For **K** - is road routing is existence of constraints [4]

$$\sum_{l=1}^{m} \nu_{kl}^{(j)} \leq \mathbf{K},$$

$$\nu_{kl}^{(j)} = \begin{cases} 1, & \text{if } \sum_{i=1}^{m} \chi_{kl}^{(i,j)} > 0; \\ 0, & \text{if } \sum_{i=1}^{m} \chi_{kl}^{(i,j)} = 0. \end{cases}$$

Here $\chi_{kl}^{(i,j)}$ – is the part of the flow γ_{ij}, which passes between junctions *k* and *l*, $0 \leq \chi_{kl}^{(i,j)} \leq 1$; γ_{ij} – is the average traffic in a junction and *i* addressed the junction of *j*.

Littl's formula for average delay of messages in the telecommunication network [4]:

$$T = \frac{1}{\gamma} \lambda_{kl} t_{kl},$$

where t_{kl} – is the average time to live of package in the branch (*kl*); γ – is the complete external traffic; λ_{kl} - is the flow in the branch (*kl*) [4]:

$$\lambda_{kl} = \gamma_{ij} \chi_{kl}^{(i,j)}.$$

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There is always the tensor decomposition of irreducible representation, as well-known from tensor analysis [5]. Examining the analogous symmetrical stress, which breaks down on the spherical (scalar) and the deviator representation, in [6] held an analysis of traffic. In general case, second-rank tensor can be decomposed in symmetric and antisymmetric tensor [5]. Antisymmetric tensor represented as a vector representation, and therefore second-rank tensor is decomposed on the scalar, the deviator (symmetric) and the vector (antisymmetric) components.

The parameters γ_{ij}, λ_{kl} and t_{kl} are second-rank tensors. The parameters γ_{ij} and t_{kl} are non-symmetric, because traffic and time to live from the junction-source in the junction-addressee in comparison with opposite directions are different. It is not important for the flow in which direction the load is transferred from (*kl*) or (*lk*). To present some correlation:

$$e_{ij} = b_{ij} + w_{ij},$$
$$b_{ij} = 1/5\,(e_{ij} + e_{ji}),$$
$$w_{ij} = 1/5(e_{ij} - e_{ji}),$$

Symmetric tensor b_{ij} describes how the parameters studied are independent of the direction of each junction (*ij*):

$$b_{ij} = I^{(b)}\delta_{ij} + D_{ij}^{(b)}, \quad I^{(b)} = 1/3 b_{ii}, \quad D_{ij}^{(b)} = b_{ii} - I^{(b)}. \qquad (1)$$

where δ_{ij} – Kronecker's symbol. From (1) see that when the scalar $I^{(b)}=0$ in the networks absent the cycle between junction. The deviator $D_{ij}^{(b)}$ is similar to the tensor b_{ij}, but without the possible loops in the network.

Antisymmetric tensor w_{ij} and the pseudovektor V_k characterize how the parameters studied differ from the direction of the branch (*ij*) and (*ji*):

$$w_{ij} = \varepsilon_{ijk} V_k,$$

where ε_{ijk} – the pseudotensor Levi-Civita.

The tensor of third rank symmetric on pair of indices, such $\nu_{kl}^{(j)}$, is decomposed

$$d_{ijk} = f_{ijk} + g_{ijk},$$

$$f_{ijk} = \frac{1}{3}(d_{ijk} + d_{jki} + d_{kij}),$$

$$g_{ijk} = \frac{1}{3}(2d_{ijk} - d_{jki} - d_{kij}),$$

where f_{ijk} – symmetrical tensor in all indices characterizes the average amount of the studied parameters between the junctions *i*, *j*, *k*, as with each other and decomposes

$$f_{ijk} = V_{(i}^{(f)}\delta_{jk)} + S_{ijk}^{(f)},$$

where $V_i^{(f)}$ – the vector, $S_{ijk}^{(f)}$ – the septor are:

$$V_i^{(f)} = \frac{3}{5} f_{ikk},$$

$$S_{ijk}^{(f)} = f_{ijk} - V_{(i}^{(f)}\delta_{jk)}.$$

The tensor g_{ijk} of symmetric on second and third indices, is decomposed on the vector $V_i^{(g)}$ and the pseudodeviator $D_{kl}^{(g)}$ parts, converted to zero at symmetrization on all indices, describes how the parameters are similar by mutual change junctions *i* on *j* or *k*:

$$g_{ijk}=(V_i^{(g)}\delta_{jk}-V_{(i}^{(g)}\delta_{jk)})+\varepsilon_{li(j}D_{k)l}^{(g)},$$

$$V_i^{(g)}=\frac{2}{3}g_{ikk}=-\frac{2}{3}g_{jji},$$

$$D_{kl}^{(g)}=\frac{2}{3}g_{ij(k}\varepsilon_{l)ij}.$$

The tensor of fourth rank symmetric one pair of the indices, such $\chi_{kl}^{(i,j)}$, it is possible decomposed:

$$r_{ijkl}=p_{ijkl}+o_{ijkl},$$

where p_{ijkl} – the tensor symmetric in each pair of indices and describes how studied the parameters are independent of the direction between junctions (*ij*) and (*kl*); o_{ijkl} – the tensor antisymmetric on the first pair of indices and describes how studied the parameters differ from the direction of branch (*ij*).

The tensor p_{ijkl} is decomposed [7]:

$$p_{ijkl}=v_{ijkl}+u_{ijkl}+w_{ijkl},$$

$$v_{ijkl}=\frac{1}{6}(p_{ijkl}+p_{ikjl}+p_{iljk}+p_{klij}+p_{jlik}+p_{jkil}),$$

$$u_{ijkl}=\frac{1}{6}(2p_{ijkl}-p_{ikjl}-p_{iljk}+2p_{klij}-p_{jlik}-p_{jkil}),$$

$$w_{ijkl}=\frac{1}{2}(2p_{ijkl}-p_{klij}).$$

Symmetrical in all indices tensor v_{ijkl} characterizes the average amount of the studied parameters between the junctions *i*, *j*, *k*, *l*, as with each other and decomposes:

$$v_{ijkl}=I^{(v)}\delta_{(ij}\delta_{kl)}+D_{(ij}^{(v)}\delta_{kl)}+N_{ijkl}^{(b)},$$

where the invariant $I^{(v)}$, the deviator $D_{ij}^{(v)}$ and the nonor $N_{ij}^{(v)}$ are:

$$I^{(v)}=\frac{1}{5}v_{iikk},$$

$$D_{ij}^{(v)}=\frac{6}{7}v_{ijkk}-\frac{10}{7}I^{(b)}\delta_{ij},$$

$$N_{ijkl}^{(v)}=v_{ijkl}-I^{(v)}\delta_{(ij}\delta_{kl)}-D_{(ij}^{(v)}\delta_{kl)}.$$

Hereinafter, generally accepted symbol indices using restricted in round or square brackets, - symmetrization or anti-symmetrization respectively, with the indices, which are limited in the vertical hyphen, does not action of brackets handles that are outside of the hyphen.

Symmetric tensor of u_{ijkl} on the first and second pair and indexes and permutations of these pairs and converted to zero at symmetric on all indices, describes how the parameters are similar to the direction between junctions and between the same pairs (*ij*), (*kl*) and differ from the averaged values parameters between the junctions *i*, *j*, *k*, *l*, as with each other and decomposes on the invariant $I^{(u)}$ and the deviator $D_{ij}^{(u)}$ parts:

$$u_{ijkl}=I^{(u)}(\delta_{ij}\delta_{kl}-\delta_{(ij}\delta_{kl)})+[\frac{1}{2}(D_{ij}^{(u)}\delta_{kl}+D_{kl}^{(u)}\delta_{ij})],$$

$$I^{(u)}=\frac{1}{4}u_{iikk},$$

$$D_{ij}^{(u)}=3u_{ijkk}-4I^{(u)}\delta_{ij}.$$

Symmetric tensor of w_{ijkl} on three indices and converted to zero as the result is symmetric for to all of indexes, characterized by the parameters differ when changing the pair of junctions (ij), (kl) between itself

$$w_{ijkl}=\varepsilon_{m(i|(k}\delta_{l)|j)}V_m^{(w)}+\frac{1}{2}(D_{ij}^{(w)}\delta_{kl}-D_{kl}^{(w)}\delta_{ij})+\varepsilon_{m(i|(k}S_{l)|j)m}^{(w)}.$$

where the pseudovector $V_m^{(w)}$, the deviator $D_{ij}^{(w)}$ and the pseudoseptor $S_{ljm}^{(w)}$ are:

$$V_m^{(w)}=\varepsilon_{mil}w_{ijjl},$$

$$D_{ij}^{(w)}=\frac{2}{3}w_{ijkk},$$

$$S_{ljm}^{(w)}=w_{(i|jk|l)}\varepsilon_{m)jk}-\frac{1}{2}V_{(i}^{(w)}\delta_{lm)}.$$

The tensor of o_{ijkl} determined by the deviator $D_{il}^{(o)}$:

$$o_{ijkl}=2\delta_{(i|[k}D_{l]j)}^{(w)},$$

$$D_{il}^{(o)}=\frac{2}{3}o_{ijjl}.$$

The irreducible representations allow define the objective function of routing. This is important elements for the analysis of symmetrical parts of tensors can be characterized by normal network and antisymmetric components detect the presence of the deviations from the optimal path.

III. CONCLUSION

The irreducible repress is proposed for the analysis of telecommunication networks. The irreducible representation for the second, third and fourth ranks tensors were written.

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