Estimation of Reliability Indices for Symmetric Ramified Systems

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Abstract - **Main reliability indices for unrestorable symmetric systems ramified to level 3 and with ageing output elements are examined in this paper. Models for the failure probability, the failure frequency and the failure rate are worked out in the case when the lifetime of ageing output elements is circumscribed by the Weibull distribution.**

Keywords – **ramified systems, symmetric systems, reliability indices, ageing elements.**

I. INTRODUCTION

Reliability theory is a subset of quality control; in it the characteristic studied is the length of life of the item. Reliability deals with products in service [1].

On output level ramified systems have sensors, printers, keyboards, disk drivers, which are exposed to aging. Lifetime of such devices is often circumscribed by the Weibull distribution. Elements of upper levels have lifetime circumscribed by the exponential distribution [2].

II. ESTIMATION OF RELIABILITY INDICES

Let us consider a symmetric complicated system, ramified to level 3, with Weibull distributed output elements , where a_1 elements of level 1 are subordinate to the element of level 0, a_2 elements of level 2 are subordinate to every element of level 1, a_3 elements of level 3 are subordinate to every element of level 2.

It was proved that lifetime of elements on upper levels (levels 0, 1 and 2) is circumscribed by the exponential distribution and lifetime of elements on the output level (level 3) is circumscribed by the Weibull or the Rayleigh distribution [4].

Main traditional reliability indices of unrestorable systems in case of ramified systems are the failure probability, the failure frequency and the failure rate in the prescribed availability condition [5].

By analogy with [6] we obtain the following generation function of the system:

$$\mathrm{S}_3(\mathrm{z}) = \mathrm{p}_0(\mathrm{p}_1(\mathrm{p}_2(\mathrm{p}_3\mathrm{z} + \mathrm{q}_3)^{\mathrm{a}_3} + \mathrm{q}_2)^{\mathrm{a}_2} + \mathrm{q}_1)^{\mathrm{a}_1} + \mathrm{q}_0 \qquad (1)$$

Let $\lambda_{3W}(k,t)$ be the failure rate of the system in the prescribed availability condition which is determined as a result of division of the failure frequency by the availability function. We obtain:

$$\lambda_{3W}(k,t) = (\sum_{x_3=k}^{a_1a_2a_3} \sum_{x_1=ceil\left(\frac{ceil\left(\frac{x_3}{a_3}\right)}{a_2}\right)}^{a_1} C_{a_1}^{x_1} \sum_{x_2=ceil\left(\frac{x_3}{a_3}\right)}^{a_2x_1} C_{a_2x_1}^{x_2} C_{a_3x_2}^{x_3} \times$$

$$\times \sum_{j_1=0}^{a_1-x_1} C_{a_1-x_1}^{j_1}(-1)^{j_1} \sum_{j_2=0}^{a_2x_1-x_2} C_{a_2x_1-x_2}^{j_2}(-1)^{j_2} \sum_{j_3=0}^{a_3x_2-x_3} C_{a_3x_2-x_1}^{j_3}(-1)^{j_3} \times$$

$$\times\left(\lambda_3(x_3+j_3)\beta_3 t^{\beta_3-1} + \lambda_0 + \lambda_1(x_1+j_1) + \lambda_2(x_2+j_2)\right)\times$$

$$\times\, e^{-(\lambda_1(x_1+j_1)+\lambda_2(x_2+j_2))} e^{-\lambda_3(x_3+j_3)t^{\beta_3}})/ \qquad (2)$$

$$/(\sum_{x_3=k}^{a_1a_2a_3} \sum_{x_1=ceil\left(\frac{ceil\left(\frac{x_3}{a_3}\right)}{a_2}\right)}^{a_1} C_{a_1}^{x_1} e^{-\lambda_1 x_1 t}\left(1-e^{-\lambda_1 t}\right)^{a_1-x_1} \sum_{x_2=ceil\left(\frac{x_3}{a_3}\right)}^{a_2x_1} C_{a_2x_1}^{x_2} \times$$

$$\times e^{-\lambda_2 x_2 t}\left(1-e^{-\lambda_2 t}\right)^{a_2x_1-x_2} C_{a_3x_2}^{x_3} e^{-\lambda_3 x_3 t^{\beta_3}}\left(1-e^{-\lambda_3 t^{\beta_3}}\right)^{a_3x_2-x_3}).$$

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III. CONCLUSIONS

The paper deals with mathematical models of main indices for unrestorable ramified systems with ageing output elements. The main thrust of this paper is to reduce the computational time and complexity when evaluating reliability indices of complicated symmetric systems.

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