Cyclostationary Processes as a Model of Forced Oscillations of Some Non-linear Systems Types

Ivan Matsko, Igor Kravets, Ihor Yavorskyy

Abstract - **In this paper the new approach for modeling of non-linear systems oscillations is given. The theory of periodically correlated random processes (PCRP) for estimation of probabilistic characteristics of such oscillations is proposed.**

Keywords – **Cyclostationary process, non-linear system, probabilistic characteristics.**

I. INTRODUCTION

In order to investigate probabilistic characteristics of oscillations of complex mechanical and telecommunication systems we should take into account interactions between its elements. In the first approximation it is enough to use second order differential equations for their description [1]. The presence of non-linearity in the system changes parameters of these differential equations and its response. Hence investigation of probabilistic characteristics of system oscillations allows us to estimate its state as a whole.

II. THE MODEL OF SYSTEM OSCILLATIONS.

Mechanical and telecommunication systems oscillations are described by similar differential equations. The main difference between these systems consists in physical interpretation of equations parameters.

In the case of mechanical system non-linearity is caused by elements cracks. During the action of external cyclic load the crack periodically opens and closes (fig.1). The cracked detail behavior is the same that non-cracked ones, if the crack is closed. But in the case of opened crack, the rigidity of detail decreases. Since oscillations of the faulty mechanical system can be described in the next form [1]:

$$\begin{cases} X'' + 2\beta_c X' + \omega_c^2 X = f(t), X \le 0; \\ X'' + 2\beta_s X' + \omega_s^2 X = f(t), X > 0, \end{cases} \qquad (1)$$

where β_c, β_s – damping coefficients and ω_c, ω_s – detail natural frequencies, when crack is opened and closed respectively. Hence using of linear systems of differential equations can leads to inaccurate information about system state.

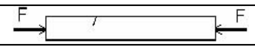	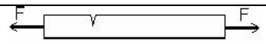
$X \le 0$	$X > 0$

Fig. 1. The behavior of cracked detail under the action of external cyclic load

The detail rigidity, if crack is opened, is defined as $k_s = k_c - \Delta k$, where k_c – non-faulty detail rigidity, and $\Delta k \ge 0$. The natural frequency of non-cracked detail can be calculated using well known formula:

$$\omega_c = \sqrt{\frac{k_c}{m}}, \qquad (2)$$

where m – detail relative mass. The natural frequency of faulty detail in the case of opened crack can be represented through the natural frequency of non-defect one in the next form:

$$\omega_s = \omega_c \, 2\sqrt{1 - \frac{\Delta k}{k_c}} \Bigg/ \left(1 + \sqrt{1 - \frac{\Delta k}{k_c}}\right). \qquad (3)$$

Its easy can be shown that the relative rigidity depends on crack length:

$$\frac{\Delta k}{k_c} \approx \frac{\Delta l}{l}, \qquad (4)$$

where Δl – crack length, l – detail length on the direct of crack growth. Therefore, the probabilistic characteristics system oscillations allow us to estimate the degree of detail faultiness.

If telecommunication or mechanical system has non-linear elements, the theory of stationary processes is not applicable for probabilistic characteristics estimation. And the PCRP estimation theory should be used. The PCRP is a class of random processes, which mean $m(t) = E\xi(t)$ and correlation function $b(t,u) = E\overset{\circ}{\xi}(t)\overset{\circ}{\xi}(t+u)$, $\overset{\circ}{\xi}(t) = \xi(t) - m(t)$, are periodical functions of time [2]:

$$m(t) = m(t+T), \; b(t,u) = b(t+T,u).$$

These probabilistic characteristics can be represented in the Fourier form

$$m(t) = \sum_{k \in \mathbb{Z}} m_k e^{ik\frac{2\pi}{T}t}, \qquad b(t,u) = \sum_{k \in \mathbb{Z}} B_k(u) e^{ik\frac{2\pi}{T}t}.$$

Coefficients m_k i $B_k(u)$ define correlation structure of PCRP and characterize the degree of non-stationarity of the signal.

III. CONCLUSIONS

Hence, the vibration response of cracked detail under the action of external cyclic load can be described within the model in the terms of the periodically correlated random processes. The appearance of crack brings to the non-linearity in the mechanical system.

REFERENCES

[1] *L.* Gelman, S. Gorpinich "Non-linear Vibroacoustical Free Oscillation Method For Crack Detection And Evaluation," in *Mechanical Systems and Signal Processing*. – 14(3) – 2000 – P. 343-351.

[2] Драган Я. П., Рожков В. А., Яворський И. Н. "Методы вероятностного анализа ритмики океанологических процессов," Львов, *Гидрометеоиздат*, 1987. – 319 с.

Ivan Matsko – Karpenko Physico-mechanical institute of NAS of Ukraine, Naukova Str., 5, Lviv, 79601, UKRAINE, E-mail:ivanmatsko@ipm.lviv.ua