Diffraction of Electromagnetic Field in Presence of Small Scatterer Bodies

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Abstract - **A numerical approach to the diffraction problem for the domain with many small particles is considered. The approach is developed under the assumptions $ka << 1$, $d >> a$, where a is the size of the particles and d is the distance between the neighboring particles. The results of numerical simulation show good agreement with the theory. They open a way to numerical simulation of the method for creating materials with a desired refraction coefficient.**

Keywords – **Diffraction Problem, Small Particle, Asymptotic Approach, Numerical Simulation.**

THE STATEMENT AND SOLUTION TO DIFFRACTION PROBLEM

The scattering problem for considered geometry described by the following set of equations

$$[\nabla^2 + k^2 n_0^2(x)]u_M = 0 \text{ in } R^3 \setminus \bigcup_{m=1}^{M} D_m , \quad (1)$$

$$\frac{\partial u_M}{\partial N} = \zeta_m u_M \text{ on } S_m ,\ 1 \le m \le M , \quad (2)$$

where

$$u_M = u_0 + v_M , \quad (3)$$

u_0 is solution to (1), (2) with $M = 0$, (i.e. in the absence of embedded particles) and with the incident field $e^{ik\alpha \cdot x}$, and v_M satisfies the radiation conditions.

It was proved in [1] that unique solution to (1) - (2) is of the form

$$u_M(x) = u_0(x) + \sum_{m=1}^{M} \int_{S_m} G(x,y)\sigma_m(y)dy , \quad (4)$$

where $G(x,y)$ is Green's function of Helmholtz equation in the case when $M = 0$.

Let us define the "effective field" u_e, acting on the m-th particle:

$$u_e(x) := u_e(x,a) := u_e^{(m)}(x) := u_M(x) - \int_{S_m} G(x,y)\sigma_m(y)dy ,$$

$$x \in R^3 , \quad (5)$$

The function $\sigma_m(y)$ solves an exact integral equation, which is solved asymptotically in [2] as $a \to 0$. Let $h(x) \in C(D)$, $\operatorname{Im} h \le 0$, be arbitrary. Let $\Delta_p \subset D$ be any subdomain of D, and $\mathrm{N}(\Delta_p)$ be the number of particles in Δ_p. We assume that

$$\mathrm{N}(\Delta_p) = \frac{1}{a^{2-\kappa}} \int_{\Delta_p} N(x)dx[1 + o(1)],\ a \to 0 , \quad (6)$$

where $N(x) \ge 0$ is a given continuous function in D. There exists the limit $u(x)$ of $u_e(x)$ as $a \to 0$:

$$\lim_{a \to 0} \| u_e(x) - u(x) \|_{C(D)} = 0 , \quad (8)$$

and $u(x)$ solves the following equation:

$$u(x) = u_0(x) - 4\pi \int_D G(x,y)h(y)N(y)u(y)dy . \quad (9)$$

This is the equation, derived in [2] for the limiting effective field in the medium, created by embedding many small particles with the distribution law (6).

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NUMERICAL SIMULATION

The results of numerical calculations demonstrate the applicability of the proposed asymptotical approach to solving the diffraction problem reduced to the respective linear algebraic system (LAS). For the numerical calculations it was taken $k = 1$, $\kappa = 0.9$. The solutions of appropriate LAS were compared with solutions corresponding to equation (4) LAS. It was proven in [3], that the solution of this LAS received by the collocation method tends to the solution of (4) when the number of collocation points tends to infinity [4].

It was established that the absolute error of solution to LAS corresponding to asymptotic formula (5) depends on change the radius a of small particle embedded in the domain D. The example with number of particles equal to 9 was considered; the number of collocation points for solution to corresponding to (4) LAS was equal to 343; the function $N(y) = 5$. The calculations show that for the considered set of input parameters the optimal value a of particle radius was reached for the values from $a = 0.04$ to $a = 0.05$. The optimal value of a depends essentially on the function $N(y)$, as well as on the distance d between particles.

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