TRANSPORT TECHNOLOGIES

Vol. 3, No. 1, 2022

Myroslav Oliskevych[1], **Viktor Danchuk**[2], **Oleksandr Mastykash**[3]
1. Lviv National University of Nature Management
1, V. Velykoho Str., Dubliany, 80381, Ukraine

2. National Transport University
1, Omelianovicha-Pavlenko Ave., Kyiv, 01010, Ukraine

3. Lviv Polytechnic National University
12, S. Bandery Str., Lviv, 79013, Ukraine

https://doi.org/10.23939/tt2022.01.038

CROSS-DOCKING CARGO DELIVERY ROUTING FOR GUARANTEED MINIMUM PERIOD

***Summary**. The article is devoted to the problem of effective use of cross-docking as a technology of cargo delivery with increased time requirements, which allows to resolve the contradictions of guaranteed delivery time ensuring and the efficiency of the existing fleet of trucks. The process of delivery organization is considered as the ordering on the transport network of many discrete freight flows in the form of their phases. If qualitative and / or quantitative changes do not occur from phase to phase with the flow, then the tact of such flow is constant. However, cross-docking flows change the size of the band of moving goods. Cargo can be moved as intended by any group size, which, however, is limited by the maximum and minimum values. A two-stage algorithm for solving the problem has been developed. The transport network is represented as a graph. The content of the route search problem is optimization, as it consists of multiple selections from the initial graph of arcs in the presence of restrictions on input and output flows. One needs to replace every each edge of the graph with an arc of the forward or reverse direction, or remove this edge. The criterion for the optimal solution of the problem, which is applied, is the minimum guaranteed duration of delivery of goods throughout the set of specified freight flows. At the first stage of the algorithm, the search for the shortest paths in the graph is performed, along which every given cargo flow can pass. The first stage of optimization is a linear problem of integer programming, the dimension of which is not too large. The initial data of the second stage is freight flows matrix, which is obtained as a result of optimization in the first stage. The content of the second stage of the algorithm is the solution of the equation of the balance of discrete goods flows. The balance equation means that all flows entering each peak including the sources of cargo flows of this peak have an average intensity equal to the intensity of the outgoing cargo flows from each source peak, including runoff. Due to the studied dependencies between the individual phases of the delivery process on the example of a cargo carrier on the transport network of Ukraine, the formulated restrictions and boundary conditions, the possibility of guaranteed accurate solution of a complex problem is obtained. At the same time, the shortest routes were found, reloading points were identified as well as time parameters of operation and the degree of loading of cars. According to the results of the research, a threefold increase in the productivity of the fleet of road trains with a reduction in the guaranteed delivery time by 30 %.*

***Key words:** cargo delivery, cross-docking, routing, discrete optimization.*

1. INTRODUCTION

If the volumes of cargo flows in the logistics chains of the transport network increase, and the goods are those that require urgent delivery, it amplifies the problem of reducing the cycles of their delivery. Simultaneously, time windows are significantly narrowed. There is a need to apply a rigid schedule of trucks with minimal costs and fixed assets. In such cases, cross-docking is used, which is a logistics technique that reduces the costs incurred in the network of supply chains, while increasing the flow of goods. This reduces the duration of delivery cycles.

2. PROBLEM STATEMENT

Despite the random nature of the origin and destination of freight flows on the highway transportation network, the formation and satisfaction of demand for goods delivery is the subject of objective laws. In particular, the delivery of goods on the network is a discrete process, so it is characterized by such parameters as flow rate, group, volume of accumulation, the front of logistics operations that are performed simultaneously. It is also known that material flows in the network are characterized by cyclicity. That is, each flow can be considered with sufficient accuracy, as one that is periodically repeated, or one that can be decomposed into a set of periodic flows. If these flow properties are taken into account, the discrepancy between customer requirements and transport efficiency can be resolved by optimization. Such optimization has a higher level of achievement than in practice.

3. RELEVANCE OF RESEARCH

Road carriers often occur with increasing volumes and diversity of orders in recent years. At the same time orders for long-distance delivery of small wholesale cargoes dominate. Consignment delivery of such goods is much lower than the nominal carrying capacity of any of the highway road train. On the transport network such orders arise chaotically, but customers require delivery just on time within time windows. There is a known tendency to reduce time windows in the cargo delivery system. In this case, the carrier that manages the fleet of trucks on the selected transport network is forced to respond flexibly to customer requests due to the high level of market competition. However, the meeting of whole requirements of independent cargo owners is accompanied by under-utilization of the transport company own fleet, which is manifested in significant downtime, incomplete loading, low speed transportation. Sometimes carriers violate the rules and technologies of cargo transportation in order to avoid such a dilemma. Transport companies allow overloading of vehicles, infringement regimes of work and rest of drivers violate the loading and securing of cargo on vehicles.

4. ANALYSIS OF RECENT RESEARCH AND PUBLICATIONS

The published research identifies benefits that can be achieved in cross-docking logistics [1]. First, it is to reduce cycle time and shipping costs. This is due to the reduction of the period of sending groups accumulation, and the groups themselves correspond to the maximum capacity of vehicles. Secondly, it is a reduction in inventories. Stocks are stored only during reloading [2]. Thirdly, it is ensuring the principle of “just on time” delivery because the requirements relate to harmonize and coordinate truck schedules arriving and serving of the transportation network nodes [3]. Due to certain advantages, cross-docking technology is used for the delivery of perishable goods, priority goods, express delivery, with increasing freight traffic on the network, and also for seasonal fluctuations. However, as the results of other studies show, cross-docking delivery technology can be ineffective if it is implemented without proper justification of logistic chains parameters [4]. For example, the size of groups sending goods may be too small or destination / dispatch of goods might too distant from other points of the route. Therefore, for the effective implementation of cross-docking technology it is necessary to make a reasonable choice of parameters of transport and technological processes that implement the delivery of goods. Currently, there are known methods to justify the location and capacity of reload hubs for the implementation of cross-docking technology [5]. However, the application of such clauses is a compromise between vehicle under-loading and schedule inconsistencies. Therefore, the application of such technology in the conditions of growth of cargo flows to critical volumes requires a more careful approach to substantiation of parameters of interaction of vehicles on the network.

All known cross-docking studies are closely related to other related delivery management tasks. Mathematical optimization is used to make complex optimal decisions, such as minimizing network flows when planning loading/unloading at hubs, improving delivery and vehicle schedules, selecting the best product groups for cross-docking [1]. Previous simulation models are based on hybrid meta-heuristic approaches, genetic algorithms, Tabu search, annealing, and nearest neighbors. However, such traditional methods do not make any new conclusions about the effectiveness of cross-docking. These methods provide an acceptable solution only for partial problems, such as multi-route routing, optimization of logistics centers, network synthesis, and so on. Examples of application and solutions of the complex problem of parametric optimization of cross-docking and routing according to the criterion of guaranteed duration when using these methods are unknown. The important conclusion from the research is that the processes that collect and deliver goods are not objectively independent. Their dependence is due to the need to coordinate the schedules of different parts of cross-docking [6].

Modern research [7, 8, 9] is devoted to the issue of coordination of input and output flow schedules in the nodes of the transport network, optimization of vehicle schedules and the work of their crews. Recently, drone observations have been used to ensure the coordination of vehicles in logistics chains. However, to achieve the goal of such coordination: reducing the duration of delivery, energy consumption of services, increasing the productivity of vehicles, meta-heuristic methods are used to find optimal solutions. In some cases, the search for the exact optimum is not guaranteed.

In general, there are four areas of cross-docking research: vehicle routing / inventory management, planning, warehousing logistics and distribution. The complex tasks choice should depend on practical production situations. According to the authors of paper [1], no single problem is of practical importance because the local optimum found does not satisfy the problem as a whole. However, studies of cargo delivery systems are asymmetric in relation to the list of solved current problems. The article review [10] found that the number of publications related to truck scheduling and scheduling is much higher than the number of articles related to cargo delivery planning in general. The most prominent of them use the methods of modified mathematical programming [11], simulation modeling [12], genetic algorithms. However, publications that would apply the principle of continuity of material flow and would consider the process of goods delivery as discrete, and as a whole are still unknown. As the results show, the main reason that freight and vehicles flows on the network cannot be reconciled with the optimal parameters is that the flows are not considered discrete, so their intensities μ is a continuous random variable that is difficult and sometimes impossible to distribute between branched streams [13]. In addition, the known studies address one or two problems of cargo delivery during the intensification of cargo turnover, but do not consider the problem as a whole.

In order to successfully coordinate trucks schedules at hubs and reduce the duration of cargo delivery, there is a lack of research on streamlining logistics operations in cross-docking centers. The vast majority of techniques for optimizing road and freight traffic are designed to coordinate trucks schedules. Most of them are based on heuristic and metaheuristic methods. In order to increase the reliability of the search for the optimal solution, such problems use two stages of solution. One such study presents a new two-stage mixed integer linear mathematical model for the transport problem of cross-dock network design, which is integrated with different freight flows and aims to minimize overall transportation costs from suppliers to customers. This model also takes into account the entry-exit plans of trucks in transport hubs and the distribution of cargo flows in the cross-docking area in the presence of physical constraints [14]. However, such a model is too complex to obtain exact solutions by deterministic methods. Therefore, genetic algorithms have been used to solve it, which, unfortunately, require a large amount of input training data, which is due to the difficulties of implementing the method in practice. However, as the results of such studies show, the two-stage model provides a comprehensive plan for managing material flows in cross-dock networks, and the proposed genetic algorithms only under such conditions allow to obtain effective solutions to a complex problem in a short computation time [15]. Operational problems of cross-docking are NP-complex tasks. Therefore, trying best to solve the problem, one should note that the exact algorithms can work effectively only for small problems [16].

In general, based on the literature, the following conclusions can be drawn regarding the problem of using cross-docking. First, the research gap in the cross-docking platform needs to be filled by focusing on planning transport tasks and incorporating the resource potential of carriers into models that are closer to real problems. The main problem of cross-docking so far has been considered to solve the problems of finding the optimal purpose in the queue of incoming and outgoing vehicles from a given node [2]. However, such a complex task was not solved on a limited network. It should be noted that scheduling tasks for cars are more relevant than scheduling tasks for transport tasks.

Based on the review of the latest known publications, which relate to the methodology of discrete material flows ordering under the condition of their cross-docking, on the one hand, complicates the tasks of schedule optimization, routing, coordination, logistics. On the other hand, discrete processes have a much larger number of constraints and boundary conditions that allow us to solve NP-hard problems by the usual methods of linear mathematical programming [17]. Linear and integer programming is the most common technique which is used in the optimization set to address vehicle routing / inventory management. However, in complex flow optimization, routing, and scheduling problems, the linearity conditions are not satisfied, and nonlinear problems do not always have a guaranteed solution due to the sensitivity of clearly defined boundary conditions.

5. PRESENTATION OF THE MAIN MATERIAL

The purpose of these studies was to develop a method of planning cargo transportation, which would resolve the contradiction in the use of cross-docking technology: to reduce the duration of totality delivery of goods provided maximum performance vehicles involved.

The transport network is given, which consists of N transport nodes and roads of communication between them. Cargo passes between nodes. Cargoes are heterogeneous and not impersonal. Each node of the network may be the source and sink of cargo flow simultaneously. It is assumed that the average duration of movement of vehicles with goods, as well as the duration of loading and unloading and reloading operations, statistical estimates of deviations of these durations are known. The average duration of traveling on the network is given by a matrix ($t_{i.j}$), which is symmetrical about the diagonal and reflects the average statistical duration of travel time between any two points of the network i and j, which are connected by a road. If there is no road, then $t_{i.j}=+\infty$. The flows on the network are discrete, characterized by the source of node b_z and by the sink which is node s_z, where $z=1\ldots P$ – the number of the specified incoming cargo flow. Numerical characteristics of flows are average stationary intensity $\mu_{i.j}$, a tact $\tau_{i.j}$, as well as the size of the cargo group $k_{i.j}$, where i, j – numbers of neighboring points, respectively, the point of departure and destination. The tact is the period between two successive phases of one flow. The tact characterizes periodic discrete flows. However, if the flow is composed of several phases, which once performed, then this flow tact is $\tau=+\infty$. If qualitative and / or quantitative changes do not occur from phase to phase with the flow, the cycle tact of such flow is constant. However, cross-docking cargo flow changes the size of the bulk when moving. Cargo can be moved as intended by any group, the size of which, however, is limited by the maximum $k_{i.\max}$ and minimum $k_{i.\min}$ values. The maximum value of a group depends on the size of a warehouse, the properties of the cargo (the possibility of its long-term storage) and other factors. The minimum value of the group of dispatch is due to the allowable size of transport packaging, loading and unloading technologies and the minimum cost-effective actual load of trucks. The same restrictions imposed on minimum $\tau_{\min}$ and maximum values $\tau_{\max}$ tact. The minimum value is technologically limited (transport packets cannot be ready for shipment faster than $\tau_{\min}$). The maximum restriction is due to the characteristics of demand (allowable time windows) [18].

The intensity of discrete freight flow is related to the group of sending the following expression:

$$\mu_{i.j} = \frac{k_{i.j}}{\tau_{i.j}}, \textit{packets per hour.} \tag{1}$$

At the constant average intensity of cargo flow $\mu_{i.j}$ between nodes q_i and q_j, the band size and tact are interdependent inverse values. The intensity of the flow $\mu_{i.j}$ does not depend on the duration of goods transportation on a given section of the transport network, because the time of departure of the next group of goods is carried out regardless of the duration of its movement. However, taking into account the tact $\tau_{i.j}$, and the duration of transportation $t_{i.j}$, the number of vehicles that simultaneously provide traffic between the nodes q_i, q_j, will be determined by their front on this section of the network:

$$f_{i.j} = \left\lceil \frac{t_{i.j}}{\tau_{i.j}} \right\rceil. \tag{2}$$

In formula (2) the division to a larger whole is applied.

During delivery, groups of goods sent on a common route, or part of a route, may be consolidated or deconsolidated. Also in the process of delivery reloading at hubs can be used if such reload is appropriate for the selected criteria. Finally, the management of cargo flows involves determining the routes of individual groups of goods in particular, and consolidated shipments in general. It is necessary to develop routes of movement of all groups of cargoes, to substantiate the size of each group, to define nodes of consolidation of departures and cross-docking, to form routes of vehicles travels and to make the schedule of their departure and arrival on purpose. These problems, given their relationship, need to be addressed comprehensively.

In order to solve such a set of problems, a two-stage algorithm with such formalization was developed and applied. The transport network is presented in the form of a graph $G(Q,V,U)$, the nodes of which are reflected with a nodes Q_i, and ribs $V_{i.j}$ – ways of nodes communication where i,j=0..N+1. The node Q_0 is virtual, formal source of all cargo flows. The node Q_{N+1} is formal sink of all flows. The ribs have a weight that is evaluated by the element of the matrix ($t_{i.j}$), which is symmetrical about the diagonal. If $t_{i.j} = t_{j.i} = +\infty$, then no rib is between nodes q_i and q_j. There are no ribs of graph G between node Q_0 and any other nodes of the graph, but there are arcs $U_{0.j}$. The weights of arcs $u_{0.i}$ and $u_{j.N+1}$ are $t_{0.j}$, or $t_{j.N+1}$, respectively is the duration of loading and unloading operations for one group of goods from the flow μ_z. The content of the route search problem is optimization, because it consists of multiple selection from the graph G of arcs in the presence of restrictions on input and output flows. It is necessary to construct such graph $G'(Q,U)$, which is a reflection of the graph $G(Q,V,U)$ with a constant number of nodes. This means that one needs to replace each rib $v_{i.j}$ of the graph G with the arc u_{ij}, or $u_{j.i}$, or delete this rib. The arcs of the graph G' are discrete freight cargo flows $\mu_{i.j}$. Each arc has four evaluation parameters: $\mu_{i.j}$ $k_{i.j}$, $\tau_{i.j}$, $f_{i.j}$. Taking into account the dependencies (1) and (2), it is also necessary to give the newly formed arcs of the graph G' weight, according to three estimation parameters, due to the predetermined flows intensity μ_z. The criterion for optimal solution of applied problem is the minimum guaranteed duration of goods delivery on all specified freight flows. This criterion is defined as the duration of the critical path in the graph G'. The critical path starts at the vertex Q_0 and ends at the vertex Q_{N+1}, and passes through such a set of arcs Ω that the chain of all these arcs is the maximum in duration:

$$T_{\max} = \sum_{i=0}^{N} \sum_{j=1}^{N+1} \tau_{i.j} \cdot f_{i.j} \to \min, \ u_{i,j} \subset \Omega. \tag{3}$$

Each value of the tact $\tau_{0.j}$ depends on the source of traffic in the j-th node of the network, so it refers to the initial data of the problem.

Restriction:

$$\sum_{i} \mu_{i.j} = \sum_{\xi} \mu_{j.\xi}, \ \text{for } j = 1 \ldots N, \tag{4}$$

where i is the number of all input flows to node j; ξ is the number of all output flows from node j; constraint (4) is written using the principles of balance and continuity of flows on the network; in the expression of the balance (4) must be present all cargo flows, including those that are outgoing and incoming; the condition of performance of all set volume of tasks on delivery of freights is carried out thanks to restriction (4);

$$k_{\min} \le k_{i.j} \le k_{\max}, \text{ for } j = 1 \ldots N, i = 1 \ldots N, \tag{5}$$

$$\tau_{\min} \le \tau_{i.j} \le \tau_{\max}, \text{ for } j = 1 \ldots N, i = 1 \ldots N, \tag{6}$$

$$\sum_{i=1}^{N} \sum_{j=1}^{N} f_{i.j} \le f_{\max}. \tag{7}$$

Expression (5) is derived from the reasoning that when for any discrete flow between two adjacent nodes q_i, q_j of the transport network the inequality $\tau_{i.j} > t_{i.j}$ holds, then the tact $\tau_{i.j}$ is a time constraint on the movement of the discrete material object (that is vehicle with a group of goods) on the path, not the duration $t_{i.j}$. In other words, the next vehicle cannot be dispatched from point q_i and arrive at point q_j if its predecessor was not dispatched and did not arrive as intended. If $\tau_{i.j} < t_{i.j}$, then number of vehicles $f_{i.j}$ will move simultaneously on the path $q_i - q_j$, which is determined by expression (2). Thus, following the entire route of movement of goods from their sources to the sinks, one can define the duration of delivery as the sum of the corresponding cycles. The longest supply chain will be critical for a given network and will determine the guaranteed delivery time. The restriction (7) concerns the total number of vehicles owned by the carrier, which may be involved in the carriage.

In order to construct a graph G' and to evaluate its arcs, it is necessary to solve the problem of ordering a graph G, which is *NP*-hard problem in a strong sense [19]. In addition, the graph G' may contain cycles, so this graph cannot unambiguously indicate the schedule of goods transportation. Because of this, we have developed and applied a two-stage algorithm, at each stage of which a guaranteed accurate solution is obtained in a reasonable time of computer search.

6. OPTIMIZATION ALGORITHM. THE FIRST STAGE

The flows to be implemented are specified. Each flow is given as the initial and final nodes of the graph G. Thus, the number of flows exiting the node q_i is a constant value r_i, i=1..N, and the number of flows entering the node j is a constant value r_j, j=1..N, while:

$$\sum_{i=1}^{N} k_i = \sum_{j=1}^{N} k_j = K. \tag{8}$$

At the first stage of the algorithm, the search for the R routes in the graph G must be fulfilled. Each of these flows must pass through such routes, the total length (in this case the duration of passage) which is minimal. In this case, each route starts at the fictitious node q_0 and ends in the fictitious node q_{N+1}. To construct a graph G' for which criterion (3) holds, one must remove from the graph G all arcs on which no shortest route passes and add multiple arcs between those nodes, which consist several shortest routes. Based on this, the problem of linear integer programming with the following attributes is formulated and solved as follows.

Variables: $x_{i.j}$, where i, j=0..N+1, are integers, non-negative values, such that:

$$x_{i.j} = \begin{cases} R / u_{i.j} \in G' \\ 0 / u_{i.j} \notin G' \end{cases}, \tag{9}$$

where R is equal to the total number of parallel paths of graph G' from q_0 to q_{N+1}. If the arc $u_{i.j}$ belongs to one of R routes, along which given flows pass, then $x_{i.j}=R$.

Search by paths in a graph *G* by minimum total duration of cargo delivery:

$$t_{\Sigma} = \sum_{i=0}^{N+1}\sum_{j=0}^{N+1} x_{i.j}\cdot t_{i.j} \to \min, \tag{10}$$

subject to:

$$\sum_{i=1}^{N} x_{i.j} \ge \sum_{j=1}^{N} x_{1.j}, \tag{11}$$

$$\sum_{j=1}^{N} x_{0.j} = \sum_{j=1}^{N} x_{i.j}, \tag{12}$$

$$\sum_{i=1}^{N} x_{i.0} = \sum_{i=1}^{N} x_{i.j}, \tag{13}$$

$$x_{i.j} \ge 0,\ \ i,j = 1\ldots N, \tag{14}$$

$$\sum_{i=0}^{N} x_{i.0} - \sum_{j=0}^{N} x_{0.j} \le f_{\max}, \tag{15}$$

$$\sum_{i=1}^{N} x_{i.j} - \sum_{j=0}^{N} x_{0.j} = 0, \tag{16}$$

$$x_{0.j} \ge r_j,\ \ j = 1\ldots N, \tag{17}$$

$$x_{i.N+1} \ge r_i,\ \ j = 1\ldots N. \tag{18}$$

The limitation (10) means that for each *j*-th node of the graph *G*' the number of flows passing through it must be not less than the number of its input flows.

The limitation (11) means that the total number of flows originating in any *j*-th node must pass and stop in nodes different from *j*.

The limitation (12) means that the total number of flows ending in each *j*-th node must be equal to the specified number of flows to be executed.

The limitation (13) means that all the required variables are positive, in addition, the problem specifies that they must all be integers.

The limitation (14) means that the total number of all freight flows should not exceed the number of vehicles serving them.

The constraint (15) is the equation of the balance of cargo flows in each nodes of the graph, except zero.

Constraints (16) and (17) mean that all specified flows must be performed in at least one ride.

Thus, the first stage of optimization is a linear optimization problem of integer programming, which can be solved by all available tools and methods. It is all because the dimension of problem is not too large. The exact solution of such a problem is guaranteed for reasonable calculation time.

The solution of the first stage is the set and the value of cargo flows $q_{i.j}$, $i,j = 1..N$, the total duration of service which is determined by the parameters of duration $t_{i.j}$, i.e. in the case when each cycle $\tau_{i.j} = t_{i.j}$. However, this situation occurs when all the duration of cargo transportation $t_{i.j}$ are multiples of cycles. This is an extremely rare situation, so one needs to be adjusted for finding optimal the cargo flows according to the specified intensities μ_R. Similarly flows must be mutually agreed on the cycles and groups sizes. To do this, the second stage of the optimization algorithm is developed. At the second stage parameters of discrete freight flows are defined and schedules of performance of all transport tasks are made.

7. OPTIMIZATION ALGORITHM. THE FIRST STAGE

The initial data for the second stage is the matrix of cargo flows ($q_{i.j}$), which is obtained as a result of first stage optimization. The incoming cargo flows are q_R, as well as the sizes of the groups of shipments at the points of departure are k_R.

The basic equation of the cargo flows balance in each j-th vertex can be written as follows:

$$\sum_{i=0}^{N}\mu_{i.j}+\sum_{i=1}^{N}\frac{k_{i.j}}{\tau_{i.j}}=\sum_{\xi=1}^{N+1}\mu_{j.\xi}+\sum_{i=1}^{N}\frac{k_{j.\xi}}{\tau_{j.\xi}}. \tag{19}$$

The balance equation means that all flows entering each node j, including the sources of freight flows q_z have an average intensity equal to the outgoing freight flows from node j, including sinks. To provide such an equation, one needs to choose the numerical value of the variables $k_{i.j}$, $\tau_{i.j}$, $k_{j.\xi}$, $\tau_{j.\xi}$, where the nodes i, j, ξ correspond to non-zero values of the matrix ($q_{i.j}$). In other words, one has to choose the parameters $k_{i.j}$, $\tau_{i.j}$ so that condition (3) is satisfied. In this case, conditions (5) and (6) must be satisfied, as well as the flow balance equations written on the basis of equality (4). It is also necessary to ensure condition (7) of sufficiency of available vehicles. This problem is nonlinear if we take into account expression (19). The search for variables $k_{i.j}$, $\tau_{i.j}$ for nonlinear problems has no guaranteed success if we use deterministic algorithms. Therefore, additional boundary conditions have been applied under which an optimal solution can be achieved. These conditions, based on expression (3) occur when the required tact $\tau_{i.j}$ are minimal, subject to restrictions, the size of the groups $k_{i.j}$ – maximum, and the fronts of vehicles $f_{i.j}$ approach to 1. If such initial conditions are established, then, it is obvious that the balances (18) may not be satisfied. Using the gradient descent method, we change the numerical values of the $k_{i.j}$ groups so that all equations of balances (19) hold. For example, according to the first stage, there are three consecutive nodes q_i, q_j, q_ξ of the graph G', through each of which the cargo flow with intensity μ_z must pass. The durations of movement of goods on the corresponding arcs are $t_{i.j}$, $t_{j.\xi}$. One has to choose $k_{i.j}$, $k_{j.\xi}$, $\tau_{i.j}$, $\tau_{j.\xi}$, $f_{i.j}$, $f_{j.\xi}$. Initially, we set the boundary conditions:

$$\tau_{i.j}:=\tau_{\min},\ \tau_{j.\xi}:=\tau_{\min},\ k_{i.j}:=k_{\max},\ k_{j.\xi}:=k_{\max},\ f_{i.j}:=1,\ f_{j.\xi}:=1.$$

Such values of variables, according to formula (3) are the most desirable. However, after setting such values, the condition of continuity of the material flow through these nodes and expression (2) may not be fulfilled, i.e. it is probable that:

$$\frac{k_{i.j}}{\tau_{i.j}}\neq\frac{k_{j.\xi}}{\tau_{j.\xi}},\ \left\lceil\frac{t_{i.j}}{\tau_{i.j}}\right\rceil\neq f_{i.j},\ \text{and}\ \left\lceil\frac{t_{j.\xi}}{\tau_{j.\xi}}\right\rceil\neq f_{j.\xi}.$$

In the required paths of the graph G' we choose the following pair of nodes q_i, q_j, for which:

$$\left|\mu_{i.j}-\frac{k_{i.j}}{\tau_{i.j}}\right|\to\max. \tag{20}$$

We change $k_{i.j}$ for the found pair of nodes, so that the balance equation (19) holds. Then the search for the next pair of nodes that meet criterion (20) continues until the balance equation for all vertices of the graph is fulfilled. As a result of such actions, the numerical value of criterion (3) will increase, but this will be a compromise to the applied changes. The search for flow imbalances on nodes is a finite process and its algorithmic complexity is $O(2n)$, where n is the number of vertices of the graph G' [19]. The solution of the global problem is provided with a minimum duration of computation.

8. TEST METHODS FOR TEST MODELS

The trial model of transport network (Fig. 1) is considered, which consists of six transport points: A, B, C, D, E, F.

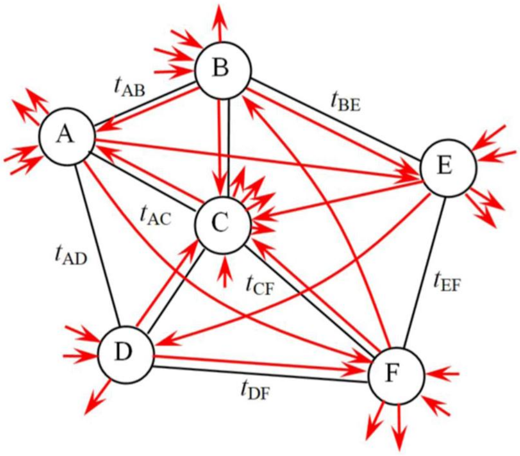

Fig. 1 Model of the transport network (black vertices and ribs) and the specified flows (red arcs)

The points are connected by roads, the average duration of which is determined by the square matrix of time connections ($t_{i,j}$) (Table 1).

Table 1

Matrix of durations

	0	A	B	C	D	E	F	N
0	###	1	1	1	1	1	1	
A	0	###	4	7	15	###	###	1
B	0	4	###	6	###	8	###	1
C	0	7	6	###	7	###	4	1
D	0	15	###	7	###	###	6	1
E	0	###	8	###	###	###	8	1
F	0	###	###	4	6	8	###	1

The notation ### refers to a large number of advance, which means that there is no connection path between the corresponding nodes. Nodes q_0, q_N are virtual. The duration of loading and unloading operations at the nodes was assumed to be constant $t_{0.i}$ =$t_{i.N}$=1 hour. Among the given freight flows for some period T are the following: 1) μ_{AE} = 2; 2) μ_{AF} = 6; 3)μ_{BA} = 12; 4) μ_{BC} = 9; 5) μ_{BE} = 4; 6) μ_{CA} = 5; 7) μ_{DC} = 8; 8) μ_{DF} = 1; 9) μ_{EC} = 3; 10) μ_{ED}=5; 11) μ_{FB}=2; 12) μ_{FC}=9 packages per hour. We will use the given digital numbering of cargo flows in the future. The maximum size of the sending group on the given flows $k_{\max}$=100 packets, the minimum – $k_{\min}$=1 packet. The minimum time of sending each group of goods is $\tau_{\min}$=1 hour, the maximum time is $\tau_{\max}$=16 hours. The number of vehicles at the carrier – f=60. As a result of the first stage, solutions were obtained – the shortest paths from the zero to the virtual end node, along which the specified freight flows should pass (Table 2).

Table 2

The result of the search for the shortest paths on the network

	0	A	B	C	D	E	F	0	*sum i*
0	0	2	3	1	2	2	2	0	12
A		0	1	1	0	0	0	2	4
B		2	0	0	0	1	0	1	4
C		0	0	0	1	0	0	4	5
D		0	0	2	0	0	0	1	3
E		0	0	0	0	0	2	2	4
F		0	0	1	0	1	0	2	4
$\sum_{i=0}^{6} x_{i.j}$	0	4	4	5	3	4	4	12	
$\sum_{i=1}^{6} x_{i.j}$		2	1	4	1	2	2		
sum flows		2	1	4	1	2	2		

In the Table 2 *sum_flows* is the total number of specified flows that must pass through a given nodes. From the received Table 2 it is possible to display the scheme of cargo flows (fig. 2).

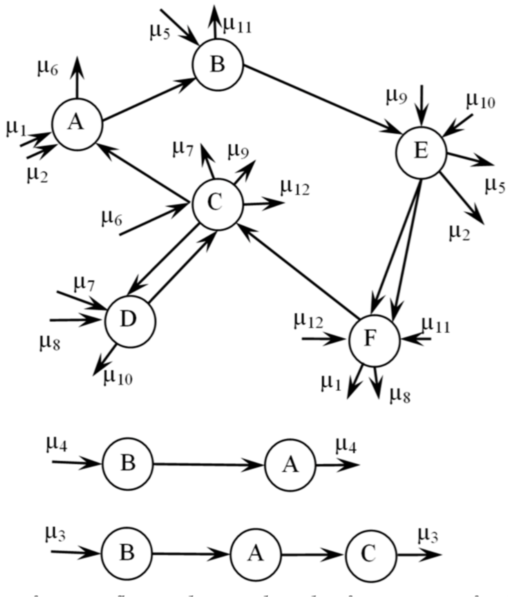

Fig. 2 The scheme of cargo flows obtained in the first stage of solving the problem

According to this scheme, it is seen that the cargo flows μ_3 and μ_4 are performed without cross-docking on simple pendulum routes and are independent of all others. Other given cargo flows form a more complex scheme that requires further processing. Subsequently, flows 3 and 4 are excluded from consideration. The dependences of the balance of flows 1, 2, 5–12 are written in such expressions:

$$\mu_{AE} + \mu_{AF} + \frac{k_{CA}}{\tau_{CA}} = \mu_{CA} + \frac{k_{AB}}{\tau_{AB}}, \tag{21}$$

$$\mu_{BE} + \frac{k_{AB}}{\tau_{AB}} = \mu_{FB} + \frac{k_{BE}}{\tau_{BE}}, \tag{22}$$

$$\mu_{EC} + \mu_{ED} + \frac{k_{BE}}{\tau_{BE}} = \mu_{BE} + \frac{k_{EF1}}{\tau_{EF1}} + \frac{k_{EF2}}{\tau_{EF2}} + \mu_{AE}, \tag{23}$$

$$\mu_{FB} + \mu_{FC} + \frac{k_{EF1}}{\tau_{EF1}} + \frac{k_{EF2}}{\tau_{EF2}} = \mu_{AF} + \mu_{DF} + \frac{k_{FC}}{\tau_{FC}}, \tag{24}$$

$$\mu_{CA} + \frac{k_{FC}}{\tau_{FC}} = \mu_{EC} + \mu_{FC} + \mu_{BC} + \frac{k_{DC}}{\tau_{DC}}, \tag{25}$$

$$\mu_{BA} = \frac{k_{BA}}{\tau_{BA}}, \tag{26}$$

$$\mu_{BC} = \frac{k_{BA}}{\tau_{BA}} = \frac{k_{AC}}{\tau_{AC}}. \tag{27}$$

It is necessary to set the numerical values of the cycles presented in expressions (21) – (27) and the sizes of groups so that the maximum guaranteed duration of delivery on all schemes was minimum. At the beginning the value of all flows tact was τ=1 hour, and it was adopted that the size of all groups is k=100. The guaranteed duration is $T_{\max}$=28 hours. However, all expressions (21) – (27) were not fulfilled. The largest difference between the planned and actual flow intensities was observed at the node *C*. Then k_{FC} was reduced to 76 and τ_{FC} increased to 4 hours. All flows between nodes *F* and *C* were agreed. The process of matching other flows of this scheme took 16 calculations, which confirms expected algorithmic complexity of the problem. The solution found is shown in Fig. 3, where each flow evaluated by required marks.

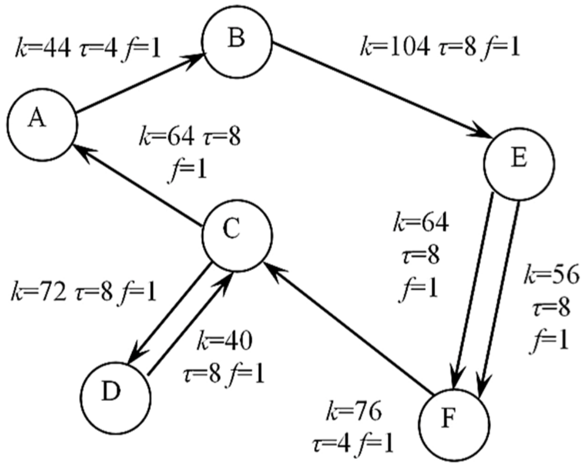

Fig. 3 The result of parametric optimization of the flow scheme

The given scheme needs to be practically implemented in such way. The truck A1 with 64 packages arrives at point A where 44 packages are reloaded on the truck A2 in the ratio: 24 packages are to delivery on the AF route, 8 packages are to the AE route, 8 packages are to FB, 4 packages are to DF. 20 packages are consumed at point A. The truck A2 moves to point B in 4 hours. And 8 packets of FB are unloaded at point B and 16 packets of BE are accumulated for sending on the route BE in 4 hours. 104 packets are sent in 8 hours from point B to point E by vehicle A3. 32 packages of route BE and 16 packages of route AE are unloaded at point E. The other 56 packages are transferred to A4 car. The A4 car goes into circulation with the tact of 8 hours. It delivers to point F 8 packets that are destined for delivery to point D and 48 packets in the AF direction. All 56 packages are unloaded from the A4 truck in point *F* Also in point A5 64 packages are loaded on routes, respectively, 24 – ES, 40 – ED. The A5 truck with 64 packages departs from point E with the tact of 8 hours. The A4 vehicle unloads 56 packages at F, 48 of which are transported on the route AF, 8 packages – on the route DF. 64 packages from the A5 vehicle at F are transferred to the warehouse, and then to the A6 with twice less frequency. Cargoes are loaded on the A6 vehicle in the direction of FC in the amount of 36 packages and in the direction of FB in the amount of 8 packages as well. The A6 is sent from point F with the tact of 4 hours and it travels on the FC route for exactly 4 hours. Therefore, the number of loads on the A6 truck will be 64/2 + 36 + 8 = 76 packages. The truck A6 unloads 48 packages in point C intended for consumption in this point and sent from points, accordingly, E – 12 packages and F – 36 packages. The other 28 delivered packages are formed in the group of departure to points D and A. 40 packages are sent from point C by truck A7 with a frequency of 8 hours to point D. Vehicle A, which transports 72 packages with a frequency of 8 hours, is sent from point D to point C at the same time. Thus, the maximum duration of cargo delivery under this scheme is 40 hours. The number of trucks involved in the scheme is 8. Their actual loads are: A1 – 64, A2 – 44, A3 – 104, A4 – 56, A5 – 64, A6 – 76, A7 – 40 packages, A8 – 72 packages.

Trucks schedule is a set of $\{t_i\}$ points of time that limit their departure / arrival at the appropriate points of the transport network for unloading / loading. Moments of time were calculated by expressions:

$$t_{j.\max} = t_i + \tau_{i.j}, \tag{28}$$

$$t_{j.\mathrm{mmin}} = t_i + t_{i.j}, \tag{29}$$

9. THE RESEARCH OF DELIVERY PROCESSES OF THE TRANSPORT COMPANY: CASE STUDY

The developed methodology was used to improve the process of cargo delivery in the central and southern regions of Ukraine. The “Trans-Service-1” Ltd. receives orders for transportation of goods in packages that are periodically repeated in one day. Such goods transported by the enterprise include food, construction materials, industrial and household equipment and others. The enterprise uses freight road trains as a part of the MAN-TGX tractor-truck with the SCHMITZ semitrailer for such transportation. The volume of cargo in such a road train is 92 m^3. The maximum load capacity of a road train is 34 standard transport packages. Groups of shipments of these cargoes range in size from 3 to 34 packages. Shipments range from 3 to 250 packages. Data on the transportation of piece goods in the south-eastern and central regions of Ukraine during the year were used. Transportation is carried out from freight stations and terminals, between different companies. Time windows for different loads range from 24 hours to 4 days. Trans-Service-1 has three vehicles bases in these regions: Haisyn, Dnipro and Odessa cities. Data on incoming orders and their execution were collected for May-July 2021. The map of routes shows the main origin and sink points and the average travel time between them (Fig. 4).

Fig. 4. Scheme of optimal cargo flows, obtained as a result of the first stage of planning: red arrows – cargo flows, black ribs – unloaded sections of the network

The depots of fleet of enterprise are marked on the scheme by black circles (on the scheme of them 3), all transport points are numbered (on the scheme of them – 12). The average travel time is determined for the average summer intensity of traffic flows in the period from 8 to 19 o'clock, on the current highways of this region of Ukraine as of May–July 2021 using the Google Map router. Based on the distances between each two transport points, as well as when using Google Map, a matrix of travel time $t_{i,j}$ is constructed. The number of fleets of tilt road trains that carry goods in packages in the defined area is 28. All trucks are serviceable, with mileage from the beginning of operation is no more than 150 thousand km. Duration of loading and unloading of a truck for one cycle is 1.0 hour, taking into account time of performance of maneuvers. Duration of work shift is 8 hours. The 4-month period of transportation performance was considered, during which the majority of orders were periodically repeated. Requests for transportation of goods are received at the head office of the enterprise and form the incoming flow within one working day. The distribution of applications between free trucks is made after that. If the application is rejected by the manager or not fulfilled by the company, the customer cancels the order. Thus the refusal is fixed. The average daily number of applications is 28. The standard deviation is 6 applications. The maximum number of orders is 47 per day. Applications are processed by both the logistics department and the commercial and financial department. One or two applications are distributed for a daily transport task for each road train. Some orders are eliminated as unprofitable for this transport company. A statement of orders was compiled after processing the daily applications for the transportation of groupage cargo, which is presented in the Table 3. The Table 3 does not indicate the maximum rate of departure of goods, as it coincides with the average duration of the time window of 24 working hours. The size of the maximum group of departure coincides with the volume of departure.

When using the first stage of the algorithm, the optimal duration of the cargo flow was obtained, which is shown in Table 3 and in Fig. 5. The total turnover, provided that the total volume of orders for the shortest routes is fulfilled, is 86.9 packages / hour. The multiplicity of flows in Table 3 means that in a given direction the flow of goods will be moved by several independent carriers. This means that the multiplicity of flows corresponds to the number of vehicles involved in a given section of the transport network.

Table 3

Initial data from the order statement

№ of order	Start point	End point	Average intensity of cargo flow, packages / hour	The minimum tact, hours	The minimum group size	The average tact, hours	The average size of group
1	2	3	4	5	6	7	8
1	5	3	1.0	3	3	8	8
2	3	6	3.8	3	1	8	30
3	3	5	0.5	3	6	24	12
4	4	12	0.3	3	10	16	4
5	4	3	0.5	3	6	24	12
6	10	3	3.0	3	1	4	12
7	7	12	6.0	3	1	4	24
8	4	8	2.7	3	1	12	32
9	1	12	0.2	3	15	48	8
10	7	10	0.1	3	30	72	8
11	1	2	2.0	3	2	4	8
12	1	3	1.3	3	2	8	10
13	11	12	1.5	3	2	8	12
14	4	11	2.0	3	2	8	16
15	12	6	0.8	3	4	12	10
16	5	12	0.3	3	10	24	8
17	4	2	0.3	3	10	24	8
18	12	9	0.5	3	6	16	8
19	12	5	0.8	3	4	24	20
20	2	10	0.6	3	5	36	20
21	10	9	5.0	3	1	4	20
22	2	3	4.0	3	1	4	16
23	7	2	0.2	3	15	34	8
24	9	12	0.2	3	15	72	12
25	8	11	0.8	3	4	16	12
26	7	9	0.8	3	4	16	12
27	12	11	1.3	3	2	12	16
28	12	2	3.0	3	1	4	12
29	12	1	1.0	3	3	8	8
30	12	4	0.3	3	10	12	4

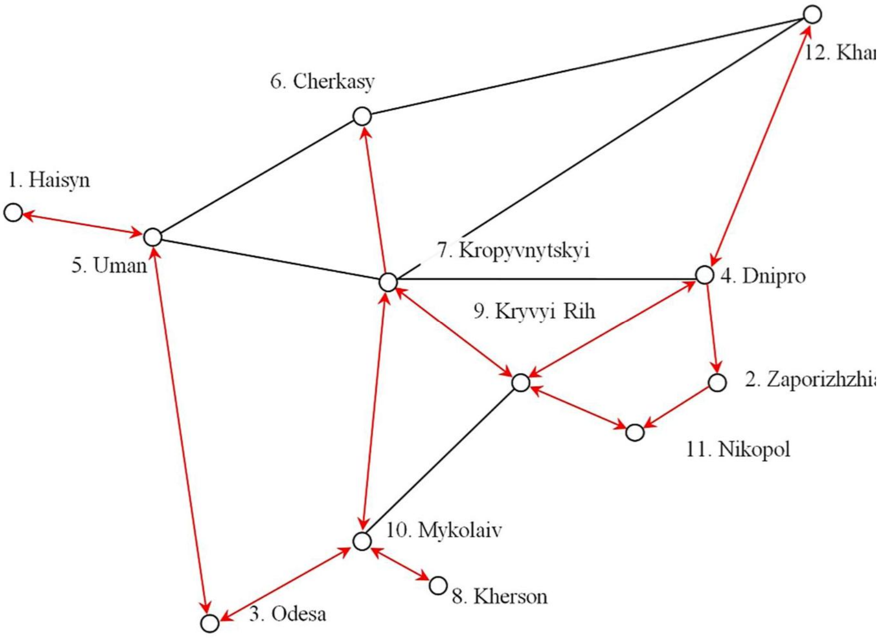

Fig. 5 Scheme of optimal cargo flows, which is obtained as a result of the first stage of planning: red arrows are cargo flows, black ribs are unloaded sections of the network

In order to obtain optimal routes and trucks timetables we optimize the parameters of discrete flows pre-applying such conditions and restrictions. Based on the scheme and initial data, we will make a balance of total cargo flows that must pass through each node of the transport network, which are: $\mu_1 = 3.6$, $\mu_2 = 5.6$, $\mu_3 = 8.4$, $\mu_4 = 24.2$, $\mu_5 = 7.4$, $\mu_6 = 4.0$, $\mu_7 = 10.6$, $\mu_8 = 1.3$, $\mu_9 = 6.5$, $\mu_{10} = 10.1$, $\mu_{11} = 2.4$, $\mu_{12} = 19.0$ packets / hour. The initial value of the tact of all clock cycles is $\tau_{i,j}$=3.0 hours. The initial value of all fronts is $f_{i,j}$= 1. The initial sizes of groups are accepted according to the Table 4.

Table 4

Optimal freight flows obtained after the first stage of optimization

From the point	To the point	Multiplicity of flows	Total average packet intensity / hour
1	5	3	3.6
5	1	1	1.2
5	3	2	6.2
3	5	1	4.0
3	10	2	4.4
10	3	3	6.6
10	8	1	1.3
10	7	1	2.4
8	10	1	1.3
7	10	1	3.8
7	9	1	2.8
7	6	2	4.0
9	7	1	2.8
9	4	1	1.3
4	9	1	2.6
9	11	1	2.4
11	9	1	2.4
4	2	4	5.6
4	12	6	16.0
12	4	7	19.0
2	11	2	2.8

Due to the complexity of the cargo flow scheme, it was not possible to single out independent transport and technological cargo delivery schemes. However, the second stage of the problem was successfully solved. All conditions and restrictions of (4)–(7) type were met. Due to the applied boundary conditions, the balances of flows in the directions indicated in Fig. 5 were harmonized. The guaranteed duration of delivery of the transport package in any direction of this scheme is 16.8 hours. At the enterprise of "Trans-Service-1" duration of execution of orders for delivery of freights in the same directions reaches 24 hours. Eight vehicles are involved in the transport-technological scheme. Compared with the real production situation at the enterprise, this result more than triples the productivity of road trains. This result was achieved due to the fact that on adjacent sections of the transport network the organizational parameters were agreed (reduced to the equality of tact) due to the variable load capacity of vehicles. The actual load capacity of vehicles ranges from 12–34 transport packages. The specific schedule for each car is given in the Table 5.

Table 5

The optimal schedule for the execution of the daily set of orders for the optimal delivery plan

Number of truck	Start point: departure time, hours; downloads, packages	Unloading / loading, packages in the intermediate point of cross-docking			End point: unloading, packages
		1st	2st	3st	
1	1: ф=4 h., 34 pack.	3: 5/–	10: –/5	–	7: 34
2	5: ф=16 h., 14 pack.	7: –/16	9: 10/6	–	2: 26
3	10: ф=10 h., 30 pack.	3: 30/15	–	–	1: 15
4	7: ф=12 h., 32 pack.	–	–	–	6: 32
5	4: ф=2 h., 34 pack.	9: 4/–	7: 12/2	10: 6/6	3: 10
6	4: ф=2 h., 12 pack.	–	–	–	12: 12
7	4: ф=8 h., 34 pack.	2: 6/3	11: 12/–	9: 12/–	8: 7
8	12: ф=6 h., 24 pack.	4: 3/–	2: 8/–	–	11: 13

According to received schedule in Table 5 it is possible to notice that work cycle of each road train is a constant corresponding to conditions of uninterrupted work. It is also seen that the cycles of cargo delivery processes on adjacent routes are multiple. Thanks to the optimization, it was found that the front of vehicles in each section of the transport network does not exceed – 1.

10. CONCLUSIONS AND PROSPECTS FOR FURTHER RESEARCH

Discrete consideration of cargo delivery processes, application of parameters that are characteristic of periodic processes, as well as the principles of continuity of flows on the network allowed to formulate the routing problem as two-stage task of flow optimization. Thanks to the considered balances of flows in vertices we managed to reduce dimension of an initial problem to apply deterministic methods of mathematical programming. The problem dimensionality is reduced due to the fact that in the first stage of the solution those traffic flows are identified for which it is impractical to use cross-docking, and which are considered in isolation from other flows. Deterministic methods can be applied to such a complex problem due to the studied balance dependencies and the established boundary conditions of optimality.

In the study of delivery processes of piece goods in packages in the central and south-eastern regions of Ukraine in the transport company revealed under-utilization of the transport capacity of the existing fleet at a steady intensity of order flows. It was found that the productivity of road trains can be increased by optimizing transport and technological schemes and work schedules more than three times. At the same time the guaranteed term of delivery of freights is reduced by 30 %.

The proposed algorithm can be used to control the freight traffic of the transport company. Its further improvement involves the study of the results obtained with increasing size of the problem and under dynamic conditions of solution obtaining.

References

1. Kiani Mavi, R., Goh, M., Kiani Mavi, N., Jie, F., Brown, K., Biermann, S., & A Khanfar, A. (2020). Cross-docking: a systematic literature review. *Sustainability*, *12(11)*, 4789 (1–19). doi: 10.3390/su12114789 (in English)

2. Santos, F. A., Mateus, G. R., & Da Cunha, A. S. (2013). The pickup and delivery problem with cross-docking. *Computers & Operations Research*, *40(4)*, 1085–1093. doi: 10.1016/j.cor.2012.11.021 (in English)

3. Theophilus, O., Dulebenets, M. A., Pasha, J., Lau, Y. Y., Fathollahi-Fard, A. M., & Mazaheri, A. (2021). Truck scheduling optimization at a cold-chain cross-docking terminal with product perishability considerations. *Computers & Industrial Engineering*, *156*, 107240. doi: 10.1016/j.cie.2021.107240 (in English)

4. Buijs, P., Vis, I. F., & Carlo, H. J. (2014). Synchronization in cross-docking networks: A research classification and framework. *European Journal of Operational Research*, *239(3),* 593–608. doi: 10.1016/j.ejor.2014.03.012 (in English)

5. Contardo, C., Hemmelmayr, V., & Crainic, T. G. (2012). Lower and upper bounds for the two-echelon capacitated location-routing problem. *Computers & operations research*, *39(12)*, 3185–3199. doi: 10.1016/j.cor.2012.04.003 (in English)

6. Wen, M., Larsen, J., Clausen, J., Cordeau, J. F., & Laporte, G. (2009). Vehicle routing with cross-docking. *Journal of the Operational Research Society*, *60(12)*, 1708–1718. doi: 10.1057/jors.2008.108 (in English)

7. Nurprihatin, F., Rembulan, G. D., Christianto, K., & Hartono, H. (2021, March). Decision support system for truck scheduling in logistic network through cross-docking strategy. *Journal of Physics: Conference Series*. *1811(1)*, 012009. doi: 10.1088/1742-6596/1811/1/012009 (in English)

8. Huang, H., Savkin, A. V., & Huang, C. (2020). Scheduling of a parcel delivery system consisting of an aerial drone interacting with public transportation vehicles. *Sensors*, *20(7)*, 2045. doi: 10.3390/s20072045 (in English)

9. Han, Y. Q., Li, J. Q., Liu, Z., Liu, C., & Tian, J. (2020). Metaheuristic algorithm for solving the multi-objective vehicle routing problem with time window and drones. *International Journal of Advanced Robotic Systems*, *17(2)*, 1729881420920031. doi: 10.1177%2F1729881420920031 (in English)

10. Buakum, D., & Wisittipanich, W. (2020). Stochastic internal task scheduling in cross docking using chance-constrained programming. *International Journal of Management Science and Engineering Management*, *15(4)*, 258–264. doi: 10.1080/17509653.2020.1764404 (in English)

11. An Integrated Routing-scheduling Model for a Hybrid UAV-based Delivery System. Retrieved from: https://www.cirrelt.ca/documentstravail/cirrelt-2020-17.pdf (in English)

12. Calabrò, G., Torrisi, V., Inturri, G., & Ignaccolo, M. (2020). Improving inbound logistic planning for large-scale real-world routing problems: a novel ant-colony simulation-based optimization. *European Transport Research Review*, *12(1)*, 1–11. doi: 10.1186/s12544-020-00409-7 (in English)

13. Oliskevych, M., Kovalyshyn, S., Magats, M., Shevchuk, V., & Sukach, O. (2020). The optimization of trucks fleet schedule in view of their interaction and restrictions of the European agreement of work of crews. *Transport Problems*, *15(2)*. 157–170. doi: 10.21307/tp-2020-028 (in English)

14. Küçükoğlu, İ., & Öztürk, N. (2017). Two-stage optimisation method for material flow and allocation management in cross-docking networks. *International Journal of Production Research*, *55(2)*, 410–429. doi: 10.1080/00207543.2016.1184346 (in English)

15. Chargui, T., Bekrar, A., Reghioui, M., & Trentesaux, D. (2019). Multi-objective sustainable truck scheduling in a rail–road physical internet cross-docking hub considering energy consumption. *Sustainability*, *11(11),* 3127 (1–23). doi: 10.3390/su11113127 (in English)

16. Gunawan, A., Widjaja, A. T., Gan, B., Yu, V. F., & Jodiawan, P. (2020). Vehicle routing problem for multi-product cross-docking. *Proceedings of the International Conference on Industrial Engineering and Operations Management (pp. 66–77).* Dubai, UAE (in English)

17. Hochbaum, D. S., & Levin, A. (2006). Cyclical scheduling and multi-shift scheduling: Complexity and approximation algorithms. *Discrete Optimization*, *3(4)*, 327–340. doi: 10.1016/j.disopt.2006.02.002 (in English)

18. Song, X., Jones, D., Asgari, N., & Pigden, T. (2020). Multi-objective vehicle routing and loading with time window constraints: a real-life application. *Annals of Operations Research*, *291(1)*, 799–825. doi: 10.1007/s10479-019-03205-2 (in English)

19. Zhang, C., Song, W., Cao, Z., Zhang, J., Tan, P. S., & Chi, X. (2020). Learning to dispatch for job shop scheduling via deep reinforcement learning. *Advances in Neural Information Processing Systems, 33*, 1621–1632. doi: 10.48550/arXiv.2010.12367 (in English)

Received 15.02.2022; Accepted in revised form 05.04.2022.

МАРШРУТИЗАЦІЯ ДОСТАВКИ ВАНТАЖІВ З КРОСС-ДОКІНГОМ ЗА ГАРАНТОВАНИХ МІНІМАЛЬНИХ ТЕРМІНІВ

Анотація. *Стаття присвячена проблемі успішного застосування кросс-докінгу, як технології доставки вантажів за підвищених вимог до термінів, що дозволяє розв’язувати суперечності між забезпеченням гарантованих термінів доставки і ефективності використання наявного парку вантажівок. Процес організації доставки розглядається як упорядкування на транспортній мережі множини дискретних вантажопотоків у вигляді його фаз. Якщо від фази до фази з потоком не відбуваються якісні, і/або кількісні зміни, то такт такого потоку є сталим. Проте вантажопотоки при кросс-докінгу змінюють за переміщення розмір гурту. Вантажі можна переміщати за призначенням довільним гуртом, розміри якого, однак, є обмежені максимальним та мінімальним значенням розмірів гурту. Розроблено двостадійний алгоритм розв’язання задачі. Транспортна мережа представлена у вигляді графа. Зміст задачі пошуку маршрутів є оптимізаційним, оскільки полягає у множинному виборі з початкового графа дуг при наявності обмежень на вхідні і вихідні потоки. Потрібно кожне ребро графа замінити на дугу прямого або зворотного напряму, або видалити це ребро. Критерій оптимальності розв’язку задачі, який застосовано – мінімальна гарантована тривалість доставки вантажів по усій сукупності заданих вантажопотоків. На першій стадії алгоритму виконано пошук найкоротших шляхів у графі, по яких може проходити кожен із заданих вантажопотоків. Перша стадія оптимізації є лінійною задачею цілочислового програмування, розмірність не є надто великою. Початковими даними для другої стадії є матриця вантажопотоків, яка отримана в результаті оптимізації на першій стадії. Зміст другої стадії алгоритму – це розв’язок рівняння балансу дискретних вантажопотоків. Рівняння балансу означає, що усі потоки, які входять у кожну вершину, включно із джерелами вантажопотоків даної вершини, мають середню інтенсивність, яка дорівнює інтенсивності вихідних вантажопотоків з кожної вихідної вершини, включно зі стоками. Завдяки дослідженим залежностям між окремими фазами процесу доставки на прикладі вантажного перевізника на транспортній мережі України, сформульованим обмеженням і крайовим умовам отримано можливість гарантованого точного розв’язання комплексної проблеми. При цьому знайдено найкоротші маршрути, визначено пункти перевантаження, а також часові параметри експлуатації і ступінь завантаження автомобілів. За результатами проведених досліджень отримано трикратне підвищення продуктивності використання парку автопоїздів із зниженням термінів гарантованої тривалості доставки на 30 %.*

Ключові слова: *доставка вантажів, кросс-докінг, маршрутизація, дискретна оптимізація.*