

ENHANCED TRANSMISSION THROUGH BELOW-CUTOFF HOLES AND EIGENOSCILLATIONS OF WAVEGUIDE OBJECTS AND PERIODICAL STRUCTURES

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Abstract

The spectral theory is used to explain the enhanced transmission phenomenon. Eigenoscillations of corresponding waveguide and periodic open resonators have been studied. Their influence on frequency response is demonstrated for various structures. It has been found that the origin of this recently discovered effect is in the existence of the eigenoscillations of the interface between the free half-space and the metal half-space perforated with double-periodic set of channels or the eigenoscillations of the waveguide plane junction.

Keywords: below-cutoff holes, double periodic metal screen, waveguide iris, eigenoscillations, total transmission

1. HISTORY AND STATEMENT OF A PROBLEM

In 1998 unexpectedly high level of light transmission through a thin metal screen perforated by holes with diameters much smaller than the wavelength of the light was revealed in the first time [1]. After that a lot of publications about explanation of this effect, hunt for new structures where it is in existence, and its practical applications appeared.

Initially this anomalous phenomenon was discovered in the optic range where metal conductivity is far from perfect one [1]. So the existence of the enhanced transmission was explained by finite metal conductivity and the excitation of surface plasmon polaritons. However, lately this effect was demonstrated in the microwave range where the metal is almost perfect conductor and the theory based on the surface plasmon polaritons was no longer valid [2]. Scientists paid their attention to periodicity of the screens and used the theory of surface waves [3, 4]. It let to explain the cause of appearance of the extraordinary transmission without dependence on the screen material parameters. Furthermore, it was applicable in the case of the perfect conductivity as well. Nevertheless, recently the enhanced transmission was discovered for waveguide irises with below-cutoff holes [5, 6], i.e. for the structures no supported the surface waves. A new explanation is required.

Lately some theories that interpret the nature of the enhanced transmission phenomenon by means of microwave engineering concepts only arose. The circuit theory is used in [6]. The authors of given publication treat eigenoscillations of periodic or waveguide open

resonators as an origin of all resonance effects [7]. This interpretation is based on classical ideas of electromagnetics. So its usage emphasizes the fundamental nature of the extraordinary transmission phenomenon.

The eigenoscillations theory (the spectral theory) was used by the authors for analysis of waveguide irises with some resonance slots and screens with several resonance slots on a periodic cell. It let to explain some specific effects [8, 9]. This conception was applied to structures with below-cutoff holes as well [10, 11]. Here we are going to summarize our previous results.

First of all, importance of considered phenomenon consists in expansion of our knowledges about main resonance effects. Nevertheless, some practical applications of the enhanced transmission appeared. There are the publications concerned new metamaterials [12] and designing frequency selective surfaces [13], for instance.

In this paper we will consider the nature of the enhanced transmission, its dependence on geometry parameters including structure of the screen cell and a number of the holes, and will describe influence of a single eigenoscillation and sets of them that lead to arise new resonance effects, in particular the pairs of "resonance-antiresonance".

2. THE REFLECTION FROM PERFORATED METAL SURFACES

Let us pay our attention to the simplest structure that supports considered effect. It is a plane junction of two uniform waveguides (one of them is the single-mode one and another is below-cutoff one) or an interface between the free space and a metal half-space perforo-

rated with below-cutoff holes. Consider the excitation from the single-mode waveguide for the plane junction and normal incidence of the plane wave from the free space for the periodic structure. It turned out, that there is abrupt changing of the phase of the reflected mode in the one-mode (single-mode) band near a cutoff frequency of the first highest modes for all irregularities presented in Fig. 1. From this point on the “single-mode band” means a range between cutoff frequencies of the incident mode and the first excited highest mode. The normalized frequency $\kappa = a_0 / \lambda$ is used for generality. Naturally the single-mode band boundaries depend on the symmetry properties of the incident mode and corresponding irregularity.

To understand a reason of sudden phase turn let us mark properties that are common for all structures. First of all, the incident mode is a mode with $E_z = 0$. As to structures shown in Fig.1 (from bottom to top) the one-mode bands are bounded by the cutoff frequencies of $\text{TM}(E)_{11}^{\square}$, TM_{11}° , $\text{TM}_{\pm 10}^{\text{Floquet}}$ modes. Hence changing of the phase takes place near a cutoff frequency of a TM mode. In the course of computational investigations it was determined that if the first excited highest mode is of the TE type, such phase behavior of the reflection coefficient had not been observed as well as in the case of incidence of the TM mode.

The phase rotation on 360° in the narrow frequency band is typical to the reflection coefficient from a resonant load on the transmission line end. Indirectly it points on the existence of an eigenoscillation. Excitation of this eigenoscillation leads to the rotation of the reflected mode phase.

It is important to emphasize that in our case the resonator is really the plane irregularity (it does not have any volume!) loaded on the regular channels (the waveguides or the free space). Until recently the investigation of the eigenoscillations of the waveguide or periodic open resonators was carried out for 2D scalar problems on 1D periodic screens or corresponding waveguide irregularities. Y. K. Sirenko and L. A. Rud proved that similar structures of “zero-size” volume have the eigenoscillations with real eigenfrequencies only. These frequencies are equal to cutoff frequencies of the modes of the regular channels [14]. So the question about the eigenoscillations of the 2D plane junctions has not been considered yet.

There is no way to realize such analytical evaluation for the vector problems of electromagnetics and it is necessary to use exact numerical approaches to hunt the eigenoscillations of such open structures.

Let us write the electromagnetic fields in the regular channels as series on their eigenmodes. By the mode matching technique we obtain an equation to calculate the resonator eigenfrequencies: $\det(A(\kappa)) = 0$. The solutions of the homogeneous matrix equation $A(\kappa)X = 0$ in the found eigenfrequencies give us a possibility to reconstruct field pattern of the eigenoscil-

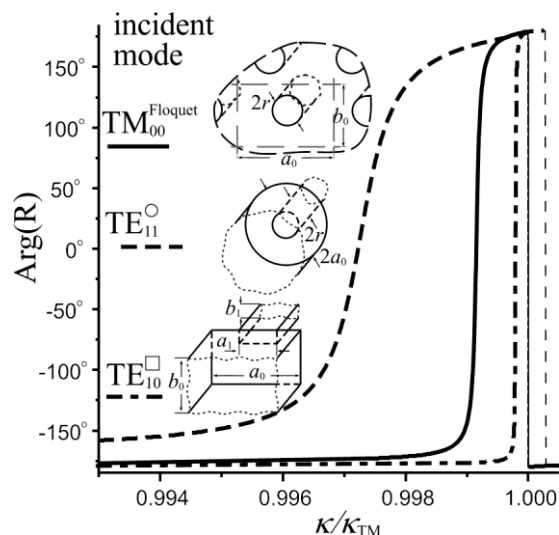


Fig. 1. The reflection from perforated metal half-space and waveguide plane junctions near cutoff frequency of the first highest TM mode. Across: the frequency parameter normalized to cutoff frequency of the TM mode. Metal half-space: $b_0/a_0 = 1$, $r/a_0 = 0.15$; circular waveguides: $r/a_0 = 0.3$; rectangular waveguides: $b_0/a_0 = 0.43$, $a_1/a_0 = b_1/a_0 = 0.13$.

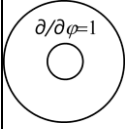
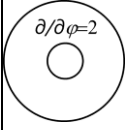
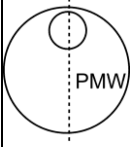
lations. Evidently an accuracy of this obtained solution depends on a truncation order of the matrix $A(\kappa)$. It is regulated by a parameter f_{cut} that is the highest limit of the cutoff frequencies of modes included in calculation. To obtain qualitatively correct physical results it is enough to choose f_{cut} so that at least three modes inside the below-cutoff waveguides are taken into account.

The complex eigenfrequencies were really found for all structures plotted in Fig. 1. Their real parts are closed to the frequencies corresponding to change of the phase of the reflection coefficient. The reconstructed field patterns of the eigenoscillations confirm the main role of the first highest TM modes in forming of the oscillations. The field lines of electrical fields of the eigenoscillations organize something similar to “hats” under the open holes.

Consequently, we can conclude that the eigenoscillations of high Q-factor with complex eigenfrequencies located near the cutoff frequency of the first highest TM mode exist in open resonators with “zero-size” volume that are perfect conductor reflectors with below-cutoff holes loaded by regular channels. Excitation of these eigenoscillations leads to abrupt 360° changing of the phase of reflection coefficient.

Number of the eigenoscillations depends on the dimensions of the regular channels, their symmetries and symmetry of the whole structure. Table presents several examples of the plane junctions of two circular waveguides with the ratio of their radii as 3:10. The exam-

Table. Eigenoscillations of plane junction of two circular waveguides.

Structure & symmetry	One-mode band	Eigenfrequency, κ
	$(\kappa_{TE_{11}}; \kappa_{TM_{01}}) = (0.29303; 0.60982)$	$0.60817 - i \cdot 2.5 \cdot 10^{-4}$
	$(\kappa_{TE_{21}}; \kappa_{TM_{21}}) = (0.4861; 0.81736)$	$0.81713 - i \cdot 1.8 \cdot 10^{-6}$
	$(\kappa_{TE_{11}}; \kappa_{TM_{01}}) = (0.29303; 0.38273)$	$0.38271 - i \cdot 3.7 \cdot 10^{-6}$

ples illustrate that decentralization of the plane junction leads to decreasing of the single mode band and display of new eigenoscillation near the cutoff

point of TM_{01}° . Hence the phase of the reflected vertically polarized TE_{11}° mode rotates on a lower frequency. We need to emphasize that this phenomenon also exists for an incident higher TE mode if the mode has the lowest cutoff frequency in corresponding mode symmetry group. The phase changing of the TE_{21}° mode takes place before the cutoff point of TM_{21}° in the case of an axial-symmetric junction, for instance. Furthermore, the symmetry field properties of the circular waveguide modes let conclude that if the TE_{nm}° mode ($n \neq 0$) is incident on the axial-symmetric junction, the phase changing of the reflected mode will appear near the cutoff frequency of TM_{nm}° one.

The modification of the objects dimensions gives us a possibility to control the eigenfrequencies to some extent. Fig. 2 demonstrates by the example of a junction of two rectangular waveguides that the width increasing of the smaller one leads to the movement of the eigenfrequency (as well as the phase changing frequency) to the low frequency range, whereas the eigenfrequency Q-factor (as well as the slope of the phase response) decreases. In the limit case of the E-plan step of the rectangular waveguide cross-section the eigenfrequency tends to the cutoff point of the TE_{10}^{\square} mode.

Due to small dimensions the electromagnetic field evanesces inside the output channel. It is evidently that the channel shorting on some distance from the junction does not tell on the result. It slightly changes the phase changing frequency only. Hence not only the perforated metal half-spaces or the waveguide plane junctions but also the cavities in the metal planes or the

waveguide plugs are able to provide resonance behavior of the reflected field. It is possible they can be used as elements of special resonators based on the eigenoscillations of the plane junctions or the “open aperture” screens.

3. ENHANCED TRANSMISSION AS THE RESULT OF COMMON EXCITATION OF PAIRED EIGENOSCILLATIONS

Now let us consider more complicated structures such as irises with below-cutoff holes and double periodic perforated metal screens. Due to longitudinal symmetry of these structures, we can suppose existence of pairs of the eigenoscillations that are formed by previous ones due to electromagnetic interaction by below-cutoff channels and have PMW or PEW in the transversal plane of the screen or iris symmetry. Using

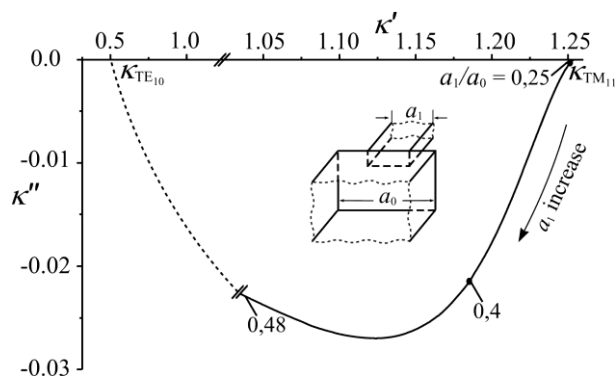


Fig. 2. Movement of eigenoscillation frequency via increase of width of smaller waveguide.

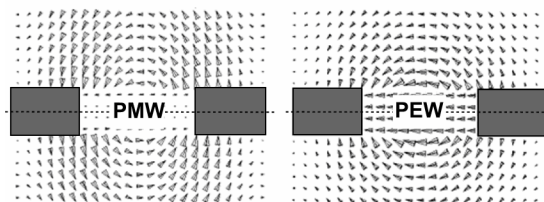


Fig. 3. Electrical field patterns of eigenoscillations of screen with narrow holes

corresponding numerical model we have found these eigenoscillations with complex-valued eigenfrequencies for a wide range of popular structures. The simplest example of field patterns are plotted in Fig. 3. As we see, the electric fields are mainly concentrated out of the holes and they are relatively weak within the regular waveguide channels.

Excitation of the revealed eigenoscillations by the incident field leads to appearance of the pair of the total transmission resonances on the frequency response (Fig. 4). This pair of the resonances is under discussion

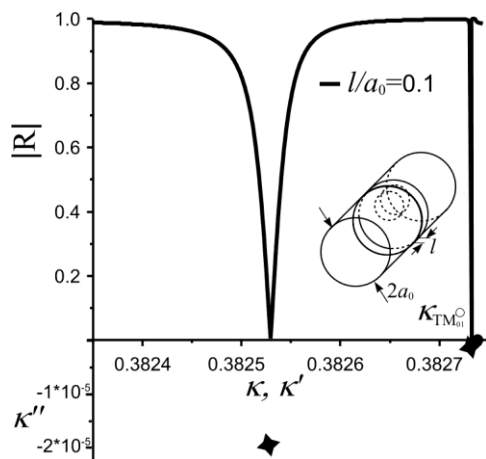


Fig. 4. Frequency response of the vertically polarized TE_{11}^O mode. The positions of two complex eigenfrequencies are marked by the diamonds.

by scientific world last ten years.

If we know the eigenoscillations, we can not only predict existence of these resonances. There is a possibility to reconstruct whole frequency curve using the approximation formula [7]. It contains the known set of the eigenfrequencies only. The comparison of numerical results obtained by MMT and the frequency response reconstruction according this formula is shown in Fig. 5.

The approximation formula has done it possible to study analytically some peculiarities of the frequency response. It has become clear that three qualitatively different situations depended on the irregularity thickness are possible:

- 1) there are two points of total transmission;
- 2) if the thickness increases, then both eigenfrequencies come to each other in exponential manner, approaching simultaneously a fixed complex point. Then there is the only one point of total transmission;
- 3) the resonance gradually vanishes with the further thickness growth.

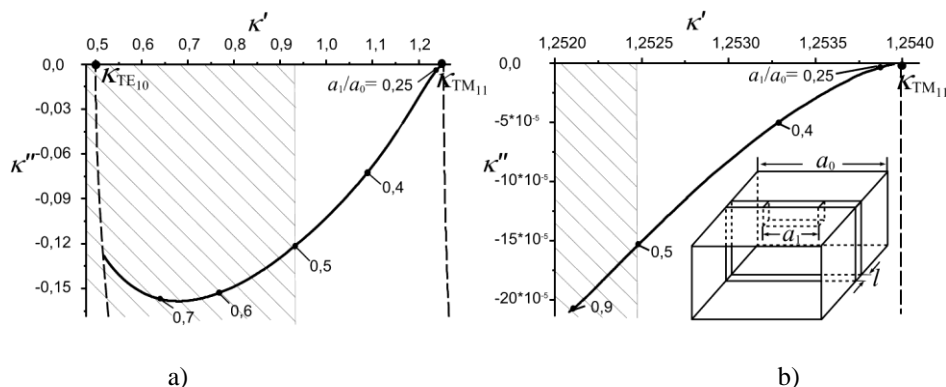


Fig. 6. Frequency change of the eigenoscillations with PMW (a) and PEW (b) in the longitudinal plane of symmetry via the slot dimension increasing. Hatched domain corresponds to resonant slots.

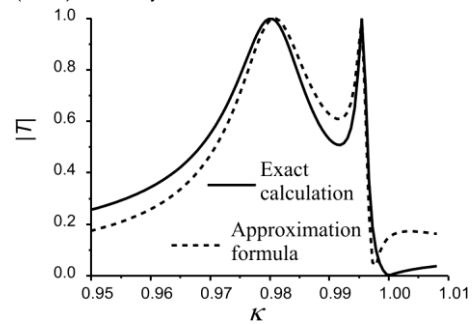


Fig. 5. Comparison of numerical results with the reconstruction of the plane wave transmission coefficient with the aid of two eigenfrequencies in the case of the screen perforated by circular holes ($b_0/a_0 = 0.83$ mm, $r/a_0 = 0.22$ mm).

The movement of the eigenfrequencies in the complex plane when the hole aperture increases from zero to the maximum size (Fig. 6) gives general idea about a possible frequency response of the irregularity. The complex-value eigenfrequency of the eigenoscillation with PMW starts from the branch point at the first highest TM mode cutoff frequency κ'' and moves to the lower frequencies with increasing κ'' . The corresponding resonance in the reflection coefficient moves to the lower frequencies gradually losing its Q-factor.

This is to emphasize when the hole becomes resonant one (as itself!), our eigenoscillation corresponds to the well-known “half-lambda” resonance of the slotted iris or perforated screen. The eigenoscillation field changes its structure in this band of geometrical parameters. It “gradually” enters into the slot with increasing of the slot dimensions. For the resonant holes the field maximum is positioned within the hole. Thus this eigenoscillation has direct relation to the well-known phenomenon of the total transmission through the resonant slots.

Unlike this, the complex frequency of another eigenoscillation does not essentially depend on the slot width. Its field pattern is also stable and is concentrated

out the hole.

Changing the slot location within a waveguide cross section gives us possibility to change the resonance frequencies as in the case of plane junctions. Unfortunately it is not possible by moving the slot location within a screen cell, because the cutoff frequency of the first highest $\text{TM}_{\pm 10}^{\text{Floquet}}$ mode is formed only by the cell period and don't depend on location of the slot.

So, there is no wonder in existence of the "enhanced transmission" from the point of view of number of resonances, their frequency locations and Q-factors, moving with geometry changing, their confluence when the screen or iris thickness increases, and other characteristic features of the frequency response topology as it is practically the same as well known "half-lambda resonance", but appeared when apertures are below-cutoff ones (see [10, 11] for details). The essential influence of the ohmic loss at optical frequency range leads, as usually, to smoothing the frequency spikes, to their frequency shifts and so on.

4. PLURALITY OF THE SCREEN MODELS, IMPLICIT EIGENOSCILLATIONS, SCREEN WITH PERIODICAL PERTURBATIONS AND NEW TYPES OF ENHANCED TRANSMISSION

In contrast to the waveguide problems the situation in the theory of periodic structures has some ambiguity. Since we can use in calculation any cell that concludes several identical elementary periods, if we consider a periodic screen. Then together with eigenoscillations corresponding to the elementary cell some other eigenoscillations appear. They are caused by the larger period and located on the real axis.

Naturally, for any computational model the scattering problem will have only one solution that don not react to these implicit eigenoscillations. Their influence will become apparent if there are some periodic perturbations of the screen increased its elementary cell. Then some of the implicit eigenoscillations will have got complex eigenfrequencies. It indicates radiation losses in the free space. Hence excitation of these eigenoscillations will have an effect on the frequency response by arising additional resonance picks. The resonances will be located in lower frequencies and their Q-factors will be determined by symmetry properties of the screen, the incident field and the eigenoscillations. If the perturbation is small, the "old" resonance effects do not practically change and we can tell about new enhanced transmission resonances located in multiple wave-lengths. Since main enhanced transmission resonance has got smaller Q-factor its influence leads to the appearance not only the new total transmission resonances but also the total reflection ones.

To illustrate the influence of these new eigenoscillations on the forming of the resonance responses let us consider the simplest example of a screen with two holes within a periodic cell (Fig. 7). We suppose the

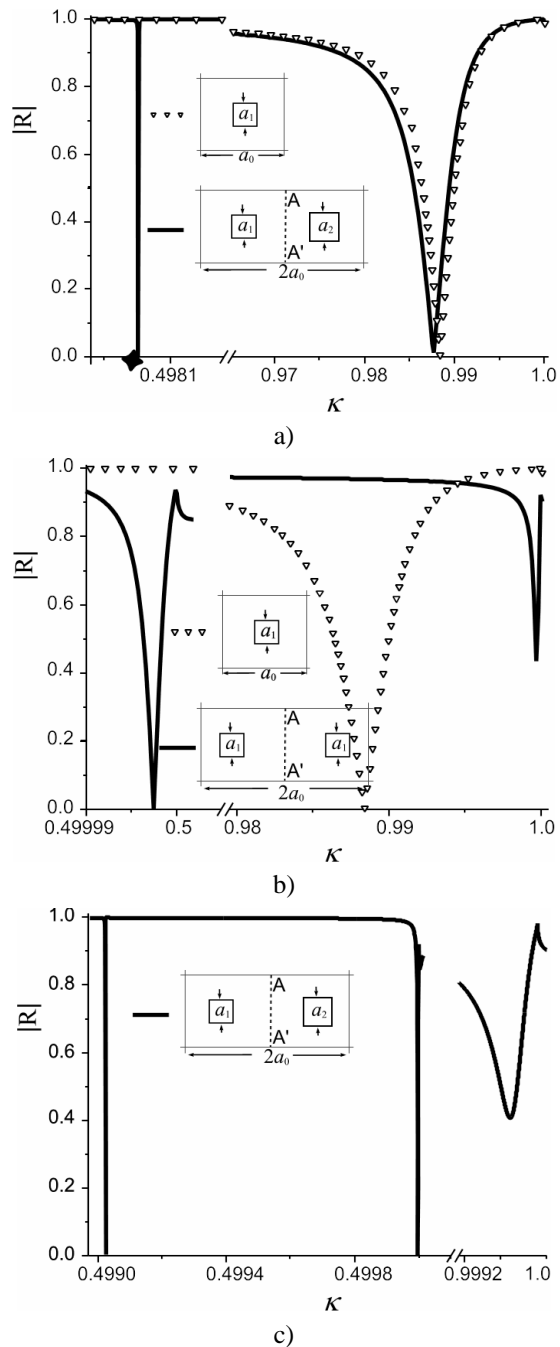


Fig. 7. Frequency response of the $\text{TM}_{00}^{\text{Floquet}}$ for several screens with periods increased by modification of element perturbation (a), space perturbation (b), both perturbation together (c). ($a_1/a_0 = 0.1396$, $a_2/a_0 = 0.1431$).

incidence of the $\text{TM}_{00}^{\text{Floquet}}$ mode polarized along the largest dimension of the cell and infinitesimal thickness of the screen. The latter lets consider only the eigenoscillations with PMW in the transversal symmetry plane of the screen. Because of the rest of the eigenoscillations have got real eigenfrequencies and don not influ-

ence on the frequency response.

Initially, in the case of uniformly distributed two equal holes the screen can be considered as a single-element one with a hole within a small period a_0 , and its single-mode band $\kappa \in (0;1)$. The enhanced transmission is observed slightly below $\kappa=1$. With changing the dimensions of one of the holes (or element perturbation) we increase the cell width twice and decrease one mode band to $\kappa \in (0;0.5)$. As it can be observed at the vicinity of $\kappa=0.5$ the frequency response acquires a new resonance of total transmission and the old enhanced transmission resonance is slightly changed (Fig. 7a). Furthermore in this case a couple of “resonance-antiresonance” appears.

The embedding of the other kind of perturbation with changing the distance between the holes (or space perturbation) results in appearance new enhanced transmission resonance too. (Fig. 7b). However there is not the total reflection resonance as in previous example.

As it is easy to see at the space perturbation the structure symmetry in relation to AA' is saved, and at the element perturbation the structure symmetry is violated. In our opinion absence of the symmetry lets interaction between the eigenoscillations with different “quasi-symmetry” properties and we see additional reflection resonances.

At last in the case of simultaneous space and element perturbations (Fig. 7c) two new resonances of the total transmission and the total reflection between them appeared on the frequency response near $\kappa=0.5$. The reason of such behavior is excitation a pair of the eigenoscillations with complex frequencies and different symmetry properties in relation to AA', whereas in the previous examples one of the oscillations had got a real eigenfrequency. Existence of the pair resonances of total transmission corresponds to the theory published in [8, 9] for structures with several resonance holes in full manner. It again emphasizes the fundamental character of the enhanced transmission phenomenon and common nature of all resonance effects.

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