№ 1 (s) (3), 2020

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INFLUENCE OF THE NUMERICAL METHOD SAMPLING ON THE DIGITAL PID-CONTROLLER BEHAVIOR

https://doi.org/10.23939/sepes2020.01s.035

Modern digital control systems make it possible to implement quite complex control strategy, the complexity of which is limited by the hardware capabilities, the available software and the implemented control algorithm. An important component of such an algorithm is the numerical method, which allows to discretize the control rule on the basis of a continuous prototype. Such application example is the classic PID controller, which has become the basis for the development of digital control systems. Two mathematical operations are performed in such a controller: integration and differentiation, which in a digital system obtain discrete equivalents in the corresponding recurrent equations form.

The article considers a digital PID controller as a digital filter, which with the use of the frequency characteristics (Bode diagrams) allowed to determine its most "narrow" place – the high-frequency diagram region that correspond to the differentiating part of the controller. This made it possible to focus the research on the practical implementation of the differential part of the digital PID controller. It is shown that the traditional way of the differential operation performing by the simple method of finite differences has some of disadvantages that make it fundamentally impracticable, as illustrated by the corresponding graphs.

To eliminate the limitations of the traditional differential operation by the finite difference method, two variants of structural schemes of a real differentiator are proposed. The first variant of the real differentiator proposed to build on the structural scheme in feedback with the integrator. The second variant of the real differentiator is proposed to build according to the structural scheme with a parallel connection of the proportional and the first order blocks. The application of explicit numerical Adams integrators (also known as Adams-Bashforth rule) from the first to the fourth order under the conditions of physical realization is proposed to perform sampling. A study of both their frequency characteristics and their behavior performed on a noisy signal for these structural schemes.

All research in the article was conducted using the Control System Toolbox library of the mathematical application MATLAB. It is shown that the use of the proposed real differentiation methods allows simple and efficient implementation of digital PID controllers.

Keywords: differentiation, digital PID controller, digital control systems, noises, numerical integration, numerical methods, real-time systems.

Introduction

The using modern digital technology in automatic control systems has made it possible to implement relatively complex regulation strategy. Such systems consist the built-in algorithms and programs, part of which is the use of certain numerical methods. For example, it is claimed [1] that up to 90% of all industrial controllers are PID controllers, where integration and differentiation operations are a controller part.

Digital control systems are real-time systems, so the use of numerical integrators in such systems is limited to explicit multi-step methods with a constant integration step due to the technical implementation [2], which do not provide in the process the next sample (not performed yet) of the measured coordinate (for the implicit method). Therefore, both implicit multi-step methods and all single-step methods that require intermediate values of the derivative for the fractional step are excluded from the studies due to the inability to get information on the integrated function behavior in the interval between samples of the operational signal.

Note that:

- on the one hand, it is known from applied mathematics about the numerical methods effects for solving ordinary differential equations on the behavior of the developed mathematical model – this is corrected by choosing the appropriate integration step (in modern algorithms this is achieved by automatic selection step);
- on the other hand, in real-time digital systems, the sampling period (integration step) is a fixed value and determined during the design of such a system, which leaves a certain imprint on its behavior [2] and limits the further adjusting possibility of the digital system behavior.

The use of PID-controller digital models, which are obtained using various classical discretization methods, allows using the control theory methods (for discrete systems) to analyze and search for causes of differences in the digital controllers behavior, considering them as digital filters [2, 3]. In this case, the use of control theory methods, in particular, methods of analysis and synthesis of discrete systems using the z-transform, has advantages that are associated with correspondence of characteristics in the frequency and time domains for linear and linearized systems [3, 4]. This method of analysis using appropriate mathematical applications [5, 6] allows to study the frequency characteristics of digital control systems.

In the plot of Fig. 1 shows the frequency characteristics (Bode diagrams) of a continuous and some implementations of a digital PID controller obtained by the Control System Toolbox of the MATLAB application [5, 6] with the following parameters: proportional gain $K_p = 1$, integrator time constant $T_i = 10$ c, differentiator time constant $T_d = 0.1$ c, the time constant of the real differentiator filter $T_d/100$. The frequency characteristics of digital implementations are built for the sampling step $h = 25$ ms and three ways to sample a continuous controller: classical z-transform (marked on the plot by the letter **z**), Tustin method (***Tustin***) and a method for matching the zeros/poles of the transfer functions (***matched***). This analysis makes it possible to determine the critical frequency range in relation to the numerical methods used to discretization the continuous system.

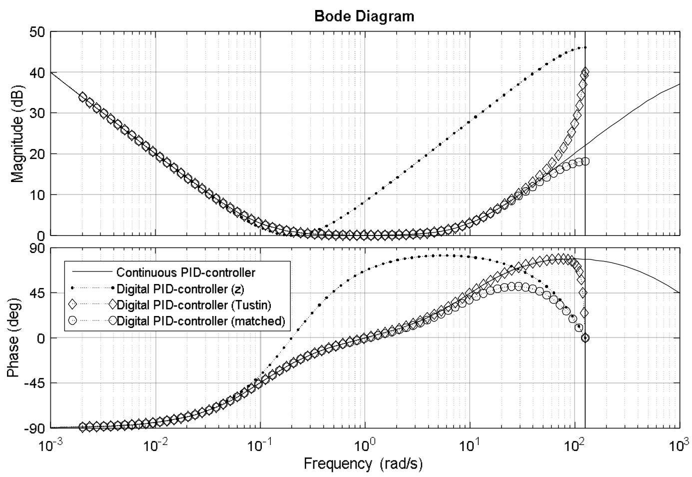

Fig. 1. Frequency response of a PID controller: a continuous controller and three types of digital realizations for a sampling step h=25 ms

Graph analysis of Fig. 1 confirms that the obtained frequency characteristics of the considered PID controller's digital implementations indicate the determining influence of sampling processes on the high-frequency part of the frequency characteristics, which corresponds to the differentiating part. Thus, in the case of a PID controller, it is rational to study the sampling processes influence of a continuous system on the differentiating part of the digital PID controller.

Problem analysis, recent research and publications

The implementation of the operation of digital signal differentiation in control systems is one of the problems of the digital systems development, which is associated with the presence of various types of high-frequency interference and noise in signal circuits, which are not always effectively eliminated by filtering. In particular, these may be the results of level quantization due to the route through an analog-to-digital converter (ADC). It should also be borne in mind that the mathematical software used in digital control systems performs finite accuracy calculations, which is often determined by the used hardware and software.

The classical and most frequently used implementation of a digital differentiator according to the known algorithm of finite differences of the first order $\frac{dx}{dt} \approx \frac{x_i - x_{i-1}}{h}$ [3, 4] produces a satisfactory derivative signal for an ideal noise-free signal [5]. Unfortunately, there are always noises and interferences in the operating signals in reality, the effect of which increases during decreasing sampling step ***h*** (see the formula above). Example of unsatisfactory differentiation of a sinusoidal signal with a noise level 1% (*note that this level of interference is not even visible on the plot*) for two sampling times 1 and 0.1 ms shown on Fig. 2 (*pay attention to the output value scale*). For this example, MATLAB generated a uniformly distributed random signal ("white noise"). It is clear that the use of such a result of digital differentiation will lead to unsatisfactory operation of the control system as a whole.

A similar result is given by the route of the signal through the ADC – quantization by level ("digitization"). Even a small change in the signal in the lower bit due to the small size of the sampling time (*again, see the formula above*) leads to a significant value of the output signal, which is shown in the case of a hypothetical 4-bit ADC (*example for illustration only*) in Fig. 3 and for a typical industrial 12-bit ADC (Fig. 4) in the case of an input signal of 1 V and two different sampling steps 1 i 0.1 ms (again, pay attention to the scale).

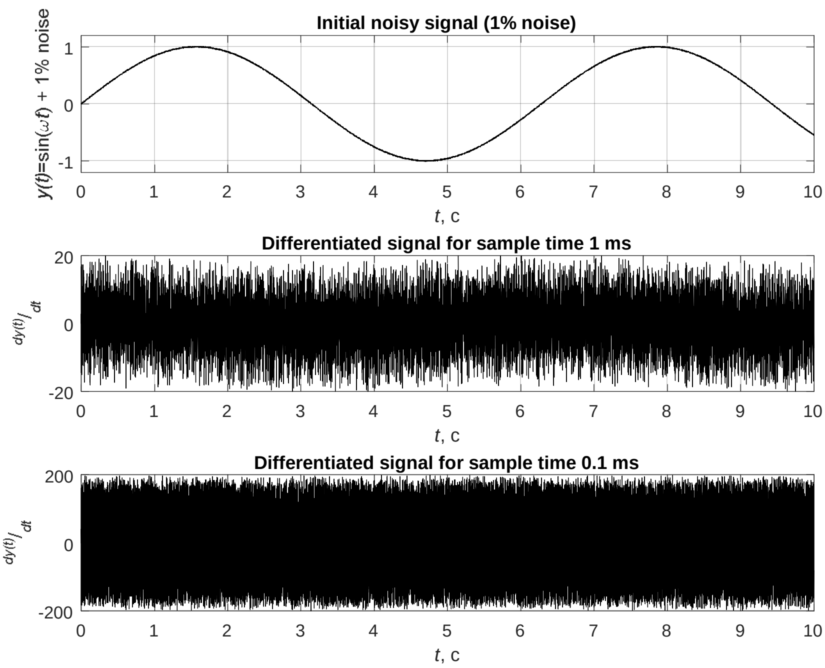

Fig. 2. The result of digital differentiation for different sampling periods by the first order method of finite differences

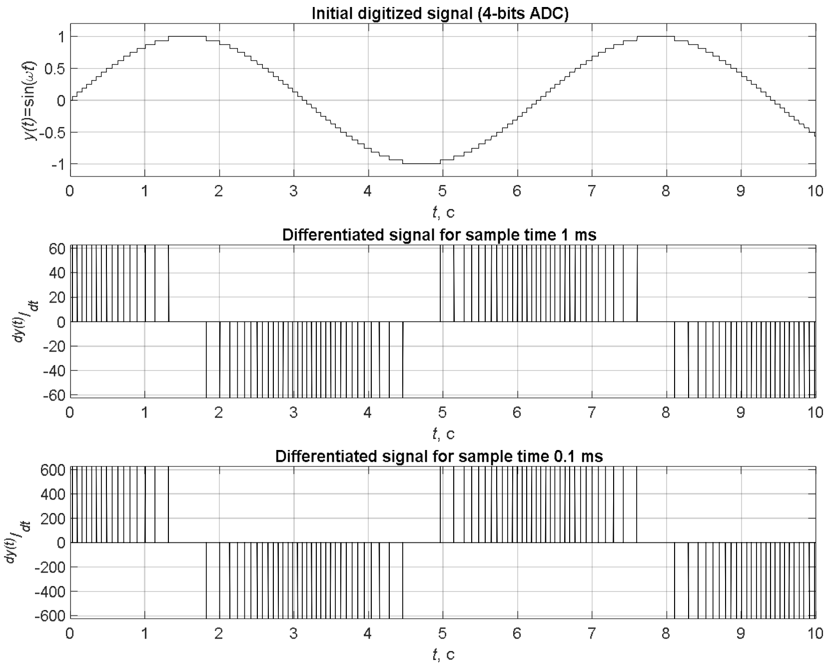

Fig. 3. The output signal of the digital differentiator according to the algorithm of finite differences after the route of the sine signal through a hypothetical 4-bit ADC

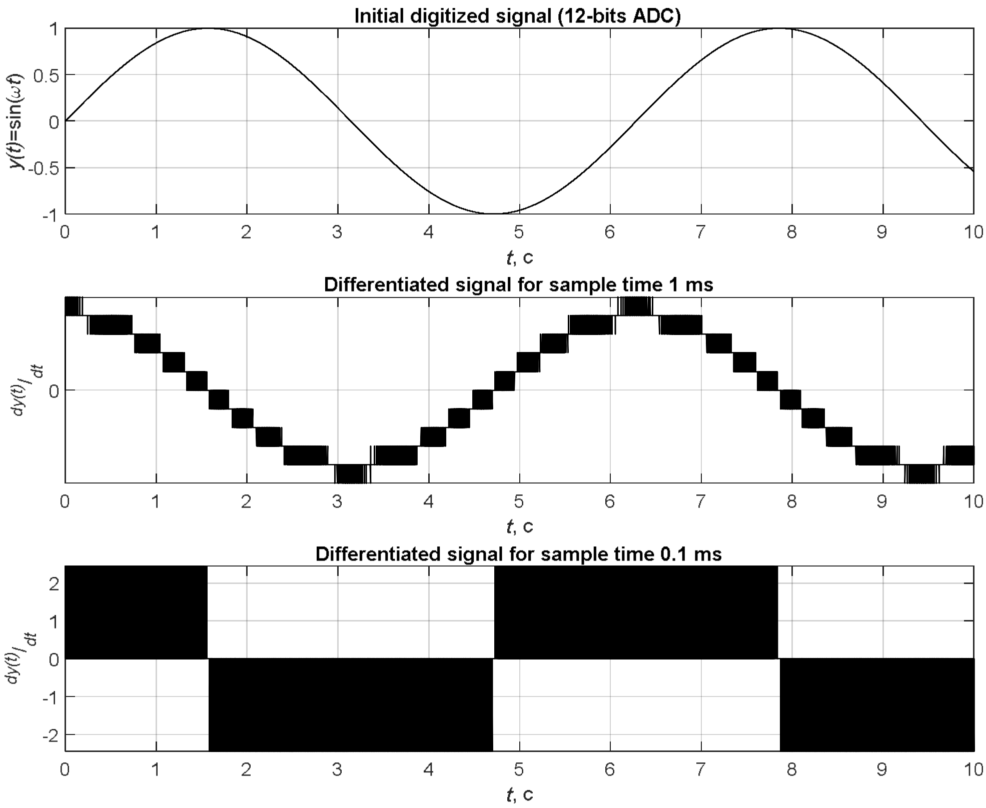

Fig. 4. The output signal of the digital differentiator according to the algorithm of finite differences after the route of the sine signal through a 12-bit ADC

It is clear that the "digitization" of real signals with noise and interference in digital control systems does not improve the situation, especially since all arithmetic operations in a digital control system occur, as we recall, with limited precision, which is determined, as mentioned above, hardware and software.

Thus, in the digital PID controller from the point of view of practical implementation the most problematic is the differentiating part.

The aim of the research is to study the influence of numerical integrators [7] in the case of practical implementation of digital control system, in particular, the operation of digital differentiation.

Main material

To reduce the impact of high-frequency interference and noise in the case of PID-regulator can be used by a real differentiator [8], one of the options for which can be implemented according to the block diagram shown in Fig. 5. The transfer function of such a real differentiator is found by simple transformations:

$$W_d(s) = \frac{K}{1 + \frac{1}{T \cdot s} K} = \frac{K \cdot T \cdot s}{T \cdot s + K} = \frac{T \cdot s}{\frac{T}{K} s + 1}.$$

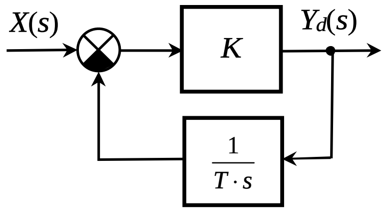

Fig. 5. Block diagram of the real differentiator

In the case of digital implementation of the PID controller according to this digital differentiator variant, it is necessary to use the appropriate digital integrator. The physical implementation of the numerical integration operation in the real-time control system is possible only for using explicit multi-step formulas, which is shown in [2]. At present, the most effective of these are still Adams' formulas [7].

The process of obtaining a discrete transfer function of a digital integrator for a real-time system and fixed integration step h shown by the example of the third order explicit Adams formula $y_{i+1} = y_i + \frac{h}{12}\left(23x_i - 16x_{i-1} + 5x_{i-2}\right)$ [7] using method [2, 3]. Taking into account the theorem [3] $\begin{cases} y_i z = y_{i+1}; \\ y_i z^{-1} = y_{i-1}; \end{cases}$ a discrete transfer function of such integrator obtained $W_3(z) = \frac{\frac{h}{12T_i}\left(23 - 16z^{-1} + 5z^{-2}\right)}{z-1} = \frac{\frac{h}{12T_i}\left(23z^2 - 16z + 5\right)}{z^3 - z^2}$ and used for next analysis. Applying the known methods [2, 4, 9], we find discrete transfer functions for other 1^{st}–4th order digital integrators, which are obtained by discretizing the operation of continuous integration by explicit Adams formulas (their discrete transfer functions are summarized in Table 1). Given the results obtained in [10, 11], it made no sense to consider discrete approximations of higher orders.

Table 1

Discrete transfer functions of numerical integration operators

Integrator order	Numerical integrator	Digital transfer function
1	$y_{i+1} = y_i + hx_i$	$W_1(z) = \frac{h/T_i}{z-1}$
2	$y_{i+1} = y_i + \frac{h}{2}\left(3x_i - x_{i-1}\right)$	$W_2(z) = \frac{\frac{h}{2T_i}(3z-1)}{z^2 - z}$
3	$y_{i+1} = y_i + \frac{h}{12}\left(23x_i - 16x_{i-1} + 5x_{i-2}\right)$	$W_3(z) = \frac{\frac{h}{12T_i}\left(23z^2 - 16z + 5\right)}{z^3 - z^2}$
4	$y_{i+1} = y_i + \frac{h}{24}\left(55x_{ii} - 59x_{i-1} + 37x_{i-2} - 9x_{i-3}\right)$	$W_4(z) = \frac{\frac{h}{24T_i}\left(55z^3 - 59z^2 + 37z - 9\right)}{z^4 - z^2}$

The obtained discrete transfer functions were used to implement the model of digital PID controller by means of Control System Toolbox (MATLAB) for further analysis of frequency characteristics and research of the output signal in the presence of noise and interference of the differentiating part of the controller (differentiator) using structure Fig. 5. The obtained frequency characteristics of the digital PID controller implementations using a differentiator (Fig. 5) are shown in Fig. 6 for 1–4th orders explicit Adams formulas and sampling time h =25 ms. A somewhat unexpected result is a better approximation to the frequency characteristics of the continuous PID controller the frequency characteristics of digital implementations, which are obtained using low-order integrators.

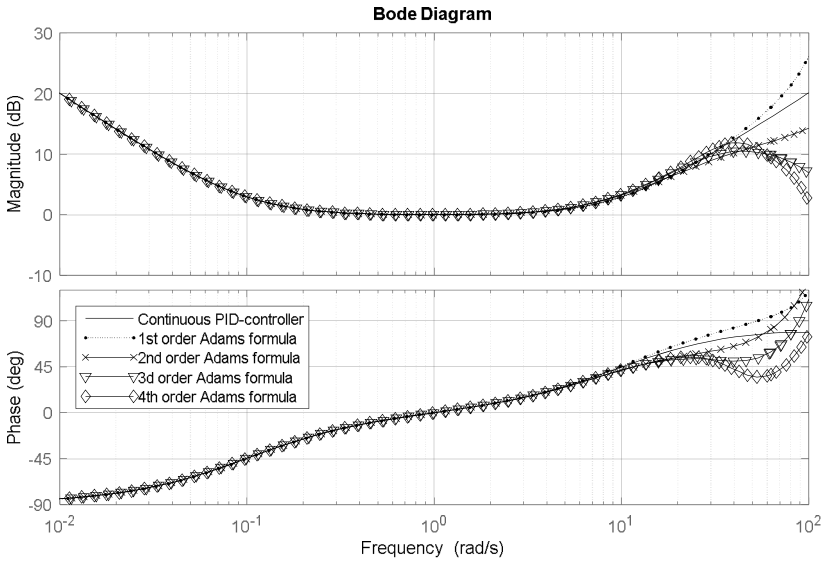

Fig. 6. Frequency responses of the PID controller: a continuous controller and its digital implementations using 1-4th orders explicit Adams formulas for a sampling time 25 ms

Checking the effectiveness of digital differentiation using structure shown in Fig. 5 is illustrated by the graphs of Figs. 7 and Fig. 8 for two cases of application of explicit Adams formulas – 1st (Fig. 7) and 4th (Fig. 8) orders and sampling times 1 and 0.1 ms. Of course, this method of numerical differentiation is much better than the classical method of finite differences, the result of which has already been presented in Fig. 2.

Regarding the efficiency of differentiation, it should be noted that there is practically no difference in the results of this operation, regardless of the applied digital integrator order, as can be seen from the plots of Figs. 7 and 8.

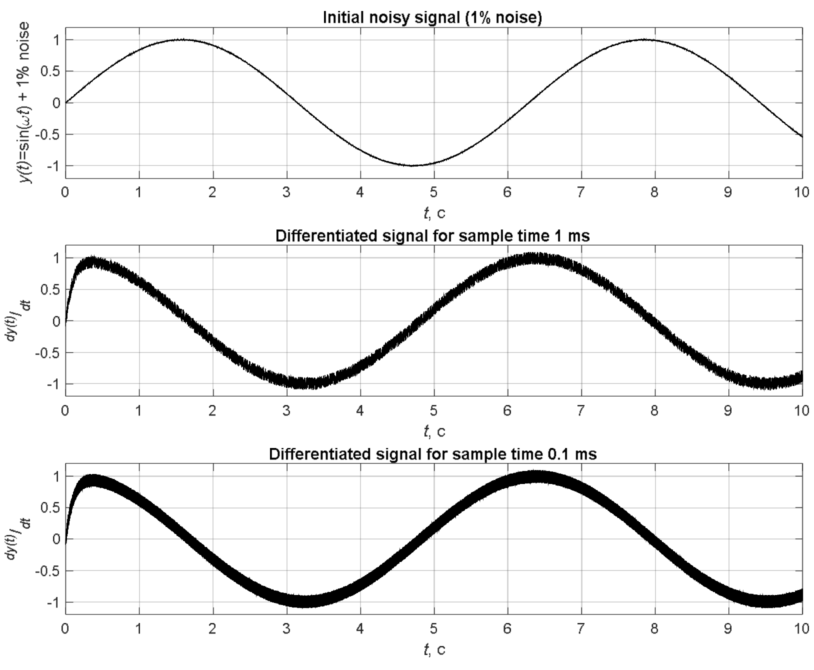

Fig. 7. The result of digital differentiation by the structure of Fig. 5 for different sampling periods and first order integrator

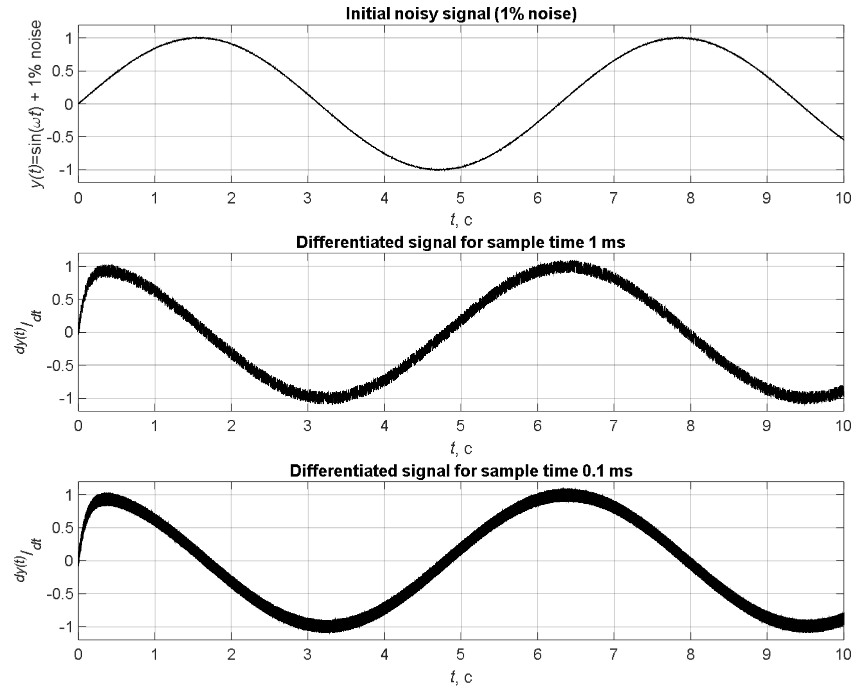

Fig. 8. The result of digital differentiation by the structure of Fig. 5 for different sampling periods and fourth order integrator

Another way to reduce the effect of high-frequency interference and disturbance is filtering. An alternative structure for the real differentiator practical implementation is shown in Fig. 9, and its transfer function is found by simple transformations:

$$W_d(s) = K - \frac{K}{T \cdot s + 1} = \frac{K \cdot T \cdot s}{T \cdot s + 1}.$$

The process to obtain a discrete model in the recurrent equation form for a real-time system for a fixed integration step h is shown by the example of an third order explicit Adams formula $y_{i+1} = y_i + \frac{h}{12}\left(23x_i - 16x_{i-1} + 5x_{i-2}\right)$ [7] using method [2, 3]. After substitution in the numerical integrator formula derivative value $y' = \frac{K \cdot x - y}{T}$ from the ordinary differential equation $T \cdot y' + y = K \cdot x$ and simple algebraic simplifications we obtain a recurrent modeling formula

$$y_{i+1} = \left(1 - \frac{23h}{12T}\right) y_i + \frac{16h}{12T} y_{i-1} - \frac{5h}{12T} y_{i-2} + \frac{h}{12T}(23x_i - 16x_{i-1} + 5x_{i-2}).$$

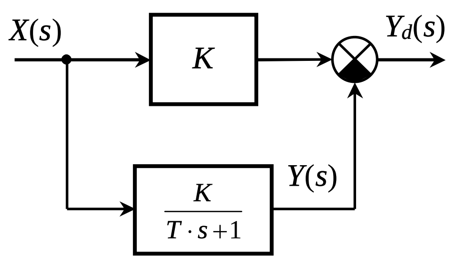

Fig. 9. Block diagram of the proposed implementation of the digital differentiator

Taking into account the theorem [3] $\begin{cases} y_i z = y_{i+1}; \\ y_i z^{-1} = y_{i-1}; \end{cases}$ a discrete transfer function of such a first-order block is obtained $W_3(z) = \frac{h\left(23z^2 - 16z + 5\right)}{12Tz^3 - (12T - 23h)z^2 - 16hz + 5h}$, which is used in further analysis. Using the mentioned methods [2, 4, 9], discrete first-order transfer functions were constructed for 1st-4th order explicit digital Adams integrators (their discrete transfer functions placed in Table 2). Again, given the results obtained in [10, 11], it made no sense to consider higher orders discrete approximations.

Table 2

Discrete transfer functions of the first-order block digital models

Integrator order	Numerical integrator	Digital transfer function
1	$y_{i+1} = y_i + hx_i$	$W_1(z) = \dfrac{h}{Tz - (T-h)}$
2	$y_{i+1} = y_i + \dfrac{h}{2}\left(3x_i - x_{i-1}\right)$	$W_2(z) = \dfrac{h(3z-1)}{2Tz^2 - (2T-3h)z - h}$
3	$y_{i+1} = y_i + \dfrac{h}{12}\left(23x_i - 16x_{i-1} + 5x_{i-2}\right)$	$W_3(z) = \dfrac{h(23z^2 - 16z + 5)}{12Tz^3 - (12T-23h)z^2 - 16hz + 5h}$
4	$y_{i+1} = y_i + \dfrac{h}{24}\left(55x_{ii} - 59x_{i-1} + 37x_{i-2} - 9x_{i-3}\right)$	$W_4(z) = \dfrac{h(55z^3 - 59z^2 + 37z - 9)}{24Tz^4 - (24T-55h)z^3 - 59hz^2 + 37hz - 9h}$

The obtained discrete transfer functions were used to implement the model of digital PID controller by means of Control System Toolbox (MATLAB) for further analysis of frequency characteristics and study of the output signal form in the presence of noise and interference of the differentiating part of the controller (differentiator) using structure Fig. 9. The obtained frequency characteristics of the digital PID controller implementations using a differentiator (Fig. 9) are shown in Fig. 10 for the 1-4th orders explicit Adams formulas and a sampling time 25 ms. Again, a somewhat unexpected result was a better approximation to the frequency characteristics of the continuous PID controller (*prototype*) frequency characteristics of digital implementations using low-order integrators.

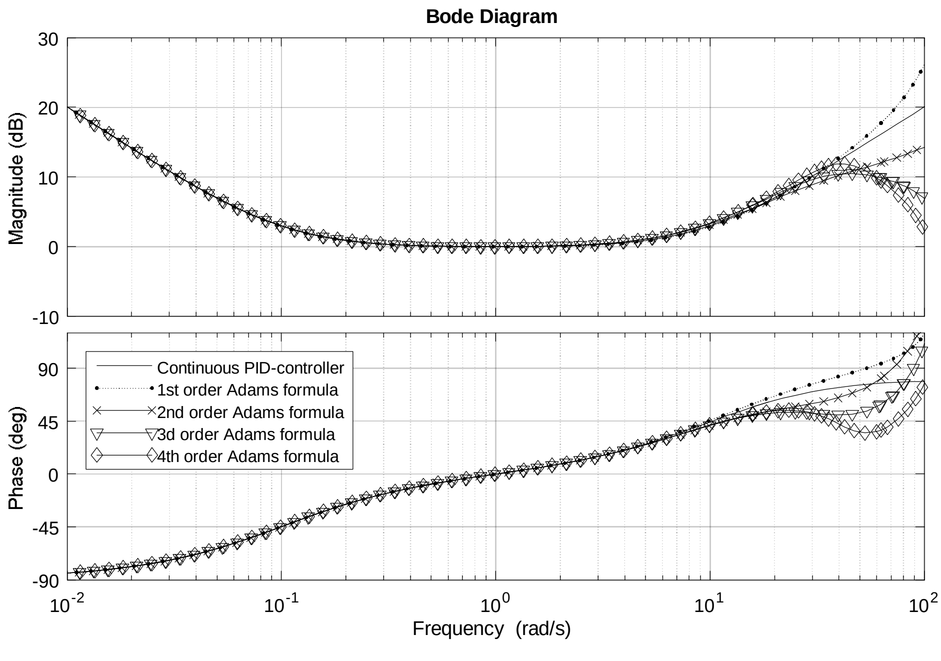

Fig. 10. Frequency responses of the PID controller: a continuous controller and its digital implementations using 1-4th orders explicit Adams formulas with a sampling time 25 ms

Checking the effectiveness of digital differentiation using the structure in Fig. 9 is illustrated by the plots of Figs. 11 and Fig. 12 for two variants of explicit Adams formulas – 1st (Fig. 11) and 4th (Fig. 12) orders and sampling time 1 and 0.1 ms. Of course, this numerical differentiation method is also much better than the classical method of finite differences, the result of which is contained in Fig. 2, and practically matches in efficiency with the previous method (Fig. 5).

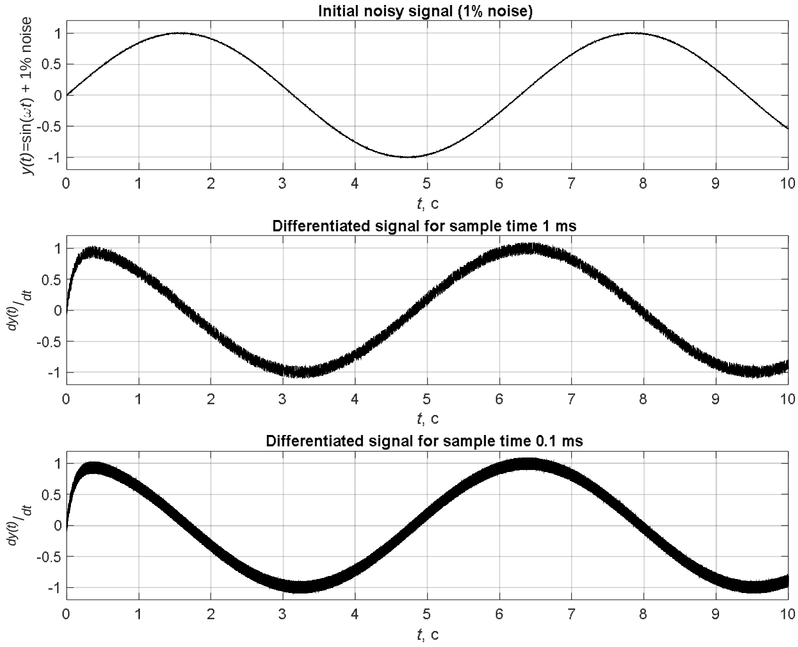

Fig. 11. The result of digital differentiation by the method of Fig. 9 for different sampling times and first order integrator

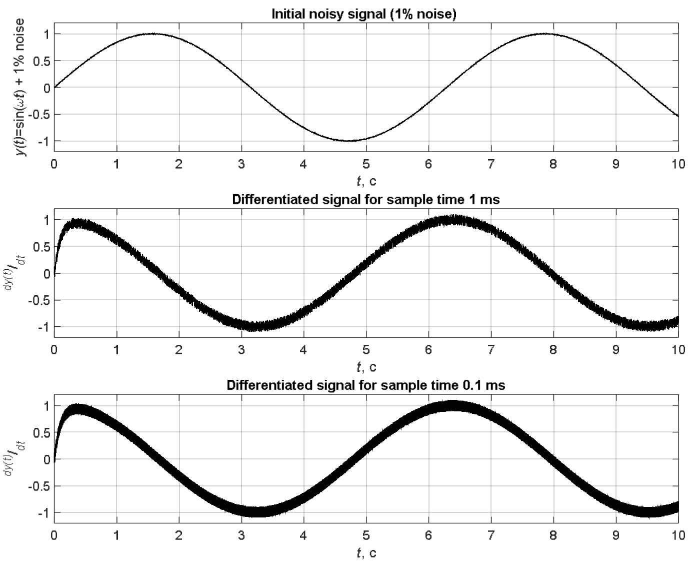

Fig. 12. The result of digital differentiation by the method of Fig. 9 for different sampling times and fourth order integrator

Regarding the differentiation efficiency, again, it should be noted practically no difference in the results of the operation, regardless of the digital integrator order, as can be seen from the plots of Figs. 11 and Fig. 12. Both types of the proposed digital differentiators implementation, as shown by research (plots because of space deficiency are not given), are effective for a wide range of input signals and are suitable for a wide range of sampling times.

Conclusions

Analysis of the research results showed:

- Traditional methods of a digital differentiator implementing by the finite difference method can be used only with certain restrictions on the type of signal and sampling time and require sufficient filtering of the input signal.
- Proposed in the article digital differentiator implementations provide sufficient differentiation accuracy for different signal types and a wide range of sampling times.

- For the practical digital differentiators' implementation according to the proposed structures, the use of low-order digital integrators is sufficient, as it is confirmed that there is no advantage of using high-order integrators. This makes it possible to simplify the digital differentiation computations and, accordingly, simplify the control algorithm and to use even low-power controllers.

Prospects for further research

As the frequency responses analysis showed, the use of numerical integrators in digital systems makes additional changes of amplitude and phase in the resulting frequency responses of the digital controller. The value of such amplitude and phase changes depends on both the applied sampling time and the numerical method order. This numerical methods' influence on frequency responses requires further research given the prospects of its use in digital control systems.

References

1. *Michael A. Johnson and Mohammad H. Moradi (Editors). PID Control: New Identification and Design Methods. © Springer-Verlag London Limited, 2005. 544 p. [ISBN-10: 1-85233-702-8; ISBN-13: 978-1-85233-702-5]*
2. *Moroz V. Chyslovi intehratory v tsyfrovykh systemakh keruvannya. Visnyk Natsionalʹnoho universytetu "Lʹvivsʹka politekhnika" // Elektroenerhetychni ta elektromekhanichni systemy". 2006. № 563. S. 99–104. (Ukr)*
3. *Elijah I. Jury. Theory and Application of the Z-Transform Method. Krieger Pub Co, 1973. [ISBN 0-88275-122-0]*
4. *Katsuhiko Ogata. Discrete-Time Control Systems, 2nd edition. Published by Pearson, 1995. [ISBN-13: 9780130342812]*
5. *MATLAB Environment. © 1994-2020 The MathWorks, Inc. URL: https://www.mathworks.com/products/matlab.html*
6. *Control System Toolbox: Design and analyze control systems. © 1994-2020 The MathWorks, Inc. URL: https://www.mathworks.com/products/control.html?s_tid=srchtitle*
7. *E. Hairer, S. Nørsett, G. Wanner. Solving Ordinary Differential Equations I: Nonstiff Problems. 2nd Edition. Springer, 2008. [ISBN 978-3-540-56670-0]*
8. *V. Moroz, V. Oksentyuk, I. Snitkov. Realizatsiya operatsiyi dyferentsiyuvannya u mikrokontrolerakh. Matematychne ta kompʹyuterne modelyuvannya. Seriya: Tekhnichni nauky : zb. nauk. pratsʹ. Instytut kibernetyky im. V. M. Hlushkova NAN Ukrayiny, Kam"yanetsʹ-Podilʹsʹkyy nats. universytet im. I. Ohiyenka. Kam"yanetsʹ-Podilʹsʹkyy : Kam'yanetsʹ-Podilʹsʹkyy natsionalʹnyy universytet im. I. Ohiyenka, 2010. Vyp. 3. [232 s.]. S. 154–159. (Ukr)*
9. *Moroz V. Pohlyad inzhenera-elektryka na chyslovi metody rozv'yazuvannya zvychaynykh dyferentsialʹnykh rivnyanʹ. Visnyk Natsionalʹnoho universytetu "Lʹvivsʹka politekhnika" // Elektroenerhetychni ta elektromekhanichni systemy. 2003. No. 485. S. 208–213. (Ukr)*
10. *Moroz V. Analiz ratsionalʹnoho poryadku aproksymatsiyi dlya vidnovlennya informatsiyi za yiyi dyskretnymy vidlikamy. RIU (Radioelektronika. Informatyka. Upravlinnya). 2008. No. 1 (19). S. 74–78. (Ukr)*
11. *Cleve Moler. Numerical Computing with MATLAB. Copyright 2004, Cleve B. Moler, MathWorks. SIAM, 2004. 336 Pp. ISBN: 0-89871-560-1.*

Список використаних джерел

1. *Michael A. Johnson and Mohammad H. Moradi (Editors). PID Control: New Identification and Design Methods. © Springer-Verlag London Limited, 2005. 544 p. [ISBN-10: 1-85233-702-8; ISBN-13: 978-1-85233-702-5]*
2. *Мороз В. Числові інтегратори в цифрових системах керування // Вісник Національного університету "Львівська політехніка" "Електроенергетичні та електромеханічні системи". 2006. № 563. С. 99-104.*
3. *Elijah I. Jury. Theory and Application of the Z-Transform Method. Krieger Pub Co, 1973. [ISBN 0-88275-122-0]*
4. *Katsuhiko Ogata. Discrete-Time Control Systems, 2nd edition. Published by Pearson, 1995. [ISBN-13: 9780130342812]*
5. *MATLAB Environment. © 1994-2020 The MathWorks, Inc. URL: https://www.mathworks.com/products/matlab.html*
6. *Control System Toolbox: Design and analyze control systems. © 1994-2020 The MathWorks, Inc. URL: https://www.mathworks.com/products/control.html?s_tid=srchtitle*
7. *E. Hairer, S. Nørsett, G. Wanner. Solving Ordinary Differential Equations I: Nonstiff Problems. 2nd Edition. Springer, 2008. [ISBN 978-3-540-56670-0]*
8. *Мороз В. Реалізація операції диференціювання у мікроконтролерах / В. Мороз, В. Оксентюк, І. Снітков // Математичне та комп'ютерне моделювання. Серія: Технічні науки : зб. наук. праць / Інститут кібернетики імені*

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9. Мороз В. Погляд інженера-електрика на числові методи розв'язування звичайних диференціальних рівнянь // Вісник Національного університету "Львівська політехніка" // Електроенергетичні та електромеханічні системи. 2003. № 485. С. 208–213.

10. Мороз В. Аналіз раціонального порядку апроксимації для відновлення інформації за її дискретними відліками / В. Мороз // РІУ (Радіоелектроніка. Інформатика. Управління). 2008. № 1 (19). С. 74–78.

11. Cleve Moler. Numerical Computing with MATLAB. Copyright 2004, Cleve B. Moler, MathWorks. SIAM, 2004. 336 Pp. ISBN: 0-89871-560-1.

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ВПЛИВ ЧИСЛОВОГО МЕТОДУ НА ПОВЕДІНКУ ЦИФРОВОГО ПІД-РЕГУЛЯТОРА

Сучасні цифрові системи керування дають змогу реалізовувати достатньо складні закони регулювання, складність яких обмежується можливостями апаратної частини, наявним програмним забезпеченням і закладеним алгоритмом регулювання. Важливою складовою такого алгоритму є застосований числовий метод, який дає змогу дискретизувати закон керування на підставі неперервного прототипу. Прикладом такого застосування є класичний ПІД-регулятор, який став базою для розробки цифрових аналогів. У такому регуляторі здійснюються дві математичні операції: інтегрування та диференціювання, які в цифровій системі отримують дискретні еквіваленти у вигляді відповідних рекурентних рівнянь.

У статті розглянуто цифровий ПІД-регулятор як цифровий фільтр, що з використанням апарату частотних характеристик дало змогу визначити найбільш "вузьке" його місце – високочастотну область, за яку відповідає саме диференціююча частина регулятора. Це дало змогу зосередити основну увагу досліджень на практичній реалізації диференціюючої частини цифрового ПІД-регулятора. Показано, що традиційний спосіб виконання операції диференціювання простим методом скінчених різниць має низку недоліків, які роблять його практично непрацездатним, що й проілюстровано відповідними графіками.

Для усунення недоліків традиційної операції диференціювання методом скінчених різниць запропоновано два варіанти структурних схем реального диференціатора. Перший варіант реального диференціатора пропоновано будувати за структурною схемою з інтегратором у зворотному зв'язку. Другий варіант реального диференціатора запропоновано здійснити за структурною схемою з паралельним з'єднанням пропорційного блока та ланки першого порядку. Для виконання дискретизації запропоноване застосування з умов фізичної реалізації явних числових інтеграторів Адамса від першого до четвертого порядків. Для вказаних структурних схем проведено дослідження як їхніх частотних характеристик, так і їх поведінки на зашумленому сигналі.

Усі дослідження в статті проведено з використанням бібліотеки Control System Toolbox математичного застосунку MATLAB. Показано, що використання пропонованих способів реального диференціювання дає змогу простої та працездатної реалізації цифрових ПІД-регуляторів.

Ключові слова: диференціювання, завади, системи реального часу, цифровий ПІД-регулятор, цифрові системи керування, числове інтегрування, числові методи.