NEW METHOD OF GRAPHIC REPRESENTATION OF ELECTROMAGNETIC FIELDS STRUCTURE

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Abstract

A method of visual graphic representation of an electromagnetic field structure, which consists in rectangular grid mapping of data calculated in a polar coordinate system, is proposed herein. As a result of the Maxwell differential equation solution, a formula is determined which allows mapping force lines of electrical component of the field radiated by a linear array in any space domain.

Keywords: Electromagnetic radiation; electromagnetic fields; Poynting vector; electromagnetic energy flow; Hertzian dipole.

1. INTRODUCTION

Modern computer technology allows solving almost any task in terms of calculation of electromagnetic fields radiated by complex antenna systems. When analyzing and interpreting calculation data, visual representation is of great importance. Electrodynamics experts normally use electromagnetic fields graphic representations designed as force lines of electric (**E**) and magnetic (**H**) components.

In order to describe radiation fields, it is convenient to use a spherical coordinate system (**r, θ, φ**). Cylindrical coordinates (**r, z**) are generally used to reflect the structure of linear antennas radiation fields in azimuth cross-section (constant **φ**). Such coordinates system is essentially a rectangular one. A typical example of its application [1] is shown in Fig.1.

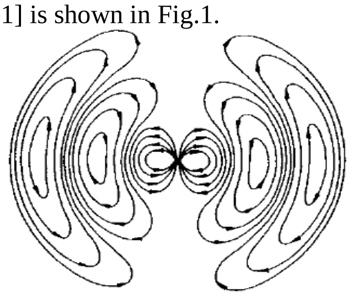

Fig. 1. Force lines pattern of the **E** field of Hertzian dipole in coordinates (**r, z**).

The key shortcoming of such representation is impossibility to image the structure of radiated fields, especially within sectors positioned at a material distance from the source.

This article provides a way to represent the structure of the radiated field force lines in any remote space domain with regard to the radiation source.

2. THEORETICAL BACKGROUND

In order to make radiation fields representation more illustrative, the following is suggested:

1. For field calculation, the spherical coordinate system (**r, θ**) variables should be used, and obtained data should be represented within a rectangular grid. To this end, r coordinates shall be plotted along axis ***X*** and θ angle values shall be plotted along axis ***Y***;
2. In order to design a pattern of linear antennas force lines, relatively simple mathematical expressions shall be used. Rigorous deduction of such is presented below.

The structure of electromagnetic field radiated by an elementary Hertzian radiator located along axis ***Z*** is shown below. This field is characterized by two electric field components, i.e. E_r, E_θ and one magnetic field component, i.e. H_φ. The H_φ component is defined by the following formula:

$$\dot{H}_\varphi = \frac{\dot{I}_m l}{4\pi}\left(\frac{1}{r^2}+\frac{ik}{r}\right)e^{-ikr}e^{i\omega t}\sin\theta . \qquad (1)$$

Electric field force lines in each domain point should comply with the following equation:

$$E_\theta dr - rE_r d\theta = 0 . \qquad (2)$$

In accordance with the Maxwell equation, with account of the harmonic law of field time variation, electric field is defined by the following formula:

$$\overline{E} = \frac{1}{i\omega\varepsilon_0}\cdot rot\overline{H} . \qquad (3)$$

Operation *rot(**H**)* performed within the spherical coordinates system results in definition of two electric field components:

$$E_r = \frac{1}{i\omega\varepsilon_0}\cdot\frac{1}{r^2\sin\theta}\cdot\frac{\partial}{\partial\theta}\left(r\cdot\sin\theta\cdot H_\varphi\right)$$
$$E_\theta = \frac{-1}{i\omega\varepsilon_0}\cdot\frac{1}{r\sin\theta}\cdot\frac{\partial}{\partial r}\left(r\cdot\sin\theta\cdot H_\varphi\right) \qquad (4)$$

Plugging of the obtained values in the differential equation (3) and removal of the multiplying constant results in the following equation:

$$\frac{\partial}{\partial r}\left(r \cdot \sin\theta \cdot H_{\varphi}\right)dr + \frac{\partial}{\partial \theta}\left(r \cdot \sin\theta \cdot H_{\varphi}\right)d\theta = 0 \quad (5)$$

which has a simple format:

$$r \cdot \sin\theta \cdot H_{\varphi} = C = const \quad (6)$$

The condition ensuring delivery of expression (6) is availability of a single component $\boldsymbol{H_{\varphi}}$. Therefore, the obtained expression is also applicable for describing radiation field structures of antennas where only this element is present in magnetic components. Such antennas include, for instance, linear antennas or phased arrays characterized by definite current distribution, with all radiators located along the **Z** axis and respective excitation currents directed along the same axis. The resulting magnetic field is a super-position of the respective components of specific radiators fields taking into account their individual remoteness from the observation point:

$$H_{\varphi} = H_0 e^{i\omega t} \sum_{m=1}^{M} A_m e^{i\varphi_m} \left(\frac{1}{r_m{}^2} + \frac{ik}{r_m} \right) \sin\theta_m e^{-ikr_m}, \quad (7)$$

where:

M – the number of radiators of the phased array or equivalent radiators that the linear antenna is conventionally divided to;

A_m, φ_m, – relative amplitude and phase of the radiator with number n;

r_m, θ_m – coordinates of the actual observation point in the space domain within the own coordinates system of the radiator with number n.

In order to analyze the structure of force lines, it is essential to get to the expression describing the real component. The final expression (6) will be as follows:

$$r \cdot \sin\theta \cdot real\{H_{\varphi}\} = C \quad (8)$$

The function of two variables at the left side of the expression (8) will be hereunder referred to as a characteristic function. Its cross-sections describe force lines of vector **E**.

3. RESULTS OF NUMERICAL SIMULATION

It is worth looking upon the use of obtained solution for analyzing radiation field of a single Hertzian radiator or linear antenna. By plugging the H_{φ}-related expression (7) in formula (8) and using mathematic simulation methods, it is possible to design force line patterns for any space domain.

Fig.3 shows a 3D characteristic function, with its cross-sections describing force lines of the Hertzian dipole electric field vector in the φ=*const* planes. The structure of the vector ***E*** force lines within the distance range of 5λ to 7λ and within the θ angle sector from 0 to π is shown. It is worth noting that the described field is more illustrative and provides deeper information content versus the traditional image within the cylindrical coordinates system (Fig.1).

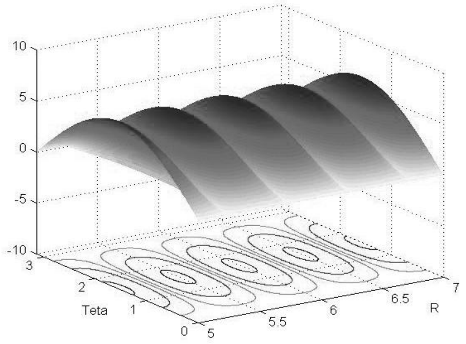

Fig. 3. Characteristic function and force lines of the Hertzian dipole vector E

Fig.4-6 show images of force lines of field ***E*** at the moment t_0, which is radiated by a linear antenna consisting of 10 Hertzian dipoles located along the ***Z*** axis at a distance of 0.5λ one from another. On Fig.4 the field is presented within a space domain limited in terms of distance from the antenna center (2λ to 9λ) and angles θ (from 0 to π).

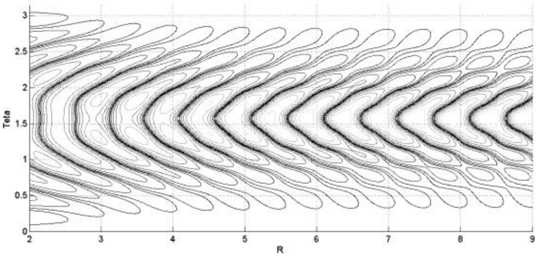

Fig. 4. Force lines of field **E** of a linear antenna at a distance from 2λ to 9λ.

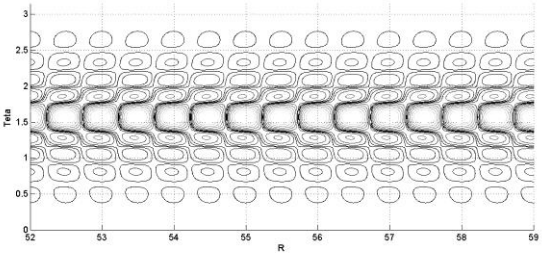

Fig. 5. Force lines of field **E** of a linear antenna at a distance from 52λ to 59λ.

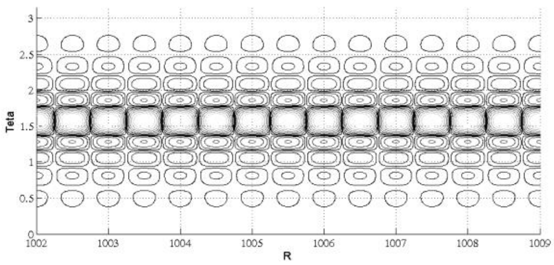

Fig. 6. Force lines of field **E** of a linear antenna at a distance from 1002λ to 1009λ.

The picture clearly demonstrates rotated V-shaped spaces united by common force lines. It is also seen how the radiated field structure is getting formed while the distance from the antenna is increased.

The field structure in the area from 52λ to 59λ is shown on Fig.5. If compared to the field structure in vicinity to the antenna, in this area narrowing of rotated V-shaped spaces is observed due to separation of zones that have their own closed force lines. However, such zones are still present and include space domains located in the vicinity of direction $\theta \approx \pi/2$.

Fig.6 shows a similar pattern of force lines within the distance range from 1002λ to 1009λ. All force lines of the electric field are closed round lines with their centers located in the clearly defined rectangular grid nodes.

It is worth noting that a similar pattern of force lines will be observed when the field is excited by means of elementary magnetic radiators. Pursuant to the duality principle, in this case the above formulas shall describe behavior of magnetic force lines.

Analysis of the field **E** component force lines structure presented on Fig.6 points to its similarity to the one drafted for waveguides. The only key difference here is the fact that in the example presented herein angle variable is plotted along the axis of ordinates. Therefore, the exposed radiation field can also be regarded as one of the types of ***E*** (***TM***) or ***H*** (***TE***) waves. The coordinates system selected herein effectually emphasizes this similarity.

When analyzing the structure of presented fields, it is essential to take into account the fact that in all cases the field magnetic component is orthogonal with respect to the design plane. Due to the fact that the Poynting vector ***P*** direction is orthogonal with respect to both components and is defined by the formula:

$$\mathbf{P} = [\mathrm{real}(\mathbf{E})\ \mathrm{real}(\mathbf{H})] =$$

$$= \mathbf{r_0}\ \mathrm{real}(E_\theta)\ \mathrm{real}(H_\varphi) - \boldsymbol{\theta_0}\ \mathrm{real}(E_r)\ \mathrm{real}(H_\varphi) \qquad (9)$$

the Poynting vector component directed along the angular component θ [2,3] will be present within the framework of the given structure of field ***E*** force lines.

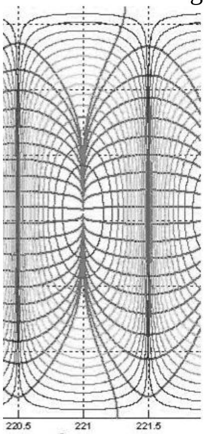

Fig.7a. Force lines of fields **E** and **P**

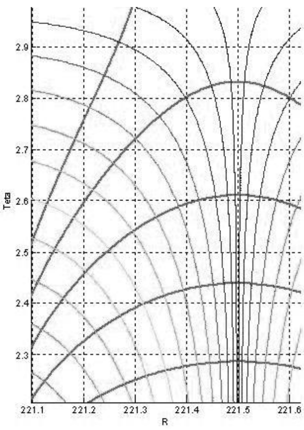

Fig.7b. Fragment of fields **E** and **P** force lines pattern

Fig.7a shows calculated patterns of ***E*** vectors and the Poynting vector ***P*** force lines orthogonal to such, as applicable to an elementary radiator. Fig.7b presents a scaled-up fragment of the space domain demonstrating orthogonal property of vectors ***E*** and ***P*** force lines. The provided data show that vector ***P*** is only directed along coordinate ***r*** within the angular space adjacent to direction $\theta=\pi/2$. It is also clearly seen that all force lines of vector ***P*** coincide in specific space points.

The provided conclusions also result from the fact that performed calculations carefully took into account all electromagnetic field components, including the weakest ones. Purely radial energy transmission occurs in case the radial component of field $\mathbf{E_r}$ direct-axis component is disregarded.

4. Conclusions

The key conclusions can be summarized as follows:

1. Herein, a way of force lines visual representation is suggested, which focuses on using a polar coordinates system (***r***, θ) within the framework of variables calculation as well as reflection of obtained data in a rectangular grid format;
2. Analytical expression of the characteristic function has been developed, which facilitates representation of force lines of linear antennas radiation field component ***E***;
3. It is demonstrated that radiation field of an exposed extended antenna can also be regarded as one of the types of ***E*** (***TM***) or ***H*** (***TE***) waves;
4. Performed calculations have proved that radiation electromagnetic field energy is mostly transmitted along radius ***r*** only in the angular domain adjacent to the normal line to the antenna aperture. In the other space domains, the Poynting vector is characterized by a component directed along angular coordinate θ.

The obtained results can be useful within the framework of antenna radiation fields study in any space domain.

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