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ASYMPTOTIC METHODS ON THE SOLUTION OF DIFFRACTION PROBLEMS ON THE CONVEX IMPEDANCE CYLINDERS

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Abstract

In work are considered problems of diffraction of electromagnetic waves by the impedance convex cylinder for two cases : the observation point and source point are located on removal from the cylinder and a source point is located in the close vicinity of the convex cilinder. Process of obtaining asymptotic interpolation formulas, suitable for calculations both in shadow and in lighted zones is described..

Keywords: diffraction, asymptotic method, interpolation.

Asymptotic methods are widely used at the solution of problems of diffraction on various objects. One of the known approaches used to the solution of problems of diffraction on convex bodies is the method of asymtotic theories of the diffraction, offered by Fok [1].

In work the given approach is used for the solution diffraction problems on the convex impedance cylinder for two practically important cases: the observation point and a source point are located on removal from the cylinder (diffraction of a flat electromagnetic wave) and one of points is located near to a surface, and another is removed on infinity.

Using Watson's transformation and asymptotic representations of Bessel and Hankel functions are received asymptotic formulas for the impedance circular cylinder, suitable for calculation of fields in a semishadow and shadow zone.

For the first case (scattering plane wave in case of E polarization) the formula looks like

$$E_z = -M\sqrt{\frac{2}{kr}}e^{ikr}\left[e^{ika\varphi}\widehat{g}(M\varphi)+e^{-ika\varphi}\widehat{g}(-M\varphi)\right], \quad (1)$$

where E_z - z component of an electric field, scattering on the circular cylinder, k – wave number, a – radius of the cylinder, r –distance from the center of the cylinder up to a observation point, φ –the angle on a observation point, $M=\left(\frac{ka}{2}\right)^{\frac{1}{3}}$ and $\widehat{g}$ represents integral of a kind

$$\widehat{g}(\xi)=\frac{e^{i\frac{\pi}{4}}}{\sqrt{\pi}}\int_{\Gamma} e^{i\xi t}\cdot\left[\frac{v'(t)-qv(t)}{w_1'(t)-w_1(t)}\right]dt, \quad (2)$$

The contour Γ – run in contour plane from $e^{i\frac{2\pi}{3}}\infty$ to 0 and from 0 to ∞ and v, v', w_1, w_1' – Airy functions and their derivatives in the Fok notation (see. [2]).

For a case when one of points is located near to a surface (for example a source point) and the second - on infinity, the formula for a full field looks

$$E_z = -\sqrt{\frac{k}{8\pi r}}e^{ikr}\left[e^{ika(\frac{\pi}{2}+\varphi)}g\left(M(\frac{\pi}{2}+\varphi)\right)+ +e^{ika(\frac{\pi}{2}-\varphi)}g\left(M(\frac{\pi}{2}-\varphi)\right)\right], \quad (3)$$

where E_z – z component of a full field, and g represents integral of a kind

$$g(\xi)=\frac{e^{-i\frac{\pi}{4}}}{\sqrt{\pi}}\int_{\Gamma} e^{i\xi t}\cdot\left[v(t-h)- -\frac{v'(t)-qv(t)}{w_1'(t)-qw_1(t)}w_1(t-h)\right]dt. \quad (4)$$

Here $h=\frac{k(\rho-a)}{M}$, where $\rho-a$ height of a source point above a surface of the cylinder.

This is necessary to note, that in expressions (1) and (3) fields are represented as two waves bypassing the cylinder on hour and counter-clockwise.

The received formulas, however, are inapplicable in the lighted area.

For obtaining of universal formulas, suitable for calculation fields both in shadow, and in depth of the lighted zone the approach described in [3] was used. The essence of it will be, that as one of two waves corresponds to the reflected field, for it is searched interpolation representation so that in depth of the lighted zone the formula transform in the formula of geometrical optics, and in a shadow zone would remain unchanged.

For a case of a flat wave interpolation formula looks

$$E_z = -M\sqrt{\frac{2}{kr}}e^{ikr}\left[e^{ika\varphi}\,\hat{g}(M\varphi) + +e^{-ika2\sin\frac{\varphi}{2}}e^{\frac{i}{12}(-2M\sin\frac{\varphi}{2})^3}\,\hat{g}(-2M\sin\frac{\varphi}{2})\right] \quad (5)$$

For a case when a source point is located near a surface, and observation point is located in a far zone interpolation formula looks like

$$E_z = -\sqrt{\frac{k}{8\pi r}}e^{ikr}\left[e^{ika(\frac{\pi}{2}+\varphi)}g\left(M(\frac{\pi}{2}+\varphi)\right) +e^{ika\cos\varphi}e^{i\frac{(M\cos\varphi)^3}{3}}g\left(M\cos\varphi\right)\right] \quad (6)$$

In the formulas obtaining above arguments of integrals represent the sizes describing phase progression, and parameters an exponent product of a wave vector on length of an arch. In case of the cylinder with variable curvature, phase progression can be represented as

$$\xi = \int_{t_1}^{t_2}\frac{1}{\rho_g}\left(\frac{k\rho_g}{2}\right)^{\frac{1}{3}}dt, \quad (7)$$

where t – length of arc and ρ_g – radius of curvature, and the length of an arch is defined as integral

$$t = \int_{t_1}^{t_2}dt \quad (8)$$

In view of above-stated, using geometrical optics approach it has been shown, that mentioned above interpolation formulas can be distributed on a case of the impedance convex cylinder with any curvature, in particular elliptic. Thus the technics of representation of the reflected wave, described in [4] was used.

The lead comparison of the results received with use interpolation formulas with results, received other methods (method of moments) has shown good concurrence [5].

REFERENCES

1. Fok V.A. Problemy diffrakcii i rasprostranenija jelektromagnitnyh voln./ Fok V.A. — M.:Sovetskoe radio, 1970. — 517 s.
2. Fok V.A. Diffrakcija radiovoln vokrug zemnoj poverhnosti./ Fok V.A. — M.: Izd. AN SSSR, 1946. —80 s.
3. Gorjainov A.S. Asimptoticheskoe reshenie zadachi o difrakcii ploskoj jelektromagnitnoj volny na provodjashhem cilindre/ Gorjainov A.S.// *Radiotehnika i jelektronika.* — 1958. — T.3, no.5. — S. 603-614.
4. Pathak, P. An analysis of the radiation from apertures in curved surfaces by the geometrical theory of diffraction / P. Pathak, R. Kouyoumjian // *Proc. IEEE.* — 1974. Vol. 62, no. 11. — Pp. 1438-1447.
5. Vyunnik V.I. Asimptoticheskij metod reshenija zadach difrakcii na vypuklyh impedansnyh cilindrah./ Vyunnik V.I., Zvjagincev A.A. // *Radiotehnika: Vseukr. mezhved. nauch.-tehn. sb.* — Harkov, 2009. — Vyp. 157.