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BUILDING OF TENSOR GREEN'S FUNCTIONS FOR SOLVIND OF RADIATION PROBLEM FROM SLOT IN WALL OF SENDING STRUCTURE

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Abstract

The building of general algorithm solving problems of radiation from a longitudinal slot in the wide wall of rectangular waveguide is considered. An algorithm is created on the basis of integral theorem of vectorial theory of diffraction where the tensor Green's function is used. It is therefore necessary to consider the general method of construction of Green's tensor function of waveguide and half-space..

Keywords: Radiation problem, function, tesor.

1. Formulation of the Problem

We will consider the system of co-ordinates, where z is a longitudinal ax of waveguide, and is a size wide; b is a size narrow wall of waveguide. In integral presentations for the complete fields of the selected areas the Green's tensor functions of endless rectangular waveguide and half-space of radiation are used.

For a rectangular waveguide there is diagonal affinor

$$\overleftrightarrow{G}(\vec{r},\vec{r}\,')=\begin{pmatrix} G_{xx} & 0 & 0 \\ 0 & G_{yy} & 0 \\ 0 & 0 & G_{zz} \end{pmatrix} \qquad (1)$$

satisfies tensor equation of Helmholtz

$$\Delta\overleftrightarrow{G}(\vec{r},\vec{r}\,')+k^2\overleftrightarrow{G}(\vec{r},\vec{r}\,')=-\overleftrightarrow{I}\,\delta(\vec{r}-\vec{r}\,') \qquad (2)$$

and boundary condition

$$\vec{n}\times\overleftrightarrow{G}_1^e(\vec{r},\vec{r}\,'),\quad \vec{r},\vec{r}\,'\in S; \qquad (3)$$

$$\vec{n}\times[\vec{\nabla}\times\overleftrightarrow{G}_2^e(\vec{r},\vec{r}\,')],$$

$$\vec{n}\cdot\overleftrightarrow{G}_2^e(\vec{r},\vec{r}\,')=0,\quad \vec{r},\vec{r}\,'\in S \qquad (4)$$

electric (index 1) and magnetic (index 2) types. Indeed an operator of Laplace Δ is a scalar operator, therefore equation (2) is equivalent nine independent scalar equations relatively component of affinor $\overleftrightarrow{G}(\vec{r},\vec{r}\,')$

$$(\Delta+k^2)G_{\alpha\beta}=-\delta_{\alpha\beta},\quad \alpha,\beta=x,y,z,$$

where $\delta_{\alpha\beta}$ is character of Kronecker. We will notice, what only three diagonal components $G_{\alpha\alpha}$ are the scalar Green's functions; other six component are satisfied homogeneous equations and the Green's functions aren't been – presentation follows from here (1).

Thus, equations (2) is equivalent three scalar equations

$$(\Delta+k^2)G_{\alpha\alpha}=-\delta(\vec{r}-\vec{r}\,'),\quad \alpha=x,y,z, \qquad (5)$$

and boundaries conditions (3) and (4) are according assume:

$$G_{xx}\perp_{y=0,b}=0;\quad G_{yy}\perp_{x=0,a}=0;$$
$$G_{zz}\perp_{x=0,a}=0;\quad G_{zz}\perp_{y=0,b}=0\quad ; \qquad (6)$$

and

$$\frac{\partial}{\partial y}G_{xx}\perp_{x=0,a}=0;\quad \frac{\partial}{\partial y}G_{xx}\perp_{x=0,b}=0;$$
$$\frac{\partial}{\partial x}G_{yy}\perp_{x=0,a}=0;\quad \frac{\partial}{\partial x}G_{yy}\perp_{x=0,b}=0;$$
$$\frac{\partial}{\partial z}G_{xx}\perp_{x=0,b}=0;\quad \frac{\partial}{\partial z}G_{yy}\perp_{x=0,b}=0; \qquad (7)$$
$$\frac{\partial}{\partial x}G_{zz}\perp_{x=0,a}=0;\quad \frac{\partial}{\partial y}G_{zz}\perp_{x=0,b}=0;$$
$$G_{xx}\perp_{x=0,a}=G_{yy}\perp_{y=o,b}=0.$$

The decision of equation (5), as a rule, is got as follows: in the beginning find the decisions of that equation, but with right part of equal to the zero. That is determined complete system of own functions of operator of Helmholtz, and then find the decision of non homogeneous equation (5) as decomposition on the found complete system of own functions.

However in practice widely apply the so-called "source" presentation for the Green's function. It consists in that the Green's function decomposes in a row on the complete system of functions, containing all of co-ordinates, except for one with coefficients, being indefinite functions of off-duty co-ordinate, with subsequent determination of these functions.

For a rectangular waveguide comfortably to find the components of Green's tensor in a kind:

$$G_{\alpha\alpha} = \sum_m \sum_n \varphi_{\alpha mn}(x,y)\,\varphi_{\alpha mn}(x',y')\,F_{\alpha mn}(z,z'),$$

$\alpha = x, y, z.$

Here $F_{\alpha mn}(z,z')$ are unknown functions, and $\varphi_{\alpha mn}(x,y), \varphi_{\alpha mn}(x',y')$ are the complete systems of transversal own functions of rectangular waveguide.

Transversal own functions for boundary condition of first kind look like (6):

$$\varphi_{xmn}(x,y) = \left(\frac{\varepsilon_m \varepsilon_n}{ab}\right)^{1/2} \cos\frac{m\pi}{a}x \sin\frac{n\pi}{b}y,$$

$$\varphi_{ymn}(x,y) = \left(\frac{\varepsilon_m \varepsilon_n}{ab}\right)^{1/2} \sin\frac{m\pi}{a}x \cos\frac{n\pi}{b}y,$$

$$\varphi_{zmn}(x,y) = \left(\frac{\varepsilon_m \varepsilon_n}{ab}\right)^{1/2} \sin\frac{m\pi}{a}x \sin\frac{n\pi}{b}y,$$

for boundary condition of second kind (7):

$$\psi_{xmn}(x,y) = \varphi_{ymn}(x,y);$$

$$\psi_{ymn}(x,y) = \varphi_{xmn}(x,y);$$

$$\psi_{zmn}(x,y) = \left(\frac{\varepsilon_m \varepsilon_n}{ab}\right)^{1/2} \cos\frac{m\pi}{a}x \cos\frac{n\pi}{b}y,$$

where $\varepsilon_m, \varepsilon_n$ is Nyman's symbol: $\varepsilon_0 = 1;\ \varepsilon_m = 2,\ m \neq 0.$

For an endless waveguide:

$$F_{\alpha mn}(z,z') = \frac{1}{2\gamma_{mn}} e^{-\gamma_{mn}|z-z'|}$$

where γ_{mn} is a longitudinal coefficient of propagation of wave.

The tensor Green's function of half-space, limited an endless ideally conducting plane, can be got by the method of mirror images.

The Green's function for free space satisfies equation

$$\Delta\overleftrightarrow{G}_0(\vec r,\vec r\,') + k^2\overleftrightarrow{G}_0(\vec r,\vec r\,') = -\overleftrightarrow{I}\,\delta(\vec r - \vec r\,')$$

and to the condition of radiation of Summerfield

$$\lim_{|\vec r|\to\infty} |\vec r|\,\langle \vec\nabla \times \overleftrightarrow{G}_0(\vec r,\vec r\,') + jk\frac{1}{|\vec r|}\,\vec r \times$$

$$\overleftrightarrow{G}_0(\vec r,\vec r\,')\rangle = 0.$$

It has a next kind

$$\overleftrightarrow{G}_0(\vec r,\vec r\,') = \overleftrightarrow{I}\ g_0(\vec r,\vec r\,') = \overleftrightarrow{I}\,\frac{e^{-jk|\vec r - \vec r\,'|}}{4\pi|\vec r - \vec r\,'|}.$$

The Green's function of the first kind, satisfying on an ideally conducting surface to boundary conditions (3), must have at z=0 difference from a zero only normal to the indicated surface components and can be presented in a kind

$$\cdot\,\overleftrightarrow{G}_1(\vec r,\vec r\,') = \overleftrightarrow{G}_0(\vec r,\vec r\,') - \overleftrightarrow{G}_M(\vec r,\vec r\,')(\overleftrightarrow{I} - 2\vec z_0 \circ \vec z_0),$$

where

$$\overleftrightarrow{G}_M(\vec r,\vec r\,') = \overleftrightarrow{I}\ g_M(\vec r,\vec r_M);\quad \vec r_M\,' = \vec r\,' - 2\vec r\,'\vec z_0 \circ \vec z_0.$$

Multiplying of affinor $\overleftrightarrow{G}_M(\vec r,\vec r\,')$ by $(\overleftrightarrow{I} - 2\vec z_0 \circ \vec z_0)$ equivalently to the change of sign at his normal components, that provides implementation of the set boundary conditions. The Green's function of the second kind, satisfying at z=0 to boundary condition (4) can be written in a kind

$$\overleftrightarrow{G}_2(\vec r,\vec r\,') = \overleftrightarrow{G}_0(\vec r,\vec r\,') + \overleftrightarrow{G}_M(\vec r,\vec r\,')\cdot(\overleftrightarrow{I} - 2\vec z_0 \circ \vec z_0).$$

Higher presented Green's functions $\overleftrightarrow{G}_1(\vec r,\vec r\,')$ and $\overleftrightarrow{G}_2(\vec r,\vec r\,')$ are the functions of potential type. The Green's functions of electric type are equal

$$\overleftrightarrow{G}^e_{1,2}(\vec r,\vec r\,') = (\overleftrightarrow{I} + \frac{\vec\nabla \circ \vec\nabla}{k^2})(g_0 \mp g_M) \pm$$

$$(\overleftrightarrow{I} + \frac{\vec\nabla \circ \vec\nabla}{k^2})\ g_M \cdot 2\vec z \circ \vec z.$$

Taking into account that

$$\vec\nabla g_M(\vec r,\vec r\,' - 2\vec r\vec z_0 \circ \vec z_0) = -\vec\nabla g_M(\vec r,\vec r\,' - 2\vec z_0 \circ \vec z_0) +$$

$$2\vec\nabla' g_M \vec z_0 \circ \vec z_0;$$

$$\vec\nabla g_0(\vec r,\vec r\,') = -\vec\nabla' g_0(\vec r,\vec r\,'),$$

where stroke above an operator a «nabla» designates differentiation on the hatched co-ordinates. Then expression for $\overleftrightarrow{G}^e_{1,2}(\vec r,\vec r\,')$ it can be written in more comfortable kind

$$\overleftrightarrow{G}^e_{1,2}(\vec r,\vec r\,') = (\overleftrightarrow{I} - \frac{\vec\nabla \circ \vec\nabla}{k^2})(g_0 \mp g_M) \pm$$

$$g_M \cdot 2\vec z \circ \vec z.$$

CONCLUSIONS

Thus, the construction of Green's functions is in-process considered for solving of radiation from a longitudinal slot in the wall of sending structure.