SYNTHESIS OF SHAPED BEAM ANTENNA PATTERN USING A NEW PHASE METHOD OF FAN PARTIAL PATTERNS

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Abstract

Theoretical basis of the proposed phase synthesis method regarding onedimensionally extended phased array beams is presented. The proposed method allows for unambiguous determination of required phase distribution on the basis of given aperture amplitude distribution and required beam shape.

Keywords: beam synthesis; phase-only pattern synthesis; phase-only beam forming, shaped beams.

1. INTRODUCTION

Antennas must shape directional patterns which shall in the best way contribute to solving issues related to radio technical systems. In most cases, while working on air targets, airborne radars use directivity patterns with beams formed as a pencil and shaped in one or two angular dimensions. Shaped beams « $\csc^2(\theta)$ » are often used for working on ground targets. In spite of numerous studies devoted to investigation of phase synthesis of beams shaped in phased arrays, e.g. $[1\div 3]$, the number of methods brought up to practical use is relatively small. Herein, the phase method of fan partial patterns [4] is presented, which allows identifying phase distributions for forming shaped beams of predetermined forms with account of known amplitude distribution within the phased array aperture.

2. THEORETICAL BACKGROUND

Let us have a look at linear phased arrays with predetermined amplitude distribution ${A_n}$, the radiators of which are arranged with certain element spacing $\{\Delta x_n\}$. It is required to form shaped beam with predetermined distribution amplitude of field F(u) in given angular sector $[u_{min}, u_{max}]$ only by means of changing phase distribution in the aperture.

It is worth looking at the Parseval formula for linear phased arrays (1). The number of the terms of series related to the right member can be arbitrary. However, the most interesting case is when the total number of terms on the left and right of the equation coincides:

$$
\frac{1}{L}\sum_{n=1}^{M}A_{n_{-}}^{2}\Delta x_{n} = \sum_{n=1}^{M}F_{n_{-}}^{2}A u_{n}
$$
 (1)

where:

M – number of radiating elements in the aperture;

 L – length of antenna;

 Δx_n – n-section of the aperture element spacing;

- $A_{n_m}^2$ average relative power radiated within the aperture section Δx_n ;
- Δu_n angular sector with number 'n';
- $u=sin(\theta)$ angle variable;
- $F_{n_m}^2$ average relative power within the angular interval Δu_n .

As geometrical parameters of the aperture and its feeds are known, all terms of the left sum are exactly determined. Angular sectors $\{\Delta u_n\}$ and its respective average values ${F^2}_{n_cp}$ are undefined.

It is obvious that the formula (1) has various solution sets. The simplest solution can be calculated by termwise equation of components:

$$
\frac{1}{L}A_{n_{-m}}^{2}\Delta x_{n} = F_{n_{-m}}^{2}\Delta u_{n}
$$
 (2)

Expression (2) is a clue for the phase synthesis method study. Physically it means that energy emitted by every n-element of the aperture located within interval Δx_n should be directed to the respective angular sector 'n' and thus determine energy density in area Δu_n (Fig.1).

If distances between phase centers of the phased array radiating elements in the aperture are equal, i.e. $\Delta x_n = \Delta x$, the latest expression is further simplified:

$$
\frac{A_{n_{-m}}^2}{F_{n_{-m}}^2 \Delta u_n} = \frac{L}{\Delta x} = const
$$
\n(3)

Finally, if a sector shaped beam is formed, with average relative power inside such beam being a constant value ($F_{n_m}^2$ =const), the simplest formula is derived:

$$
\frac{A_{n_{-m}}^2}{\Delta u_n} = \frac{F_{n_{-m}}^2 L}{\Delta x} = const
$$
\n(4)

The resulting mathematical solutions have a very simple format and allow calculating angular sectors

 $\{\Delta u_n\}$ and average field power $\{F^2_{n_m}\}\$ according to predetermined parameters of aperture excitation and required beam shape.

Fig. 1. Forming of a shaped beam.

3. PHYSICAL INTERPRETATION

It is possible to conventionally break down a phased array aperture to subbarrays and present the whole field emitted by antenna as amount of subbarrays partial patterns, where angular location of maximums can be controlled. Angular zone of the shaped beam should also be subdivided into angular sectors, the number of which shall be equal to that of partial patterns. By means of controlling phase distributions, maximums of partial patterns should be directed into the centers of relevant angular sectors (Fig.2)

Fig. 2. Forming of partial patterns. a)shaped beam; b)partial patterns

Locations of such angular sectors and their sizes are defined by using formulas $(1\div 4)$. The position of the partial pattern "m" maximum is calculated by formula:

 Δ

 $\sqrt{ }$

$$
u_m = \left(\frac{u_{\text{max}} - u_{\text{min}}}{\sum\limits_{i=1}^{M-1} F_i^2(0)}\right) \left(\sum\limits_{i=1}^{m-1} F_i^2(0) + 0.5F_m^2(0)\right) + u_{\text{min}}\tag{5}
$$

4. SYNTHESIS PROCEDURE

The directional pattern of a linear phased array consisting of M isotropic radiators located with 'd' element spacing is determined by the following formula:

$$
F(u) = \sum_{n=1}^{M} A_n e^{ikux_n} = \sum_{n=1}^{M} A_n e^{i\varphi_n} e^{ikux_n} = \sum_{n=1}^{M} A_n e^{i(\varphi_n + kux_n)}
$$
(6)

 A_n - amplitude of the n-radiator;

 φ_n – phase of the n-radiator.

It is further assumed that an individual element does not have directional properties and emits a spherical wave. Directional properties are possessed only by radiator clusters including at least of two elements.

Fig. 3. Presentation of antenna as subbarrays of two elements.

For forming such clusters according to the superposition principle, one should break down each inherent radiator into two equal ones located at an infinitely close distance, and integrate into pairs similar half-bats of adjacent radiators (Fig.3). Thus, the pattern of antenna array consisting of M radiators can be presented as superposition M-1 of partial diagrams.

Mathematical operations corresponding to such partitioning algorithm are presented below:

$$
F(u) = (\dot{A}_1 e^{ikux_1} + \frac{1}{2} \dot{A}_2 e^{ikux_2}) + (\frac{1}{2} \dot{A}_2 e^{ikux_2} + \frac{1}{2} \dot{A}_3 e^{ikux_3}) + \dots
$$

\n
$$
\dots + (\frac{1}{2} \dot{A}_{M-2} e^{ikux_{M-2}} + \frac{1}{2} \dot{A}_{M-1} e^{ikux_{M-1}}) +
$$

\n
$$
+(\frac{1}{2} \dot{A}_{M-1} e^{ikux_{M-1}} + \dot{A}_M e^{ikux_M}) =
$$

\n
$$
= F_1(u) + F_2(u) + F_3(u) + \dots + F_{M-2}(u) + F_{M-1}(u)
$$

\nwhere $E_1(u)$ - partial pattern of subharrav formed by

where $F_m(u)$ – partial pattern of subbarray formed by two m-elements.

The expression for partial pattern $F_m(u)$ of a paired subbarray will be as follows:

$$
F_m(u) = \frac{1}{2} \dot{A}_m e^{ikux_m} + \frac{1}{2} \dot{A}_{m+1} e^{ikux_{m+1}}
$$

= $\frac{1}{2} e^{ikux_m} (A_m + A_{m+1} e^{ikud})$ (8)

For setting maximums of partial patterns to directions $\{u_m\}$ according to (5), it is essential that phases of paired subbarrays right radiators should be made equal: A_0 _{m+1} =-kdu_m (9)

$$
\Delta \varphi_{m+1}
$$
 =- $\kappa \alpha \iota_m$ (9)
aximums of partial patterns $F_m(0)$ is

The level of ma calculated by means of amplitude distribution in the aperture and location of the relevant elements pair: ϵ

$$
A_{m_{-}m} = F_m(0) = \begin{cases} A_1 + \frac{1}{2} A_2, & m = 1\\ \frac{1}{2} A_m + \frac{1}{2} A_{m+1}, & m \in [2, M - 2] \\ \frac{1}{2} A_{M-1} + A_M, & m = M - 1 \end{cases}
$$
(10)

It is also possible to perform "integration" of partial subbarrays into a united array, which is inverse to the "partitioning" operation (7).

To provide only phase solution with a continuous law of phase modification, it is necessary to sum up partial patterns according to the following formula:

$$
F(u) = F_1(u) + e^{i\Delta\varphi_1} F_2(u) + e^{i(\Delta\varphi_1 + \Delta\varphi_2)} F_3(u) + \dots + e^{\int_{u=1}^{M-2} A\varphi_m} F_{M-1}(u)
$$
\n(11)

By inserting definitions of partial patterns (7) and (8) into the formula, we get a regular expression calculating phased array pattern formed by M elements.

$$
F(u) = A_1 e^{ikux_1} + A_2 e^{i\Delta\varphi_1} e^{ikux_2} + A_3 e^{i(\Delta\varphi_1 + \Delta\varphi_2)} e^{ikux_3} + \cdots + A_{M-2} e^{\sum_{m=1}^{M-3} \Delta\varphi_m} e^{ikux_{M-2}} + A_{M-1} e^{\sum_{m=1}^{M-3} \Delta\varphi_m} e^{ikux_{M-1}} + A_M e^{\sum_{m=1}^{M-1} \Delta\varphi_m} e^{ikux_M}
$$
\n(12)

By comparing the original expression (7) and resultant (12), it is possible to note that the resultant amplitude distribution ${A_m}$ corresponds to the original and only phase distribution has changed.

Taking into account that the phase of the first (leftmost) element cannot be changed, the required phase of m-element (m>1) will be calculated by the formula:

$$
\varphi_m = \sum_{i=1}^{m-1} \Delta \varphi_i = -kd \sum_{i=2}^{m} u_{i-1}
$$
 (13)

Thus, for calculating phase distribution forming the predetermined one-dimensional shaped beam according to this method, it is necessary:

- 1. to calculate equivalent amplitude of paired subbarrays $F_m(0)$ excitation in accordance with (10);
- 2. calculate angular intervals $\{\Delta u_n\}$ related to beams of partial diagrams by using expressions (2), (3) and (4) depending on the array structure and beam shape. This operation is described in more detail in [4];
- 3. determine directions of partial patterns $\{\Delta u_n\}$ by using formula (5);
- 4. calculate phases $\{\Delta \varphi_m\}$ of the array aperture radiators in accordance with (13).

5. COMPUTER SIMULATIONS

As an example of shaped beams formation, Fig. 4,5 show phase distributions determined in accordance with the given method, as well as respective patterns with sector beams. Simulation was delivered for a phased array consisting of 40 elements located with 0.4λ element spacing and characterized by amplitude distribution of the cosine-type on the pedestal with the edge field level equal to 0.25.

The width of the shaped sector beams is 20° , 40° and 90° respectively. For comparison purposes, the initial pattern is also shown in the figure.

Fig. 4. Phase distributions for shaping beams

Fig. 5. Pattern of shaped beams

It s worth noting that phase distributions characterized by faster changing rates along the aperture with larger phase modification ranges form wider beams.

6. CONCLUSIONS

The key advantage of the method proposed herein is the fact that phase distribution calculation is brought down to simple mathematical expressions providing unambiguous solutions.

This method has passed experimental testing and is successfully applied for design purposes.

REFERENCES

- 1. Kautz G.M. Phase-only shaped beam synthesis via technique of approximated beam addition. – Antennas and Propagation, IEEE Transactions on, Issue: 5, May 1999. - Vol.47. – Pp.887-894.
- 2. Trastoy, Ares F., Moreno E. Phase-only control of antenna sum and shaped patterns through null perturbation. – IEEE Antennas and Propagation Magazine, Dec. 2001. – Vol.43. N6. –pp.45-54.
- 3. Gatti R.V., Marcaccioli L., Sorrentino R. A novel phase-only method for shaped beam synthesis and adaptive nulling. - 33rd European Microwave Conference, 2003, 7-9 Oct.2003.- Vol.2. – pp.739-742.
- 4. Gribanov A.N. An effective method of phase synthesis of one-dimensionally extended beams of phased array. – Antennas, 2007, N6(121). – pp.26- 29[in russian].