IEEE Second International Conference on Data Stream Mining & Processing
August 21-25, 2018, Lviv, Ukraine

A Generalized Description for the Perceived Contrast of Image Elements

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***Abstract*—In this paper, the problem of assessing the perceived contrast of image elements for no-reference measurement of the global contrast of complex (multi-element) images is considered. A new method for assess the perceived contrast of elements of complex image is proposed on the basis of measuring of the contrast of these elements on a pre-normalized image with subsequent correction of the contrast value taking into account the dynamic range of the primary (original) image. A new generalized description of the perceived contrast of the image elements for different definitions of the contrast kernel is suggested. New definitions of the weighted and relative contrast of the image elements are proposed. A comparative analysis of the proposed and known definitions of the weighted and relative contrast of image elements was carried out.**

Keywords—image, perceived contrast, image elements, global contrast, weighted contrast, relative contrast.

I. INTRODUCTION

Wide applying of modern technologies in imaging and image processing requires the solution of the task of no-reference assessing of the perceived quality of image [1].

Global contrast is the most important quantitative characteristic, which largely determines the overall perception of image [2, 3, 4]. At present, the development of new effective technologies for no-reference measuring the perceived contrast of complex images is relevant as never before [5].

The global (generalized) contrast of a complex (multi-element) image is determined by the contrast of its elements (objects and background). The contrast of the image elements (two objects or the object and the background) determines the difference in their objective characteristics. The contrast of the two image elements is usually defined on the basis of the difference in their brightness values [6]. The method of measuring the contrast of image elements appreciably defines the accuracy of measuring the perceived contrast for the image as a whole. Currently, there are various approaches to measuring the contrast of image elements [6, 7]. However, the known definitions of the contrast of image elements have a number of disadvantages that significantly reduce the effectiveness of their practical use [7, 8, 9]. To address these disadvantages, we propose a new method of measuring the perceived contrast of elements of complex image on the basis of measuring of the contrast of these elements for normalized image and of the dynamic range of the initial image.

The problem of increasing the accuracy of measuring the perceived contrast of elements of complex image is considered in this paper. The object of the study is the process of measuring the contrast of complex images to assess their quality. The purpose of the work is to increase the accuracy of measurement the perceived contrast of elements of complex image. The subject of the study is methods of measuring the perceived contrast of elements of complex image. The main known approaches to measuring the image contrast are considered (Section II). A new method of measuring the perceived contrast of two image elements (objects and background) is proposed by measuring their contrast on a normalized image with subsequent correction of the contrast value taking into account the dynamic range of the initial image (Section III). A generalized description of the perceived contrast of elements of complex images for various definitions of the contrast kernel is proposed. New definitions of perceived contrast of image elements for weighted and absolute contrast are proposed. The research of known and proposed definitions of a weighted and relative contrast to evaluate the efficiency of measuring of perceived contrast of image elements was carried out (Sections IV and V).

II. THE DEFINITION OF IMAGE CONTRAST

The global contrast of a complex multi-element image is determined on the basis of contrast values for all pairs of its elements (objects and background) [2]. The contrast of the two elements of the image (two objects or an object and a background) characterizes the distinction in their objective quantitative characteristics [6, 7, 10, 11].

A. The definition of global contrast for complex images

At present there are various approaches to assessing the global contrast of a complex multi-element image on the basis of measuring the values of contrast for all pairs of its elements.

The global (generalized) contrast of a complex multi-element image is usually defined as the average value of contrast for all pairs (i, j) of image elements (of objects and background) [2]:

$$C_{gen} = \int_{-1}^{1} \left|C_{ij}\right| \cdot h(C_{ij}) dC_{ij} \,, \qquad (1)$$

where C_{ij} – contrast of a pair (i, j) of image elements; $h(C_{ij})$ - probability density function for contrast C_{ij}.

However, the assessment of the distribution of $h(C_{ij})$ itself is quite a challenge. Therefore expression (1) is often represented in the form [2]:

$$C_{gen} = \int_0^1 \int_0^1 \left| C_{ij} \right| p(L_i, L_j)\, dL_i dL_j \;, \tag{2}$$

where L_i, L_j - brightness of the image elements *i* and *j*; $p(L_i, L_j)$ - two-dimensional distribution of brightness of image elements.

In [2] the definition of global contrast of multi-element image as the average value of the contrast of pairs of image elements relative to a preset adaptation level was proposed:

$$C'_{gen} = \int_0^1 \int_0^1 \left| C_{ij0} \right| p(L_i, L_j)\, dL_i dL_j \;, \tag{3}$$

where C_{ij0} – contrast of two image elements at a preset adaptation level value L_0.

In [6] the value L_0 of adaptation level is equal to the average value of brightness of the current image.

To the practical implementation of these approaches (2) and (3), it is necessary to solve the problems of estimating the two-dimensional distribution $p(L_i, L_j)$ of brightness and of choosing the definition for contrast of two image elements.

To simplify the calculations, in [2] the estimate of the two-dimensional distribution $p(L_i, L_j)$ of the brightness of the image elements (objects and background) has been suggested:

$$p\left(L_i, L_j\right) = p\left(L_i\right)\, p\left(L_j\right), \tag{4}$$

where $p(L_i)$ - probability density function of brightness.

In this case, expressions (1) and (2) on the basis of (4) take the form [2]:

$$C_{com} = \int_0^1 \int_0^1 \left| C_{ij} \right| p(L_i) p(L_j)\, dL_i dL_j \;, \tag{5}$$

$$C'_{com} = \int_0^1 \int_0^1 \left| C_{ij0} \right| p(L_i) p(L_j)\, dL_i dL_j \;, \tag{6}$$

where C_{com}, C'_{com} - definitions of complete integral contrast of image.

In [2] another approach to the estimation of the two-dimensional distribution of brightness of image elements has been suggested:

$$p\left(L_i, L_j\right) = p\left(L_i\right) \cdot \delta\left(L_i - L_j\right), \tag{7}$$

where $\delta(\cdot)$ - delta function.

In [2] for (3) on the basis (7) the definition of incomplete integral contrast of image was proposed:

$$C_{inc} = \int_0^1 \left| C_{i0} \right| p\left(L_i\right) dL_i \;, \tag{8}$$

where C_{i0} - contrast of *i*-th element of image relative to adaptation level L_0.

Expressions (5), (6) and (8) are no-reference histogram-based metrics of global contrast for multi-element images.

However, for the practical implementation of the examined approaches (5), (6) and (8) very important problem is also the choice of concrete definition of contrast for two image elements (two objects or an object and a background).

The choice of the definition of contrast for two image elements appreciably defines the accuracy of measurement of the global contrast for complex multi-element images.

It is assumed that the definition of contrast must satisfy the following basic requirements.

B. *The basic requirements to the contrast definition*

It is traditionally supposed that the contrast of two image elements is a dimensionless function and must satisfy the following basic requirements [7, 8]:

1) conditions for equality and asymmetry of the influence the arguments L_1 and L_2 [7]:

$$\left| C\left(L_1, L_2\right) \right| = \left| C\left(L_2, L_1\right) \right|, \tag{9}$$

$$C\left(L_1, L_2\right) = -C\left(L_2, L_1\right); \tag{10}$$

2) unambiguity and certainty of conditions under which the equality to zero is achieved [7]:

$$C\left(L_1, L_2\right) = 0 \text{ only when } L_1 = L_2; \tag{11}$$

3) condition of limitations of the range of contrast values [7]:

$$C\left(L_1, L_2\right) \le C_{\max}, \forall L_1, L_2 \in [0,1], \tag{12}$$

where $| C_{max} |$ - maximum absolute value of contrast, it is usually assumed that:

$$C_{\max} = 1, \left| C\left(L_1, L_2\right) \right| \in [\,0,1\,]; \tag{13}$$

4) unambiguity and certainty of the conditions under which the maximum absolute value of the contrast is achieved [7]:

$$\left|C\left(L_1, L_2\right)\right| \to \begin{cases} = \left|C_{\max}\right|, & if \left|L_1 - L_2\right| = L_{\max} - L_{\min} \\ < \left|C_{\max}\right|, & otherwise \end{cases}, \tag{14}$$

where L_{min}, L_{max} - minimum and maximum values of brightness on image.

In [8] the requirements on the invariance of the definitions of contrast relative to linear transformations of the brightness scale were discussed:

$$C\left(k \cdot L_1 + b, k \cdot L_2 + b\right) = sign\left(k\right) \cdot C\left(L_1, L_2\right), \tag{15}$$

$$k \ne 0 \;\forall\; L_1, L_2, \left(k \cdot L_1 + b\right), \left(k \cdot L_2 + b\right) \in [0,1].$$

The requirement (15) implies a number of important consequences:

a) condition of invariance to image inversion:

$$C\left(\bar{L}_1, \bar{L}_2\right) = -C\left(L_1, L_2\right), \tag{16}$$

$$\bar{L} = 1 - L\,; \tag{17}$$

b) condition of invariance to linear stretching of the dynamic range of image brightness:

$$C\left(\tilde{L}_1, \tilde{L}_2\right) = C\left(L_1, L_2\right), \tag{18}$$

$$\tilde{L}_i = \frac{L_i - L_{\min}}{L_{\max} - L_{\min}}\,. \tag{19}$$

Expressions (9)-(16), (18) define the basic requirements for contrast definition of image elements.

C. Known definitions of contrast of image elements

There are various approaches to definition the contrast of simple two-element images.

The contrast of two elements of a simple image is most often characterized by a difference in their brightness.

In [2] the definition of weighted contrast of two elements of complex image relative to a preset adaptation level has been suggested:

$$C^{wei_1}\left(L_1, L_2\right) = \frac{L_1 \cdot L_2 - L_0^2}{L_1 \cdot L_2 + L_0^2}\,. \tag{20}$$

In [6] the definition of weighted contrast of the image elements on the basis of the contrast law of light perception has been proposed:

$$C^{wei_2}\left(L_1, L_2\right) = \frac{L_1^2 - L_2^2}{L_1^2 + L_2^2}\,. \tag{21}$$

The most widely used at present definition of weighted contrast is defined as [3]:

$$C^{wei_3}\left(L_1, L_2\right) = \frac{L_1 - L_2}{L_1 + L_2}\,. \tag{22}$$

In [9], as an assessment of the perceived contrast, a weighted contrast (22) of the elements of the pre-inverted image (17) was proposed:

$$C^{wei_4}\left(L_1, L_2\right) = \frac{L_1 - L_2}{2 - L_1 - L_2}\,. \tag{23}$$

Another known definition of the contrast of image elements is the relative contrast, which is most often defined as [7, 11]:

$$C^{rel_1}\left(L_1, L_2\right) = \frac{L_1 - L_2}{\max\left(L_1,\, L_2\right)}, \tag{24}$$

$$C^{rel_2}\left(L_1, L_2\right) = \frac{L_1 - L_2}{1 - \min\left(L_1,\, L_2\right)}\,. \tag{25}$$

3D-graphs of surfaces for the weighted (20)-(23) and relative (24)-(25) contrast for primary image Lena (Fig.1) [12] are shown in Fig. 2 - Fig. 7.

However, known definitions (20)-(23) and (24)-(25) of weighted and relative contrast have significant disadvantages.

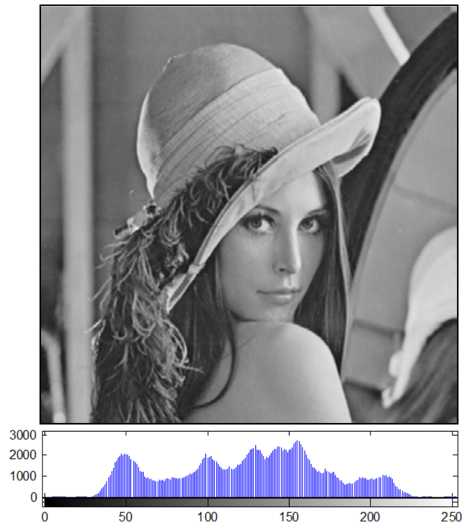

Fig. 1. Appearance of the primary image Lena [12] and its histogram (L_{min} = 0.0980; L_{max} = 0.9608; L_0 = 0.4864)

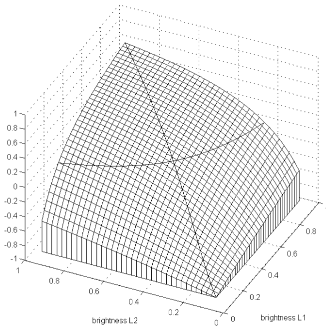

Fig. 2. $C^{wei_1}\left(L_1, L_2\right)$ (20)

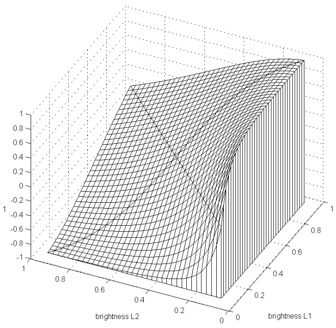

Fig. 3. $C^{wei_2}\left(L_1, L_2\right)$ (21)

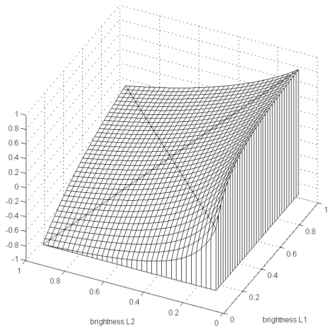

Fig. 4. $C^{wei_3}\left(L_1, L_2\right)$ (22)

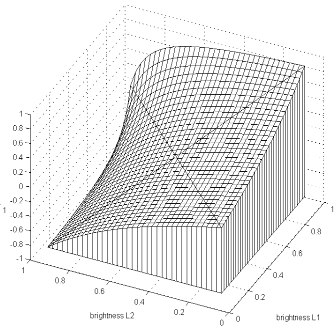

Fig. 5. $C^{wei_4}\left(L_1, L_2\right)$ (23)

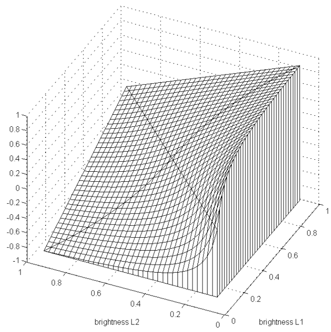

Fig. 6. $C^{rel_1}\left(L_1, L_2\right)$ (23)

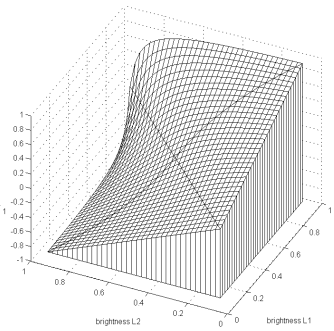

Fig. 7. $C^{rel_2}\left(L_1, L_2\right)$ (25)

A significant disadvantage of the known definitions of weighted and relative contrast is their relatively low efficiency in measuring the perceived global contrast for complex multi-element images with limited dynamic range of brightness, since when measuring the contrast of image elements the characteristics (size and location) of the dynamic range of image brightness are not taken into account [9].

To address this disadvantage, we propose a new method for assessment the contrast of elements of complex image on the basis of measuring of the contrast of these elements on a pre-normalized image with subsequent correction of the contrast value taking into account the dynamic range of the initial (original) image.

III. The Proposed Method

In this paper a new method for measuring the perceived contrast of image elements of complex multi-element images is proposed based on measuring the contrast of image elements on a pre-normalized image and estimating the dynamic range of the original image.

To define the contrast of two image elements on multi-element image for the chosen (specified) definition of contrast $C(L_1, L_2)$ (of contrast kernel), we propose a generalized description of the perceived contrast of image elements based on an analytical definition of the contrast of two image elements on a pre-normalized image and the value of maximum contrast for the original image:

$$\tilde{C}\left(L_1, L_2\right) = \left|C_{\max}\right| \cdot C\left(\tilde{L}_1, \tilde{L}_2\right), \qquad (26)$$

where $C\left(\tilde{L}_1, \tilde{L}_2\right)$ - contrast of two image elements on a normalized image using (19); $\left|C_{\max}\right|$ - maximum absolute value of contrast for the original image, normalizing factor, $\left|C_{\max}\right| \le 1$.

Taking into account that according to (14):

$$\left|C_{\max}\right| = C(L_{\max}, L_{\min}), \qquad (27)$$

and considering that according to (19):

$$C\left(\tilde{L}_1, \tilde{L}_2\right) = C\left(\frac{L_1 - L_{\min}}{L_{\max} - L_{\min}}, \frac{L_2 - L_{\min}}{L_{\max} - L_{\min}}\right), \qquad (28)$$

the expression (1) can be written in the form:

$$\tilde{C}\left(L_1, L_2\right) = C\left(L_{\max}, L_{\min}\right) \cdot C\left(\frac{L_1 - L_{\min}}{L_{\max} - L_{\min}}, \frac{L_2 - L_{\min}}{L_{\max} - L_{\min}}\right). \qquad (29)$$

Expressions (26) and (29) describe the proposed method of assessing the perceived contrast of the elements of complex image for specified definition $C\left(L_1, L_2\right)$ of contrast kernel.

To demonstrate the possibilities of the proposed method (26), let us consider known definitions of weighted (22) and relative (23), (25) contrast, which are most often used to estimate the perceived contrast of image elements.

A. *The proposed definition for weighted contrast*

At present, the known definition (22) of weighted contrast is most often used to define the perceived contrast of image elements.

Expression (28) for the weighted contrast (22) takes the form:

$$C^{wei_3}\left(\tilde{L}_1, \tilde{L}_2\right) = \frac{L_1 - L_2}{L_1 + L_2 - 2L_{\min}}. \qquad (30)$$

In this case, the maximum value of the weighted contrast (22) for the original image is equal to:

$$\left|C_{\max}^{wei_3}\right| = C^{wei_3}(L_{\max}, L_{\min}) = \frac{L_{\max} - L_{\min}}{L_{\max} + L_{\min}}. \qquad (31)$$

Taking into account (26), (30) and (31), the proposed definition of the perceived contrast of the image elements on the basis of the weighted contrast kernel (22) takes the form:

$$\tilde{C}^{wei_3}\left(L_1, L_2\right) = \frac{L_{\max} - L_{\min}}{L_{\max} + L_{\min}} \cdot \frac{L_1 - L_2}{L_1 + L_2 - 2L_{\min}}. \qquad (32)$$

By analogy with (30) - (32), the proposed definition of perceived contrast using the definition (23) of weighted contrast has the form:

$$C^{wei_4}\left(\tilde{L}_1, \tilde{L}_2\right) = \frac{L_1 - L_2}{2L_{\max} - L_1 - L_2}, \qquad (33)$$

$$\left|C_{\max}^{wei_4}\right| = C^{wei_4}(L_{\max}, L_{\min}) = \frac{L_{\max} - L_{\min}}{2 - L_{\max} - L_{\min}}, \qquad (34)$$

$$\tilde{C}^{wei_4}\left(L_1, L_2\right) = \frac{L_{\max} - L_{\min}}{2 - L_{\max} - L_{\min}} \cdot \frac{L_1 - L_2}{2L_{\max} - L_1 - L_2}. \qquad (35)$$

Expressions (26) and (29) describe the proposed method for assessing the perceived contrast of elements of complex image for known definitions (22) and (23) of weighted contrast.

B. *The proposed definition for relative contrast*

Expressions (27)-(29) for the definition (24) of relative contrast take the form:

$$C^{rel_1}\left(\tilde{L}_1, \tilde{L}_2\right) = \frac{L_1 - L_2}{\max\left(L_1, L_2\right) - L_{\min}}, \qquad (36)$$

$$C^{rel_1}\left(L_{\max}, L_{\min}\right) = \frac{L_{\max} - L_{\min}}{L_{\max}}, \qquad (37)$$

$$\tilde{C}^{rel_1}\left(L_1, L_2\right) = \frac{L_{\max} - L_{\min}}{L_{\max}} \cdot \frac{L_1 - L_2}{\max\left(L_1, L_2\right) - L_{\min}}. \qquad (38)$$

The proposed definition of perceived contrast using the definition (25) of relative contrast has the form:

$$C^{rel_2}\left(\tilde{L}_1, \tilde{L}_2\right) = \frac{L_1 - L_2}{L_{\max} - \min\left(L_1, L_2\right)}, \qquad (39)$$

$$C^{rel_2}\left(L_{\max}, L_{\min}\right) = \frac{L_{\max} - L_{\min}}{1 - L_{\min}}, \qquad (40)$$

$$\tilde{C}^{rel_2}\left(L_1,L_2\right)=\frac{L_{\max}-L_{\min}}{1-L_{\min}}\cdot\frac{L_1-L_2}{L_{\max}-\min\left(L_1,L_2\right)}. \quad (41)$$

Proposed assessments (32), (35), (38), (41) of perceived contrast on the basis of the known definitions (22) - (25) of contrast are the basis for the no-reference metrics of global contrast for multi-element images (Section II.A).

3D-graphs of surfaces for the proposed assessments (32), (35), (38), (41) of weighted and relative contrast for primary image Lena (Fig.1) [12] are shown in Fig. 8 - Fig. 11.

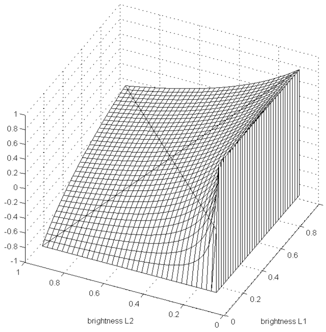

Fig. 8. $\tilde{C}^{wei_3}\left(L_1,L_2\right)$ (32)

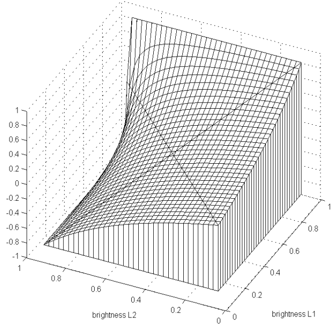

Fig. 9. $\tilde{C}^{wei_4}\left(L_1,L_2\right)$ (35)

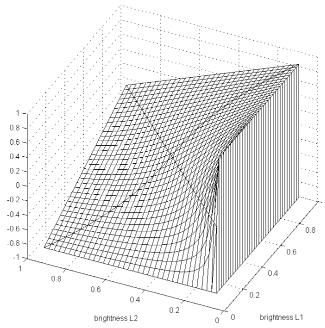

Fig. 10. $\tilde{C}^{rel_1}\left(L_1,L_2\right)$ (38)

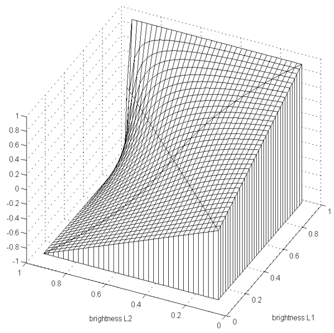

Fig. 11. $\tilde{C}^{rel_2}\left(L_1,L_2\right)$ (41)

Comparative analysis of known and proposed definitions of contrast of image elements was carried out in Section IV and Section V.

IV. RESEARCH

Experimental research was carried out by measuring of global contrast using known and proposed definitions of contrast of image elements for a group of nine test images.

The group of test images consists of nine real images with a complex structure and a limited dynamic range of brightness, the appearance of which is shown in Fig. 12.

Research was carried out by measuring of complete and incomplete integral contrast using known and proposed definitions of contrast of image elements, namely:

1) complete contrast (6) using weighted contrast (20);
2) complete contrast (5) using weighted contrast (22);
3) incomplete contrast (8) using weighted contrast (21);
4) known definition of incomplete contrast (8) using linear contrast [7]:

$$C_{inc}^{lin}=\int_0^1\left|\frac{L-L_0}{LMAX}+\frac{1}{2}-\left|\frac{L-L_0}{LMAX}-\frac{1}{2}\right|\right|p(L)dL, \quad (42)$$

where $LMAX$ - maximum possible value of brightness;

5) complete integral contrast (6) using proposed definition (32) of the perceived contrast for weighted contrast;
6) incomplete integral contrast (8) using proposed definition (32) of the perceived contrast for weighted contrast.

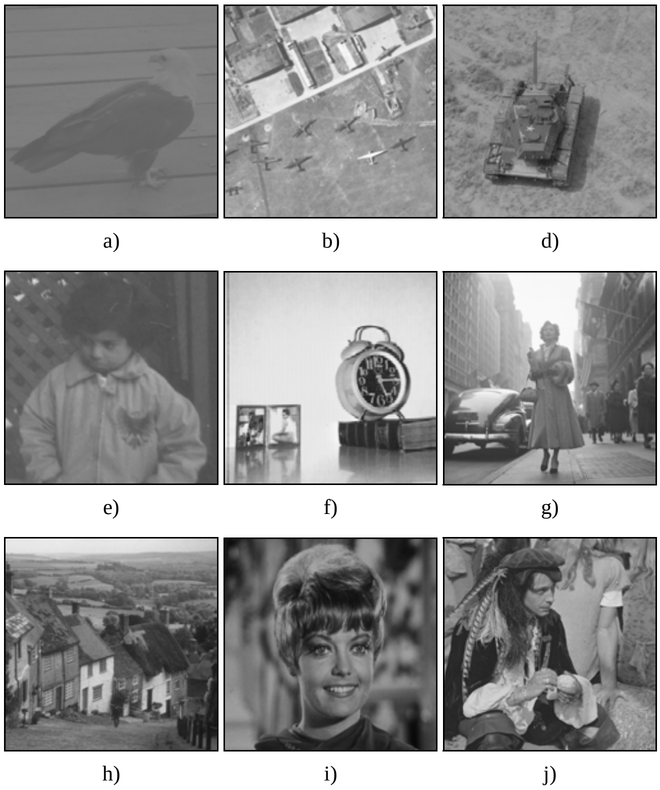

Fig. 12. The appearance of test images

The results of measuring of global contrast for test images (Fig. 12.a – Fig. 12.j) are shown in Table 1.

TABLE I. RESULTS OF CONTRAST MEASUREMENT FOR TEST IMAGES

	test images								
	a	b	d	e	f	g	h	i	j
$C_{com}^{wei_1}$	0.050	0.104	0.131	0.115	0.223	0.224	0.253	0.271	0.278
$C_{com}^{wei_3}$	0.044	0.101	0.119	0.113	0.190	0.218	0.246	0.260	0.262
$C_{inc}^{wei_2}$	0.068	0.134	0.162	0.175	0.264	0.298	0.325	0.347	0.363
C_{inc}^{lin}	0.062	0.174	0.162	0.153	0.372	0.339	0.304	0.256	0.323
$\tilde{C}_{com}^{wei_3}$	0.048	0.109	0.120	0.190	0.189	0.236	0.251	0.260	0.321
$\tilde{C}_{inc}^{wei_3}$	0.036	0.074	0.085	0.153	0.143	0.172	0.182	0.193	0.241

Graphs of values of global contrast for test images (Fig. 12.a - Fig. 12.j) are shown in Fig. 13.

Analysis of results of the research is carried out in Section V and Section VI.

V. DISCUSSION

The results of measurements of contrast for test images show that the value of the integral contrast significantly depends on the choice of the definition of contrast kernel

(Fig. 13). The values of the known definitions (20) - (25) of the weighted and relative contrast heavily depend on the changes in the average value of the brightness under the additive transformations of the brightness scale of image.

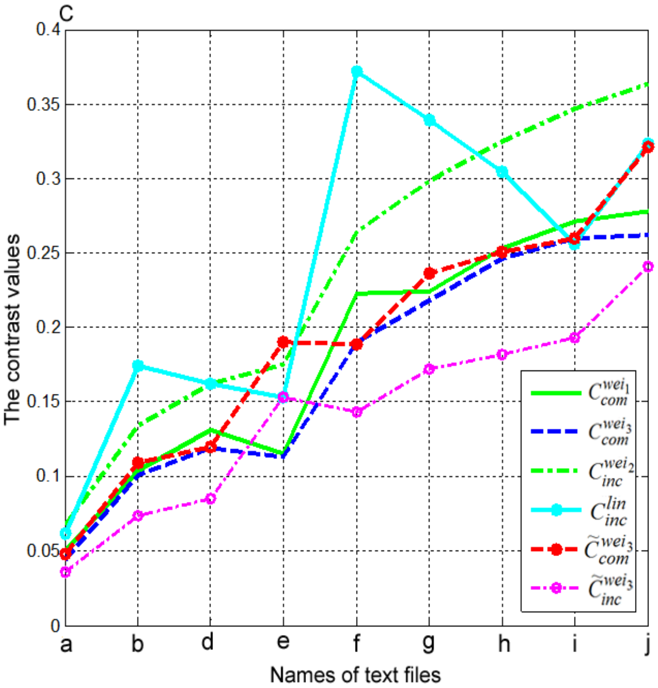

Fig. 13. Contrast values for test images (Fig. 12).

A proposed generalized description (26), (28) of the perceived contrast of elements of complex images is invariant to linear transformations of image brightness scale.

Values of known (22), (23), (24), (25) and proposed (32), (35), (38), (41) definitions for weighted and relative contrast coincide for pre-normalized images, for which $L_{min} = 0$ and $L_{max} = 1$.

The values $\tilde{C}^{wei_3}_{com}$ of the complete integral contrast (5) on the basis of the proposed definition (32) of weighted contrast are proportional to the values of the incomplete integral contrast $\tilde{C}^{wei_3}_{inc}$ (8) using weighted contrast (32).

The assessments (5), (8) on the basis of integral contrast of image using proposed definitions of weighted contrast (32) are the closest to the expert estimates of image contrast and are best suited to quantitative assessment of global contrast of images with complex structure and limited dynamic range of brightness.

VI. Conclusion

The problem of increasing the accuracy of measuring the perceived contrast of elements (objects and background) of a complex multi-element image was considered.

A new method of measuring the perceived contrast of two image elements (objects and background) by measuring their contrast on a normalized image with subsequent correction of the contrast value taking into account the dynamic range of the initial image was proposed.

A new generalized description of the perceived contrast of elements of complex images for various definitions of the contrast kernel was proposed.

The proposed generalized description of the perceived contrast satisfies the basic requirements for the definition of contrast and provides a sufficiently accurate quantitative assessment of the contrast of image elements for complex images, also allow predict the perceived contrast of the image when subjective expert assessments.

New definitions of the weighted and relative contrast of the image elements were proposed.

The proposed definitions of weighted and relative contrast allow to increase the accuracy and reliability of measuring the global contrast for multi-element monochrome images with limited dynamic range of brightness.

The proposed definitions of weighted and relative contrast can be recommended to estimate the generalized contrast of images in imaging, image processing and analysis in automatic mode.

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