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# Logic-Mathematical Model for Recognition the Dangerous Flight Events

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***Abstract*—The paper deal with increasing the data reliability by the use of parallel informational reservation. The mathematical models of systems for parallel informational reservation are obtained in the work. The conducted researches have shown that parallel reservation of information allows creating reliable information systems at low information possibilities of separate information sources.**

***Keywords—data reliability; mathematical models; parallel informational reservation; information-control system.***

## I. Introduction

Real sensors have the finite accuracy of representing the controlled information. At the same time, the accuracy and reliability of information is determined both by the design features and technical reliability of the sensors and, as a rule, does not satisfy or weakly satisfy the requirements for the accuracy and reliability of information, which fed to the inputs of computer systems for automatic control of technological processes.

It is known that statistical processing can significantly increase both the accuracy and reliability of the monitored information. It is possible by feeding information to computationally controlled systems simultaneously from several sensors connected in parallel or from the same sensor in series with a given rate. Such information input methods are called respectively parallel and sequential information reservation, which in principle allow to significantly increase the accuracy and reliability of the monitored information.

## II. Existing methods of estimating the efficiency

The analysis of informational and control systems (ICS) structures shows that their effectiveness depends on the reliability of the information upon which the relevant decisions are made. Therefore, it is necessary to take adequate actions to increase the reliability of information in the ICS. Solutions to the problem of validating the validity of data during the transmission and processing of information in process control systems are proposed in [1, 2, 3], where are considered the principles and methods of using statistic data redundancy in solving problems, related to control of information validity based on the minimum mean-square error criterion for different distribution laws of controlled parameters. This and other statistical methods stipulate the collection a large array of data on the operation parameters of the ICS and don't allow to evaluate their effectiveness during operation. The use of information reservation allows to reduce the shortcomings of statistical methods and to shorten the evaluation time.

## III. Problem statement

Informational reservation is a way to ensure the effectiveness of information and control systems by introducing reservation based on the system's information characteristics. Particularly high requirements for information are demanded to the process of control, recognition and localization the dangerous flight situations in aircraft's ICS. One of the commonest method to ensuring the reliability of information is the parallel informational reservation. The parallel informational reservation provides for use few measuring channel. On the one hand, this leads to an increase the accuracy and reliability of information, and on the other hand, this leads to a deterioration the technical and economic characteristics of the system: increasing the mass, dimensions and the cost of the system. Therefore, there is a need for evaluating parallel reservation systems with a different coefficient of majority in order to determine the optimal structure of a parallel reservation system. The main objective of paper is to ensure and evaluate the reliability of information with the use of parallel informational reservation. The objects of the research are the information and control systems of aircraft and processes of control, recognition and localization of dangerous flight situations.

## IV. Problem solution

Parallel informational reservation is a way to ensure the effectiveness of information and control systems, when data comes from several sources and a decision is made about the presence of a controlled parameter by the majority principle "*m* of *n*", that means that the m of n sources confirm the occurrence of a controlled parameter.

In accordance with the physical representation of the work the information sources (IS), a real IS can be in one of three incompatible random states: correct detection, false alarm and non-detection, determined by probabilities *a*, *b* and *d*, respectively. Such system can be represented using a trinomial probability distribution [4, 5], according to which the probability $p(n-m, m-k, k)$ that the *n* of *k* IS will not detect controlled phenomena at all, the *m*–*k* IS will work with false alarm, and *n*–*m* IS will provide the correct information about the controlled phenomenon. The probability $p(n-m, m-k, k)$ is described by the following expression:

$$P_{(n-m,m-k,k)} = C_n^{n-m} a^{n-m} C_m^{m-k} b^{m-k} d^k , \qquad (1)$$

wherein $a + b + d = 1$.

The application of the above-mentioned general theoretical premises can be shown on specific examples of the implementation the fire recognition systems inside of aircraft engines [6, 7].

Let the data collection system consist of two IS and be organized so that at the output of this system a signal $F_{1,2}$ will appear when at least one of the input IS is triggered. Such system is shown in Fig. 1.

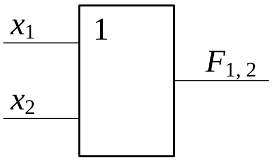


Fig. 1. The data collection system at Q = 1.

Through $F_{1,2}$ is designated the function of the system reaction, which consist from n IS, on incoming signals $x_1$, $x_2$. Through Q is designated the majority index, which shows the number of IS from n, which are necessary to make a decision.

The reliability of information, obtained with such system can be estimated by three probabilistic characteristics, namely, $p_{cd1,2}$ – the probability of correct detection, $p_{fa1,2}$ – probability of false alarm and $p_{nd1,2}$ – probability of non-detection. Of course, condition $P_{cdQ,n} + P_{faQ,n} + P_{ndQ,n} = 1$ is always satisfied. It is easy to show that the probability $p_{cd1,2}$ is determined by the following probabilities: $p(a^2)$ – the probability that both IS will work correctly $p(ab)$ – the probability that one IS will work correctly and the second IS will give false information and $p(ad)$ – the probability that one IS will work correctly and the second IS will not detect necessary information. Similarly, the probability $p_{fa1,2}$ includes the probability that both IS will give false information, the probability that one IS will work false, and the second will not detect necessary information. Finally, the probability $p_{nd1,2}$ for this scheme is estimated by the probability that both IS will not detect necessary information.

If all IS are the same in their characteristics, then we can get the following dependencies:

$$\left.\begin{aligned} P_{cd1,2} &= a^2 + 2ab + 2ad; \\ P_{nd1,2} &= d^2; \\ P_{fa1,2} &= b^2 + 2bd. \end{aligned}\right\} \qquad (2)$$

Let the data collection system, consisting of two IS, and be organized so that at the output of this system a signal $F_{2,2}$ will appear when both IS are triggered at the input. Such system is shown in Fig. 2.

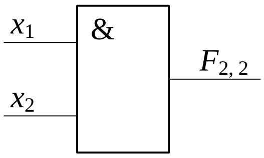


Fig. 2. The data collection system at Q = 2.

The probabilistic characteristics: $p_{cd2,2}$, $p_{fa2,2}$, $p_{nd2,2}$ of such system, for the identical in characteristics IS, can be described by the following expressions:

$$\left.\begin{aligned} P_{cd2,2} &= a^2 + 2ab; \\ P_{nd2,2} &= d^2 + 2ad + 2bd; \\ P_{fa2,2} &= b^2. \end{aligned}\right\} \qquad (3)$$

On the basis of the mathematical models of formulas (2) and (3), we can construct graphical dependencies of the probabilistic characteristics $p_{cd}$, $p_{fa}$, $p_{nd}$ from probabilities of correct detection by IS *a*, which are shown in Fig. 3

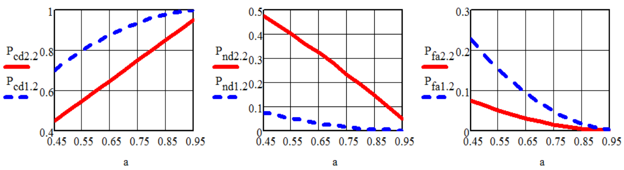


Fig. 3. The graphical dependencies for information system at n = 2.

Consider now a data collection system, which consist of three IS. Let this system be organized so that at the output of this system a signal $F_{1,3}$ will appear when at least one of the input IS is triggered. Such system is shown in Fig. 4.

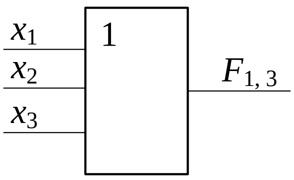


Fig. 4. The data collection system at Q = 1.

In view of the above the probabilistic characteristics: $p_{cd1,3}$, $p_{fa1,3}$, $p_{nd1,3}$ of such system, for the identical in characteristics IS, can be described by the following expressions:

$$
\left.\begin{aligned}
&P_{cd1,3} = a^3 + 3a^2 b + 3a^2 d + 3ab^2 + 3ad^2 + 6abd;\\
&P_{nd1,3} = d^3;\\
&P_{fa1,3} = b^3 + 3db^2 + 3bd^2.
\end{aligned}\right\} \quad (4)
$$

Let the data collection system, consisting of three IS, and be organized so that at the output of this system a signal $F_{2,3}$ will appear when at least two IS are triggered at the input. Such system is shown in Fig. 5.

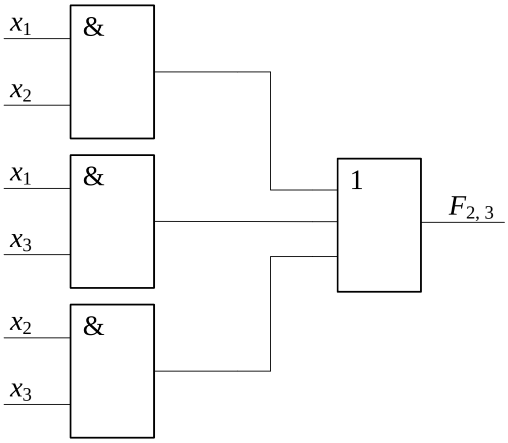


Fig. 5. The data collection system at Q = 2.

The probabilistic characteristics: $p_{cd2,3}$, $p_{fa2,3}$, $p_{nd2,3}$ of such system (Fig.5), for the identical in characteristics IS, can be described by the following expressions:

$$
\left.\begin{aligned}
&P_{cd2,3} = a^3 + 3a^2 b + 3a^2 d + 3ab^2 + 6abd;\\
&P_{nd2,3} = d^3 + 3ad^2 + 3bd^2;\\
&P_{fa2,3} = b^3 + 3db^2.
\end{aligned}\right\} \quad (5)
$$

Let the data collection system, consisting of three IS, and be organized so that at the output of this system a signal $F_{3,3}$ will appear when at least three IS are simultaneous triggered at the input. Such system is shown in Fig. 6.

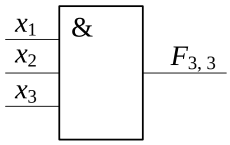


Fig. 6. The data collection system at Q = 3.

The probabilistic characteristics: $p_{cd3,3}$, $p_{fa3,3}$, $p_{nd3,3}$ of such system (Fig.6), for the identical in characteristics IS, can be described by the following expressions:

$$
\left.\begin{aligned}
&P_{cd3,3} = a^3 + 3a^2 b + 3ab^2;\\
&P_{nd3,3} = d^3 + 3ad^2 + 3bd^2 + 3db^2 + 3a^2 d + 6abd;\\
&P_{fa3,3} = b^3.
\end{aligned}\right\} \quad (6)
$$

On the basis of the mathematical models of formulas (4), (5) and (6) we can construct graphical dependencies of the probabilistic characteristics $p_{cd}$, $p_{fa}$, $p_{nd}$ from probabilities of correct detection by IS *a*, which are shown in Fig. 7.

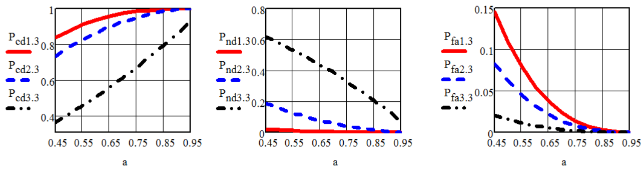

Fig. 7. The graphical dependencies for information system at n = 3.

Consider now a data collection system, which consist of four IS. Let this system be organized so that at the output of this system a signal $F_{1,4}$ will appear when at least one of the input IS is triggered. Such system is shown in Fig. 8.

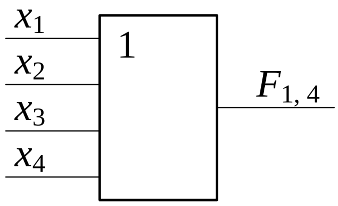


Fig. 8. The data collection system at Q = 1.

The probabilistic characteristics: $p_{cd1,4}$, $p_{fa1,4}$, $p_{nd1,4}$ of such system (Fig.8), for the identical in characteristics IS, can be described by the following expressions:

$$
\left.\begin{aligned}
&P_{cd1,4} = a^4 + 4a^3 d + 4ad^3 + 4a^3 b + 4ab^3 + 6a^2 d^2 +\\
&+ 6a^2 b^2 + 12a^2 bd + 12ab^2 d + 12abd^2;\\
&P_{nd1,4} = d^4;\\
&P_{fa1,4} = b^4 + 4db^3 + 4bd^3 + 6b^2 d^2.
\end{aligned}\right\} \quad (7)
$$

Let the data collection system, consisting of four IS, and be organized so that at the output of this system a signal $F_{2,3}$ will appear when at least two IS are triggered at the input. Such system is shown in Fig. 9.

The probabilistic characteristics: $p_{cd2,4}$, $p_{fa2,4}$, $p_{nd2,4}$ of such system (Fig.9), for the identical in characteristics IS, can be described by the following expressions:

$$
\left.\begin{aligned}
&P_{cd2,4} = a^4 + 4a^3 d + 4a^3 b + 4ab^3 + 6a^2 d^2 + 6a^2 b^2 +\\
&+ 12a^2 bd + 12ab^2 d + 12abd^2;\\
&P_{nd2,4} = d^4 + 4bd^3 + 4ad^3;\\
&P_{fa2,4} = b^4 + 4db^3 + 6b^2 d^2.
\end{aligned}\right\} \quad (8)
$$

Let the data collection system, consisting of four IS, and be organized so that at the output of this system a signal $F_{2,3}$ will appear when at least three IS are triggered at the input. Such system is shown in Fig. 10.

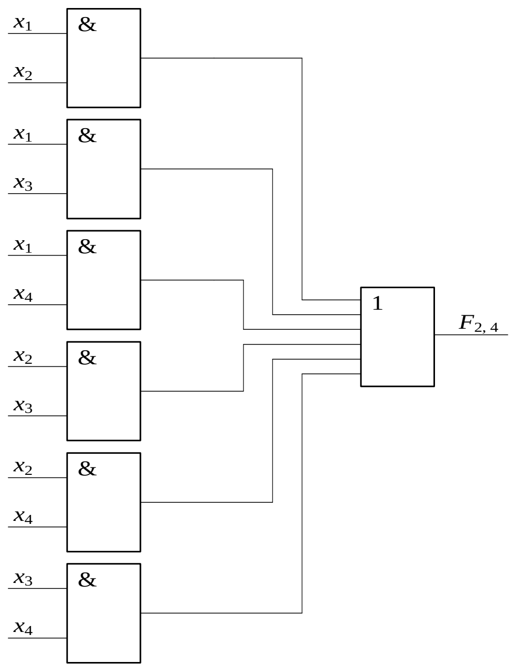


Fig. 9. The data collection system at Q = 2.

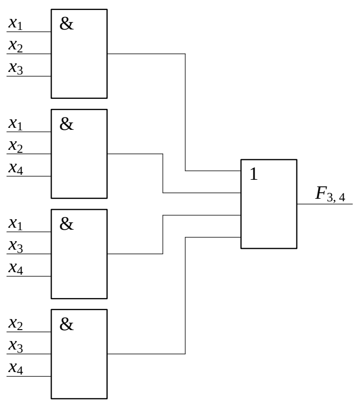


Fig. 10. The data collection system at Q = 3.

The probabilistic characteristics: $p_{cd3,4}$, $p_{fa3,4}$, $p_{nd3,4}$ of such system (Fig.10), for the identical in characteristics IS, can be described by the following expressions:

$$\left.\begin{aligned} &P_{cd3,4} = a^4 + 4a^3 d + 4a^3 b + 4ab^3 + 6a^2 b^2 + 12a^2 bd + \\ &+12ab^2 d; \\ &P_{nd3,4} = d^4 + 4bd^3 + 6b^2 d^2 + 4ad^3 + 6a^2 d^2 + 12abd^2; \\ &P_{fa3,4} = b^4 + 4db^3. \end{aligned}\right\} \quad (9)$$

The data collection system, which consist of four IS can be arranged and so that the output signal $F_{4,4}$ appears only when at least 4 sensors are triggered at the input. Such system is shown in Fig. 11.

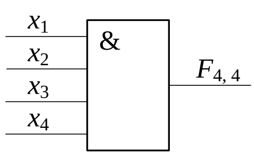


Fig. 11. The data collection system at Q = 4.

The probabilistic characteristics: $p_{cd4,4}$, $p_{fa4,4}$, $p_{nd4,4}$ of such system (Fig.10), for the identical in characteristics IS, can be described by the following expressions:

$$\left.\begin{aligned} &P_{cd4,4} = a^4 + 4a^3 b + 4ab^3 + 6a^2 b^2; \\ &P_{nd4,4} = d^4 + 4bd^3 + 4a^3 d + 4db^3 + 6b^2 d^2 + 4ad^3 + \\ &+6a^2 d^2 + 12a^2 bd + 12ab^2 d + 12abd^2; \\ &P_{fa4,4} = b^4. \end{aligned}\right\} \quad (10)$$

On the basis of the mathematical models of formulas (7), (8), (9) and (10) we can construct graphical dependencies of the probabilistic characteristics $p_{cd}$, $p_{fa}$, $p_{nd}$ from probabilities of correct detection by IS *a*, which are shown in Fig. 12.

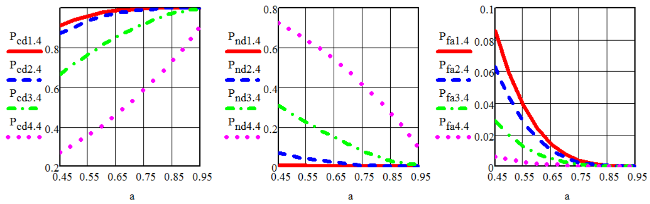


Fig. 12. The graphical dependencies for information system at n = 4.

The results of calculation the given probability characteristics $p_{cd}$, $p_{fa}$, $p_{nd}$ are shown in Tables 1, 2 and 3.

The use of systems with majority logic and increasing the requirements for the IS (the operation threshold due to the use of comparators and the "restoration" the level of parametric redundancy in operation) can significantly increase the probability of correct detection $p_{cd}$ and false alarm $p_{fa}$. So, with *a=0,9, b=0,05, d=0,05*, we obtain $p_{cd}$ =0,994, $p_{nd} = 0{,}003$, $p_{fa} = 0{,}003$, and when $a = 0{,}95$, $b = d = 0{,}0025$ the probability of a correct detection is $p_{cd} = 0{,}999$, with a further decrease $p_{nd}$ and $p_{fa}$.

TABLE I. RESULTS OF CALCULATION THE PROBABILITY CHARACTERISTICS. PART 1.

| *a* | b | d | $F_{\upsilon,n}$ | $P_{cd\upsilon,n}$ | $P_{nd\upsilon,n}$ | $P_{fa\upsilon,n}$ |
|---|---|---|---|---|---|---|
| 1/3 | 1/3 | 1/3 | $F_{1,2}$ | 0,556 | 0,111 | 0,333 |
| | | | $F_{2,2}$ | 0,333 | 0,556 | 0,111 |
| | | | $F_{1,3}$ | 0,705 | 0,035 | 0,26 |
| | | | $F_{2,3}$ | 0,592 | 0,26 | 0,148 |
| | | | $F_{3,3}$ | 0,26 | 0,705 | 0,035 |
| | | | $F_{1,4}$ | 0,80247 | 0,01235 | 0,18519 |
| | | | $F_{2,4}$ | 0,75309 | 0,11111 | 0,13580 |
| | | | $F_{3,4}$ | 0,53086 | 0,40741 | 0,06173 |
| | | | $F_{4,4}$ | 0,18518 | 0,80247 | 0,01235 |

TABLE II. RESULTS OF CALCULATION THE PROBABILITY CHARACTERISTICS. PART 2.

| *a* | b | d | $F_{\upsilon,n}$ | $P_{cd\upsilon,n}$ | $P_{nd\upsilon,n}$ | $P_{fa\upsilon,n}$ |
|---|---|---|---|---|---|---|
| 1/2 | 1/4 | 1/4 | $F_{1,2}$ | 0,75 | 0,062 | 0,108 |
| | | | $F_{2,2}$ | 0,5 | 0,437 | 0,062 |
| | | | $F_{1,3}$ | 0,876 | 0,016 | 0,108 |
| | | | $F_{2,3}$ | 0,782 | 0,156 | 0,062 |
| | | | $F_{3,3}$ | 0,406 | 0,58 | 0,014 |
| | | | $F_{1,4}$ | 0,996093 | 0,000244 | 0,0036621 |
| | | | $F_{2,4}$ | 0,9902344 | 0,0070801 | 0,0026855 |
| | | | $F_{3,4}$ | 0,919921 | 0,078857 | 0,001221 |
| | | | $F_{4,4}$ | 0,6914063 | 0,3083496 | 0,0002441 |

TABLE III. RESULTS OF CALCULATION THE PROBABILITY CHARACTERISTICS. PART 3.

| \ | b | d | $F_{\upsilon,n}$ | $P_{cd\upsilon,n}$ | $P_{nd\upsilon,n}$ | $P_{fa\upsilon,n}$ |
|---|---|---|---|---|---|---|
| 3/4 | 1/8 | 1/8 | $F_{1,2}$ | 0,94 | 0,0156 | 0,0444 |
| | | | $F_{2,2}$ | 0,75 | 0,234 | 0,016 |
| | | | $F_{1,3}$ | 0,986 | 0,002 | 0,012 |
| | | | $F_{2,3}$ | 0,948 | 0,043 | 0,009 |
| | | | $F_{3,3}$ | 0,669 | 0,33 | 0,001 |
| | | | $F_{1,4}$ | 0,93750 | 0,00391 | 0,05859 |
| | | | $F_{2,4}$ | 0,90625 | 0,05078 | 0,04297 |
| | | | $F_{3,4}$ | 0,71875 | 0,26172 | 0,01953 |
| | | | $F_{4,4}$ | 0,31250 | 0,68359 | 0,00391 |

Analysis of the graphs shown in Fig. 3, Fig. 7, and Fig.12 allows to make the following conclusions. With increasing the requirements for the reliability of information systems, namely, when Q is the maximum, then the probability of $p_{nd}$ becomes higher than the probability of $p_{fa}$. That is, such system successfully suppresses the probability of $p_{fa}$ and, at the same time, has no significant effect on $p_{nd}$. When Q is minimal, the information system successfully suppresses the probability of $p_{nd}$ and has little effect on the probability $p_{fa}$. At $Q_{m,n}$, where m lies between 1 and n, the information system will successfully suppress both $p_{fa}$ and $p_{nd}$ simultaneously. Thus, depending on the technical and economic requirements, we can select the necessary structure for the system of recognizing dangerous flight events.

Analysis of the probability characteristics $p_{cd}$, $p_{fa}$, $p_{nd}$ determined by formulas (2) - (10) allows us to make the following conclusions:

– To improve the quality of information systems consisting of n IS, in the sense of increasing the reliability of information, can be done at least by three ways: to increase in the number *n* of information sources; improving the IS characteristics *a*, *b*, *d*; choosing the optimal information structure, by choosing the correct majority index Q.

– For information systems made of n information sources with identical probabilistic characteristics, the most acceptable is such structure, when the majority index $Q = n/2$.

## V. CONCLUSION

The question of increasing the data reliability by the use of parallel informational reservation and mathematical models of system for parallel informational reservation has examined in the paper. The method of parallel informational reservation significantly reduces the probability of non-detection and has little effect on reducing the probability of false alarm. Application of the majority principle, allows reducing the probability of false alarm, but it is necessary to increase the number of parallel channels, which is due to economic constraints. The probability of false alarm can be reduced by the method of "hardening" (reducing sensitivity of sensors by raising their threshold) of individual information sources, while increasing the number of IS compensates the lacks of this method.

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