IEEE Second International Conference on Data Stream Mining & Processing
August 21-25, 2018, Lviv, Ukraine

Computational Library of the Direct Analytic Models for Real-Time Fuzzy Information Processing

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***Abstract*—This paper reveals the computational library of the analytic models for the results of fuzzy arithmetic operations with fuzzy sets. In particular, the focus is on the synthesis of the universal inverse and direct models for maximum of triangular fuzzy numbers with different masks of their parameters. The results of the study verify the efficiency of the suggested computational library with soft computing models for fuzzy information processing in real-time control and decision making.**

***Keywords*—computational library, fuzzy number, arithmetic operation, maximum, fuzzy information processing**

I. INTRODUCTION

The development of the efficient methods for big data analysis and dynamic information processing in the real-time is one of the most salient responsibilities in the signal processing, as well as control and decision making in uncertainty [1-4].

The big volume of data and high speed of its appearance requires using special mathematical approaches developed in the theory of artificial intelligence and computational optimization [5]. In some cases when the complexity of developing analytical models to ensure efficient functioning of processes and systems in the conditions of uncertainty, it is necessary to advance and develop new mathematical methods and algorithms [6]. One of these approaches, flexible to solving real-world problem, is a theory of fuzzy sets and fuzzy logic, initially developed in 1965 by Lotfi A. Zadeh [7].

Ever since, the theory of fuzzy sets has set grounds for compelling scientific and technological developments, in particular, in terms of its mathematical methods and their diverse applicability. These theoretical advancements in the theory of fuzzy sets and fuzzy logic receive a substantial attention from the global academic community [8-13].

We further proceed with analyzing a fuzzy set $\underset{\sim}{A}$ as pairs $\left(x, \mu_{\underset{\sim}{A}}(x)\right)$, specified on the universal set [7,14-16] and any element $x, x \in E$ of the fuzzy set $\underset{\sim}{A}$, that corresponds to the specific value of the membership function (MF) $\mu_{\underset{\sim}{A}}(x) \in [0,1]$.

Fuzzy sets and fuzzy logic allow solving different tasks in uncertain conditions in the field of complex systems control and decision-making in economics, management, engineering and logistics [5,6,8,17-19], in particular, in marine transportation [19-21], investment [6], finances [22] and other fields. Special attention is paid to data analysis using fuzzy mathematics and soft computing [5,23,24].

In many cases, developing the solution to the problems require fulfilling diverse fuzzy arithmetic operations, such as addition, subtraction, multiplication division, minimum and maximum calculations [4,17,25-28].

Research cautions using the inverse (horizontal) models of resulting membership functions (MFs) based on using α-cuts. It appears that using these models in solving control tasks in real time often results in compromised quality of computing operations performance [2,8,29-31].

Hence, our study aims to offer advancements in the field of universal direct analytic models that grant improvement in operating speed and accuracy of fuzzy arithmetic operations. This paper contributes to the literatures on the fuzzy information processing and data analysis [4,27]. We further proceed with developments in one of the most complex fuzzy arithmetic operations, an operation of maximum of the fuzzy numbers (FNs-maximum).

II. PROBLEM STATEMENT

Arithmetic algorithms for the FNs- maximum operations based on the α-cuts [14-16] possess high computational complexity, considering it is executed in turn for all α-levels with $\Delta\alpha$ discreteness level, which value strongly influences the computational processes' accuracy and operating speed [4,15,27,28].

Therefore, α-cuts of the fuzzy set $\underset{\sim}{A} \in R$ is ordinary subset $A_\alpha = \{x | \mu_{\underset{\sim}{A}}(x) \ge \alpha\}, \quad \alpha \in [0,1]$, that contains (Fig. 1) elements $x \in R$ whose degree of membership to a set $\underset{\sim}{A}$ is not less than α. The subsets A_α та B_α that determine the appropriate α-cuts of fuzzy sets $\underset{\sim}{A}, \underset{\sim}{B} \in R$ can be written as

$$A_\alpha = [a_1(\alpha),\ a_2(\alpha)],\ B_\alpha = [b_1(\alpha),\ b_2(\alpha)],\ \alpha \in [0,1],$$

where R is real numbers set.

The apiration of this work is to grant the synthesis of the computational library of universal analytical models of resulting MFs for the FNs-maximum of triangular fuzzy numbers (TrFNs) with different combinations of their parameters (Fig. 1) in order (a) to increase operating speed and (b) to lower the volume, complexity and accuracy of fuzzy information processing. The TrFNs $\underset{\sim}{A}=(a_1,a_0,a_2)$ and $\underset{\sim}{B}=(b_1,b_0,b_2)$ have MFs $\mu_{\underset{\sim}{A}}(x)$ and $\mu_{\underset{\sim}{B}}(x)$ with parameters $\mu_{\underset{\sim}{A}}(a_1)=\mu_{\underset{\sim}{A}}(a_2)=\mu_{\underset{\sim}{B}}(b_1)=\mu_{\underset{\sim}{B}}(b_2)=0.$ $\mu_{\underset{\sim}{A}}(a_0)=\mu_{\underset{\sim}{B}}(b_0)=1$. The inverse A_α , B_α and direct $\mu_{\underset{\sim}{A}}(x)$, $\mu_{\underset{\sim}{B}}(x)$ models of the TrFNs $\underset{\sim}{A},\underset{\sim}{B}\in R$ can be

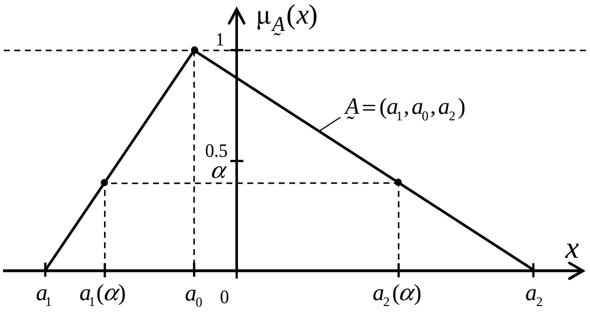

Fig. 1. Triangular Fuzzy Number $\underset{\sim}{A}$, $\underset{\sim}{A}\in R$

determined [4,14-16,27,28] by the corresponding dependencies (1)-(4):

$$A_\alpha=\left[a_1(\alpha),a_2(\alpha)\right]=\left[a_1+\alpha(a_0-a_1),a_2-\alpha(a_2-a_0)\right], \quad (1)$$

$$B_\alpha=\left[b_1(\alpha),b_2(\alpha)\right]=\left[b_1+\alpha(b_0-b_1),b_2-\alpha(b_2-b_0)\right], \quad (2)$$

$$\mu_{\underset{\sim}{A}}(x)=\begin{cases}0,\forall(x\le a_1)\cup(x\ge a_2)\\ F_{Al}(x,a_1,a_0),\forall(a_1<x\le a_0),\\ F_{Ar}(x,a_0,a_2),\forall(a_0<x<a_2)\end{cases} \quad (3)$$

$$\mu_{\underset{\sim}{B}}(x)=\begin{cases}0,\forall(x\le b_1)\cup(x\ge b_2)\\ F_{Bl}(x,b_1,b_0),\forall(b_1<x\le b_0),\\ F_{Br}(x,b_0,b_2),\forall(b_0<x<b_2)\end{cases} \quad (4)$$

where $F_{Al}(x,a_1,a_0)=(x-a_1)/(a_0-a_1);$

$$F_{Bl}(x,b_1,b_0)=(x-b_1)/(b_0-b_1);$$

$$F_{Ar}(x,a_0,a_2)=(a_2-x)/(a_2-a_0);$$

$$F_{Br}(x,b_0,b_2)=(b_2-x)/(b_2-b_0).$$

Using Max-Min or Min-Max convolutions for FNs-maximum realization [14-16] in some cases results in increased complexity and lower speed of processing or to the resulting fuzzy sets with violation of the convexity and normality properties.

The operation of FNs- maximum $\left(\underset{\sim}{C}=\underset{\sim}{A}(\vee)\underset{\sim}{B}\right)$ based on α -cuts [14] can be written as

$$\begin{aligned}C_\alpha=A_\alpha(\vee)B_\alpha&=[a_1(\alpha),\ a_2(\alpha)](\vee)[b_1(\alpha),\ b_2(\alpha)]=\\ &=[a_1(\alpha)\vee b_1(\alpha),\ a_2(\alpha)\vee b_2(\alpha)]=[c_1(\alpha),\ c_2(\alpha)].\end{aligned} \quad (5)$$

III. Synthesis of Inverse and Direct Resulting Models

Firstly, let us analyze the separate intersections of the left branches of TrFNs

$$F_{left}(x,a_1,a_0)\cap F_{left}(x,b_1,b_0):\underset{\sim}{A},\underset{\sim}{B}\in R \quad (6)$$

and right branches of TrFNs

$$F_{Ar}(x,a_0,a_2)\cap F_{Br}(x,b_0,b_2):\underset{\sim}{A},\underset{\sim}{B}\in R\,. \quad (7)$$

If the condition (6) exists for $\alpha\in[0,1]$ then

$$a_1(\alpha)=b_1(\alpha)=c_1(\alpha), \quad (8)$$

and we can write

$$a_1+\alpha(a_0-a_1)=b_1+\alpha(b_0-b_1), \quad (9)$$

taking into account that

$$a_1(\alpha)=a_1+\alpha(a_0-a_1)$$

and

$$b_1(\alpha)=b_1+\alpha(b_0-b_1).$$

From (8), (9) we can find intersection parameter

$$\alpha=(b_1-a_1)/(a_0-a_1-b_0+b_1). \quad (10)$$

The value (10) corresponds to the vertical coordinate α^* of intersection point (6)

$$\alpha^*=\mu_{\underset{\sim}{A}}(x^*)=\mu_{\underset{\sim}{B}}(x^*)=\mu_{\underset{\sim}{C}}(x^*), \quad (11)$$

where x^* is a horizontal coordinate of the intersection point (6). In this case, two pairs of coordinates $\left(a_1(\alpha^*),\alpha^*\right)$ for inverse model and $\left(x^*,\mu_{\underset{\sim}{A}}(x^*)\right)$ for direct model are corresponding to the intersection point (6), where $x^*=a_1(\alpha^*)$, $\mu_{\underset{\sim}{A}}(x^*)=\alpha^*$. Using (1) and (3) we can find

$$a_1\left(\alpha^*\right)=a_1+\alpha^*\left(a_0-a_1\right)=a_1+\frac{\left(b_1-a_1\right)\left(a_0-a_1\right)}{a_0-a_1-b_0+b_1}, \quad (12)$$

where $a_1\left(\alpha^*\right)\in\left[\max\left(a_1,b_1\right),\max\left(a_0,b_0\right)\right]$.

If the condition (7) exists for $\alpha\in[0,1]$, then analyzing right branches of the TrFNs (3), (4) and intersection condition (7) we can find, in the same way, two pairs of the intersection point's (7) coordinates for inverse model $\left(a_2\left(\alpha^{**}\right),\alpha^{**}\right)$ and for direct model $\left(x^{**},\mu_{\underset{\sim}{A}}\left(x^{**}\right)\right)$, where $x^{**}=a_1\left(\alpha^{**}\right)$ and $\mu_{\underset{\sim}{A}}\left(x^{**}\right)=\alpha^{**}$:

$$\alpha^{**}=\left(b_2-a_2\right)/\left(b_2-b_0-a_2+a_0\right), \quad (13)$$

$$a_2\left(\alpha^{**}\right)=a_2-\alpha^{**}\left(a_2-a_0\right)=a_2-\frac{\left(b_2-a_2\right)\left(a_2-a_0\right)}{b_2-b_0-a_2+a_0}, \quad (14)$$

where $a_2\left(\alpha^{**}\right)\in\left[\max\left(a_0,b_0\right),\max\left(a_2,b_2\right)\right]$.

Thus, the coordinates $\left(a_1\left(\alpha^*\right),\alpha^*\right)$ and $\left(a_2\left(\alpha^{**}\right),\alpha^{**}\right)$ for the intersections (6) and (7) can be calculated using universal models (11)-(14) and parameters of the TrFNs $\underset{\sim}{A}=\left(a_1,a_0,a_2\right)$ and $\underset{\sim}{B}=\left(b_1,b_0,b_2\right)$. In case, if $a_1<b_1, a_0>b_0, a_2<b_2$, the inverse and direct models of resulting MF can be presented as

$$C_\alpha=A_\alpha(\vee)B_\alpha=[a_1(\alpha)\vee b_1(\alpha),\ a_2(\alpha)\vee b_2(\alpha)]= [c_1(\alpha),\ c_2(\alpha)]= \left[\begin{Bmatrix}b_1(\alpha),\forall\alpha|\alpha\in\left[0,\alpha^*\right]\\ a_1(\alpha),\forall\alpha|\alpha\in\left[\alpha^*,1\right]\end{Bmatrix},\ \begin{Bmatrix}a_2(\alpha),\forall\alpha|\alpha\in\left[\alpha^{**},1\right]\\ b_2(\alpha),\forall\alpha|\alpha\in\left[0,\alpha^{**}\right]\end{Bmatrix}\right], \quad (15)$$

$$\mu_{\underset{\sim}{C}}(x)=\begin{cases}0,\forall\left(x\le b_1\right)\cup\left(x\ge b_2\right)\\ F_{Bl}\left(x,b_1,b_0\right),\forall\left(b_1<x\le a_1\left(\alpha^*\right)\right)\\ F_{Al}\left(x,a_1,a_0\right),\forall\left(a_1\left(\alpha^*\right)<x\le a_0\right)\\ F_{Ar}\left(x,a_0,a_2\right),\forall\left(a_0<x<a_2\left(\alpha^{**}\right)\right)\\ F_{Br}\left(x,b_0,b_2\right),\forall\left(a_2\left(\alpha^{**}\right)<x<b_2\right)\end{cases}, \quad (16)$$

where $c_1(0)=b_1; c_2(0)=b_2; c_1(1)=c_2(1)=a_0$;

$$c_1(\alpha)=\begin{Bmatrix}b_1+\alpha\left(b_0-b_1\right),\forall\alpha|\alpha\in\left[0,\alpha^*\right]\\ a_1+\alpha\left(a_0-a_1\right),\forall\alpha|\alpha\in\left[\alpha^*,1\right]\end{Bmatrix};$$

$$c_2(\alpha)=\begin{Bmatrix}a_2-\alpha\left(a_2-a_0\right),\forall\alpha|\alpha\in\left[\alpha^{**},1\right]\\ b_2-\alpha\left(b_2-b_0\right),\forall\alpha|\alpha\in\left[0,\alpha^{**}\right]\end{Bmatrix}.$$

IV. Computational Library of Direct Resulting Models

The inverse C_α (15) and the direct $\mu_{\underset{\sim}{C}}(x)$ (16) models for the FNs-maximum are validated only for TrFNs $\underset{\sim}{A}=\left(a_1,a_0,a_2\right)$ and $\underset{\sim}{B}=\left(b_1,b_0,b_2\right)$ under the following conditions:

$$a_1<b_1, a_0>b_0, a_2<b_2.$$

Simultaneously much of the real input values for fuzzy processing can be conferred as TrFNs with diverse relations $\mathbb{R}$, $\mathbb{R}\in\{(<),(>)\}$ between parameters:

$$a_1\mathbb{R}b_1, a_0\mathbb{R}b_0, a_2\mathbb{R}b_2. \quad (17)$$

Therefore, for each special case it is necessary to develop a separate analytic model of resulting fuzzy set for the performance of "FNs-maximum" if the TrFNs $\left(\underset{\sim}{A},\underset{\sim}{B}\right)$ have diverse relations $\mathbb{R}$ between parameters $\left(a_1,b_1;a_0,b_0;a_2,b_2\right)$.

Let us form the set of direct analytic models of the resulting fuzzy sets $\underset{\sim}{C}$ for execution of the "maximum" as arithmetic operation with TrFNs $\underset{\sim}{A}$ and $\underset{\sim}{B}$ for diverse combinations of the relations $\mathbb{R}$. For evaluaion of the relations $\mathbb{R}$ and following [4,15,27,28], we can determine a mask

$$\text{Mask}\left(\underset{\sim}{A},\underset{\sim}{B}\right)=\{d,g,p\} \quad (18)$$

for any pair of the TrFNs $\underset{\sim}{A}$ and $\underset{\sim}{B}$, where indicators d, g and p are defined as

$$d=\begin{cases}0, & \text{if } a_1>b_1\\ 1, & \text{if } a_1<b_1\end{cases}; \quad g=\begin{cases}0, & \text{if } a_0>b_0\\ 1, & \text{if } a_0<b_0\end{cases}; \quad p=\begin{cases}0, & \text{if } a_2>b_2\\ 1, & \text{if } a_2<b_2\end{cases}. \quad (19)$$

The Mask (18) is a the basis for creating a 8-component's library of the resulting mathematical models $\{M_1...M_8\}$ for FNs-maximum with all possible $\mathbb{R}$ combinations of TrFNs $\left(\underset{\sim}{A},\underset{\sim}{B}\right)$. The computational library $\{M_1,M_2,...,M_8\}$ of the developed direct models $\mu_{\underset{\sim}{C}}(x)$ is represented in the Table I.

TABLE I. LIBRARY OF THE RESULTING DIRECT MODELS

Mask	M_i	Model description
$\{1,1,1\}$	M_1	$\begin{cases} 0,\forall(x\le b_1)\cup(x\ge b_2) \\ F_{Bl}(x,b_1,b_0),\forall(b_1<x\le b_0) \\ F_{Br}(x,b_0,b_2),\forall(b_0<x<b_2) \end{cases}$
$\{1,1,0\}$	M_2	$\begin{cases} 0,\forall(x\le b_1)\cup(x\ge a_2) \\ F_{Bl}(x,b_1,b_0),\forall(b_1<x\le b_0) \\ F_{Br}(x,b_0,b_2),\forall(b_0<x<a_2(\alpha^{**})) \\ F_{Ar}(x,a_0,a_2),\forall(a_2(\alpha^{**})\le x<a_2) \end{cases}$
$\{1,0,1\}$	M_3	$\begin{cases} 0,\forall(x\le b_1)\cup(x\ge b_2) \\ F_{Bl}(x,b_1,b_0),\forall(b_1<x\le a_1(\alpha^{*})) \\ F_{Al}(x,a_1,a_0),\forall(a_1(\alpha^{*})<x\le a_0) \\ F_{Ar}(x,a_0,a_2),\forall(a_0<x<a_2(\alpha^{**})) \\ F_{Br}(x,b_0,b_2),\forall(a_2(\alpha^{**})\le x<b_2) \end{cases}$
$\{1,0,0\}$	M_4	$\begin{cases} 0,\forall(x\le b_1)\cup(x\ge a_2) \\ F_{Bl}(x,b_1,b_0),\forall(b_1<x\le a_1(\alpha^{*})) \\ F_{Al}(x,a_1,a_0),\forall(a_1(\alpha^{*})<x\le a_0) \\ F_{Ar}(x,a_0,a_2),\forall(a_0<x<a_2) \end{cases}$
$\{0,1,1\}$	M_5	$\begin{cases} 0,\forall(x\le a_1)\cup(x\ge b_2) \\ F_{Al}(x,a_1,a_0),\forall(a_1<x\le a_1(\alpha^{*})) \\ F_{Bl}(x,b_1,b_0),\forall(a_1(\alpha^{*})<x\le b_0) \\ F_{Br}(x,b_0,b_2),\forall(b_0<x<b_2) \end{cases}$
$\{0,1,0\}$	M_6	$\begin{cases} 0,\forall(x\le a_1)\cup(x\ge a_2) \\ F_{Al}(x,a_1,a_0),\forall(a_1<x\le a_1(\alpha^{*})) \\ F_{Bl}(x,b_1,b_0),\forall(a_1(\alpha^{*})<x\le b_0) \\ F_{Br}(x,b_0,b_2),\forall(b_0<x<a_2(\alpha^{**})) \\ F_{Ar}(x,a_0,a_2),\forall(a_2(\alpha^{**})\le x<a_2) \end{cases}$
$\{0,0,1\}$	M_7	$\begin{cases} 0,\forall(x\le a_1)\cup(x\ge a_2) \\ F_{Al}(x,a_1,a_0),\forall(a_1<x\le a_0) \\ F_{Ar}(x,a_0,a_2),\forall(a_0<x<a_2(\alpha^{**})) \\ F_{Br}(x,b_0,b_2),\forall(a_2(\alpha^{**})\le x<b_2) \end{cases}$
$\{0,0,0\}$	M_8	$\begin{cases} 0,\forall(x\le a_1)\cup(x\ge a_2) \\ F_{Al}(x,a_1,a_0),\forall(a_1<x\le a_0) \\ F_{Ar}(x,a_0,a_2),\forall(a_0<x<a_2) \end{cases}$

V. EXAMPLE OF COMPUTATIONAL LIBRARY APPLICATION

Let's consider an example with realisation of the arithmetic operation "maximum" for the pair $(\underset{\sim}{A},\underset{\sim}{B})$ of TrFNs: $\underset{\sim}{A}=(3,10,17)$, $\underset{\sim}{B}=(5,7,24)$. In this case, we have: $a_1=3$; $b_1=5$; $a_0=10$; $b_0=7$; $a_2=17$; $b_2=24$.

Using (18), (19), we can automatically determine (a) the corresponding Mask $(\underset{\sim}{A},\underset{\sim}{B})=\{d,g,p\}=\{1,0,1\}$ for the conditions $a_1<b_1$; $a_0>b_0$; $a_2<b_2$ and (b) the corresponding model M_3 from the computational library of models $\{M_1,M_2,...,M_8\}$ (Table I). Let's calculate the coordinates $(a_1(\alpha^*),\alpha^*)$ and $(a_2(\alpha^{**}),\alpha^{**})$ for intersection points (6) and (7) of the given fuzzy numbers $(\underset{\sim}{A},\underset{\sim}{B})$ according to (12), (11), (14) and (13):

$$a_1(\alpha^*)=3+\frac{(5-3)(10-3)}{10-3-7+5}=5.8;$$
$$\alpha^*=\frac{5-3}{10-3-7+5}=0.4;$$

$$a_2(\alpha^{**})=17-\frac{(24-17)(17-10)}{24-7-17+10}=12.1;$$
$$\alpha^{**}=\frac{24-17}{24-7-17+10}=0.7.$$

Then (for recognized M_3) we can choose the corresponding direct model $\mu_{\underset{\sim}{C}}(x)$ from the computational library (Table I). We further present the resulting inverse $C_\alpha=A_\alpha(\vee)B_\alpha$ and direct $\mu_{\underset{\sim}{C}}(x)$ models (Fig.2) for FNs-maximum $\underset{\sim}{C}=\underset{\sim}{A}(\vee)\underset{\sim}{B}$:

$$C_\alpha=A_\alpha(\wedge)B_\alpha=\begin{bmatrix}\begin{Bmatrix}5+2\alpha,\forall\alpha\,|\,\alpha\in[0,0.4]\\3+7\alpha,\forall\alpha\,|\,\alpha\in[0.4,1]\end{Bmatrix},\\ \begin{Bmatrix}24-17\alpha,\forall\alpha\,|\,\alpha\in[0,0.7]\\17-7\alpha,\forall\alpha\,|\,\alpha\in[0.7,1]\end{Bmatrix}\end{bmatrix},$$

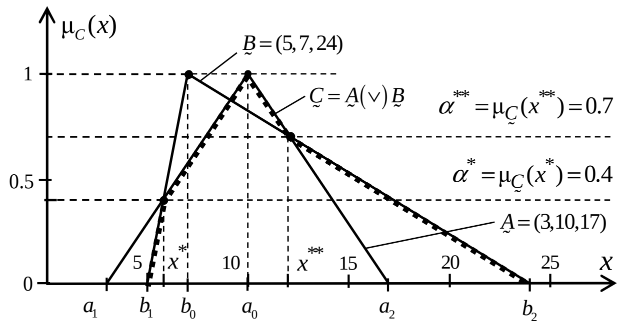

Fig. 2. FNs-Maximum $\underset{\sim}{C}=\underset{\sim}{A}(\vee)\underset{\sim}{B}$ of the TrFNs $\underset{\sim}{A}\in R$ and $\underset{\sim}{B}\in R$

$$\mu_{\underset{\sim}{C}}(x)=\begin{cases}0,\forall(x\le b_1)\cup(x\ge b_2)\\F_{Bl}(x,b_1,b_0),\forall(b_1<x\le a_1(\alpha^*))\\F_{Al}(x,a_1,a_0),\forall(a_1(\alpha^*)<x\le a_0)\\F_{Ar}(x,a_0,a_2),\forall(a_0<x<a_2(\alpha^{**}))\\F_{Br}(x,b_0,b_2),\forall(a_2(\alpha^{**})\le x<b_2)\end{cases},$$

$$\mu_{\underset{\sim}{C}}(x)=\begin{cases}0,\forall(x\le 5)\cup(x\ge 24)\\(x-5)/2,\forall(5<x\le 5.8)\\(x-3)/7,\forall(5.8<x\le 10)\\(17-x)/7,\forall(10<x<12.1)\\(24-x)/17,\forall(12.1\le x<24)\end{cases},$$

VI. Conclusion

The maximum of fuzzy sets is an essential fuzzy arithmetic operation, which is often time consuming in terms of its realization. The execution of the developed direct analytic models' library (Table I) advances current research by allowing the usage of one step automation mode for "FNs-maximum" $\underset{\sim}{C}=\underset{\sim}{A}(\vee)\underset{\sim}{B}$ operation. In some instances, it is necessary to aggregate different data streams (big data), which are presented as random time series or random consequences [32]. For fuzzy information processing of such random streams or consequences, we can use three steps algorithm:

Step 1. Each random stream or consequence can be evaluated by interval value and presented as fuzzy set or fuzzy number [16,32]. For example, in [16] the realization of such random sequences is presented as triangular fuzzy number of such types - "approximate A" or "between B and C";

Step 2. For each pair of triangular fuzzy numbers it is necessary to determine the mask [4,27,28], for example (18), according to the relations between considered TrFNs parameters;

Step 3. Using corresponding computational library for desired fuzzy arithmetic operation [33-38] (addition, subtraction, multiplication, division, minimum or maximum) it is possible to find the resulting MF based on TrFNs mask and TrFN parameters. For FNs-maximum we can use corresponding computational library, presented in Table I.

Modeling results confirm the efficiency of proposed universal direct analytic models for different applications. In some cases, such direct analytic models $\mu_{\underset{\sim}{C}}(x)=\mu_{\underset{\sim}{A}(\vee)\underset{\sim}{B}}(x)$ provide an efficient solution to the fuzzy processing in evaluation, control and decision-making processes, in particular, for the financial analysis [22], automatic evaluation of the student's knowledge [39], group anonymity [40] and partner selection [41,42], model design process [43], soft computing based on reconfigurable technology [44], analysis of the big data during testing of computer systems and their components [5,45], optimization of tanker or truck routes in the conditions of fuzzy demands at nodes [10,21,46], redesigning social inquiry [47], fuzzy-algorithmic reliability analysis of complex systems in economics, management and engineering [18, 48,49], fuzzy control in industrial processes and robotics [2,11,17,50], and others.

References

[1] D. Simon, "Design and rule base reduction of a fuzzy filter for the estimation of motor currents," International Journal of Approximate Reasoning, 25, pp. 145-167, 2000.

[2] Y. P. Kondratenko, and E. Y. M. Al Zubi, "The Optimisation Approach for Increasing Efficiency of Digital Fuzzy Controllers,' 20th Int. DAAAM Symp. "Intelligent Manufacturing and Automation", Published by DAAAM International, Vienna, Austria, pp. 1589-1591, 2009.

[3] S. Encheva, Y. Kondratenko, M. Z. Solesvik, S. Tumin, "Decision Support Systems in Logistics," AIP Conference Proc. 1060, 254-256, 2008, https://doi.org/10.1063/1.3037065

[4] Y. P. Kondratenko, N. Y .Kondratenko, "Soft Computing Analytic Models for Increasing Efficiency of Fuzzy Information Processing in Decision Support Systems," Chapter in book: Decision Making: Processes, Behavioral Influences and Role in Business Management, R. Hudson (Ed.), Nova Science Publishers, New York, 2015, pp. 41-78.

[5] J. Kacprzyk, S. Zadrozny, and G. De Tré, "Fuzziness in database management systems: Half a century of developments and future prospects," Fuzzy Sets and Systems vol, 281, pp. 300-307, 2015, https://doi.org/10.1016/j.fss.2015.06.011

[6] J Gil-Aluja,. Investment in Uncertainty. Kluwer Academic Publishers, Dordrecht, Boston, London, 1999.

[7] L. A. Zadeh, "Fuzzy Sets," Information & Control, 8, pp. 338-353, 1965.

[8] R.Yager, and D. Filev, Essential of Fuzzy Modeling and Control. Wiley, New York, 1994.

[9] M. Jamshidi, V. Kreinovitch, J. Kacprzyk, (Eds), Advance Trends in Soft Computing. Proceedings of WCSC 2013, San Antonio, Texas, USA, STUDFUZZ 312, Springer, 2013.

[10] B. Werners, and Y. Kondratenko, "Alternative Fuzzy Approaches for Efficiently Solving the Capacitated Vehicle Routing Problem in Conditions of Uncertain Demands," Complex Systems: Solutions and Challenges in Economics, Management and Engineering, C. Berger-Vachon et al. (Eds.), Studies in Systems, Decision and Control, vol. 125, Berlin, Heidelberg: Springer, 2018, pp. 521-543. DOI: https://doi.org/10.1007/978-3-319-69989-9_31

[11] J. M. Merigo, A. M. Gil-Lafuente, and R. R. Yager, "An overview of fuzzy research with bibliometric indicators," Applied Soft Computing, no. 27, pp. 420-433, 2015.

[12] D. E. Tamir, N. D. Rishe, A. Kandel, (Eds), Fifty Years of Fuzzy Logic and its Applications. Studies in Fuzziness and Soft Computing, vol. 326, Cham-Heidelberg-New Yorl-Dordrecht-London, Springer International Publishing Switzerland, 2015.

[13] Lodwick, W.A., Kacprzhyk, J. (Eds): Fuzzy Optimization. Studies in Fuzziness and Soft Computing 254, Springer-Verlag, Berlin, Heidelberg, 2010.

[14] A. Kaufmann, and M. Gupta, Introduction to Fuzzy Arithmetic: Theory and Applications. Van Nostrand Reinhold Company, New York, 1985.

[15] Y. Kondratenko, and V. Kondratenko, "Soft Computing Algorithm for Arithmetic Multiplication of Fuzzy Sets Based on Universal Analytic Models," In: Information and Communication Technologies in Education, Research, and Industrial Application. Communications in Computer and Information Science, vol. 469, Ermolayev, V. et al. (Eds): ICTERI'2014, Springer International Publishing, 2014, pp. 49–77. DOI: 10.1007/978-3-319-13206-8_3

[16] A. Piegat, Fuzzy Modeling and Control. Springer, Heidelberg, 2001.

[17] L. E. S. Pereira, and V. M. da Costa, "Interval analysis applied to the maximum loading point of electric power systems considering load data uncertainties," International Journal of Electrical Power & Energy Systems, vol. 54, pp. 334, 2014.

[18] M. Hanss, Applied Fuzzy Arithmetics: An Introduction with Engineering Applications. Springer, Berlin, Heidelberg, NewYork, 2005.

[19] M. Solesvik, Y. Kondratenko,, G. Kondratenko, et al., "Fuzzy decision support systems in marine practice," IEEE International Conference on Fuzzy Systems (FUZZ-IEEE), Naples, Italy, pp. 1-6, 9-12 July 2017. DOI: 10.1109/FUZZ-IEEE.2017.8015471

[20] P. Toth, D. Vigo, (Eds), The vehicle routing problem. SIAM, Philadelphia, 2002

[21] D. Teodorovic, and G. Pavkovich, "The fuzzy set theory approach to the vehicle routing problem when demand at nodes is uncertain," Fuzzy Sets and Systems, vol. 82, pp. 307-317, 1996

[22] A. M. Gil-Lafuente, Fuzzy Logic in Financial Analysis. Studies in Fuzziness and Soft Computing, vol. 175, Springer, Berlin, 2005.

[23] Y. Bodyansky, O. Vynokurova, I. Pliss, and P. Mulesa, "Multilayer Wavelet-Neuro-Fuzzy Systems in Dynamic Data Mining Tasks," In: Soft Computing: Developments, Methods and Applications, Alan

Casey (Ed), Series: Computer Science, technology and applications, Nova Science Publishers, Hauppauge, NY, 2016, pp. 69-146.

[24] M. Hanss, "Fuzzy Arithmetic for Uncertainty Analysis," In: Seising R., Trillas E., Moraga C., Termini S. (Eds), On Fuzziness. Studies in Fuzziness and Soft Computing, vol 298. Springer, Berlin, Heidelberg, 2013, pp, 235-240. DOI https://doi.org/10.1007/978-3-642-35641-4_36

[25] S. Gao, and Z. Zhang, "Multiplication Operation on Fuzzy Numbers," Journal of Software, vol. 4(4), pp. 331-338, JUNE 2009. http://ojs.academypublisher.com/index.php/jsw/article/download/040 4331338/1061.

[26] A. Klimke, "An efficient implementation of the transformation method of fuzzy arithmetic," 22nd International Conference of the North American Fuzzy Information Processing Society, NAFIPS 2003, pp. 468-473, 2003. doi: 10.1109/NAFIPS.2003.1226830

[27] Y. P. Kondratenko, and N. Y. Kondratenko, "Reduced library of the soft computing analytic models for arithmetic operations with asymmetrical fuzzy numbers," In: Soft Computing: Developments, Methods and Applications, Alan Casey (Ed), Series: Computer Science, technology and applications, Nova Science Publishers, Hauppauge, NY, 2016, pp. 1-38.

[28] Y. P. Kondratenko, and N. Y. Kondratenko, "Synthesis of Analytic Models for Subtraction of Fuzzy Numbers with Various Membership Function's Shapes," In: Gil-Lafuente A. et al. (Eds), Applied Mathematics and Computational Intelligence. FIM 2015. AISC, vol 730. Springer, Cham, 2018, pp.87-100, DOI https://doi.org/10.1007/978-3-319-75792-6_8

[29] M. Pasieka, N. Grzesik, and K. Kuźma, "Simulation modeling of fuzzy logic controller for aircraft engines," International Journal of Computing, vol. 16(1), pp. 27-33, 2017. Retrieved from http://computingonline.net/computing/article/view/868

[30] Y. P. Kondratenko, O. V. Kozlov, O. S. Gerasin, and Y. M. Zaporozhets, "Synthesis and research of neuro-fuzzy observer of clamping force for mobile robot automatic control system," IEEE First International Conference on Data Stream Mining and Processing (DSMP), pp.90-95, 2016, DOI: 10.1109/DSMP.2016.7583514

[31] P. Bykovyy, Y. Pigovsky, A. Sachenko, and A. Banasik, "Fuzzy inference system for vulnerability risk estimation of perimeter security," 5th IEEE International Workshop on Intelligent Data Acquisition and Advanced Computing Systems: Technology and Applications, IDAACS'2009, pp. 380-384, 2009. DOI 10.1109/IDAACS.2009.5342956

[32] Y. Bodyansky, O. Vynokurova, I. Pliss, and D. Peleshko, "Hybrid Adaptive Systems of Computational Intelligence and Their On-line Learning for Green IT in Energy Management Tasks," In: Green IT Engineering: Concepts, Models, Complex Systems Architectures, Studies in Systems, Decision and Control, V. Kharchenko et al. (Eds.), Vol. 74. Berlin, Heidelberg: Springer International Publishing, 2017, pp. 229-244. DOI: 10.1007/978-3-319-44162-7_12

[33] M. Hanss, "An Approach to Inverse Fuzzy Arithmetic," 22nd International Conference of the North American Fuzzy Information Processing Society, NAFIPS 2003, pp. 474-479, 2003. doi: 10.1109/NAFIPS.2003.1226831

[34] L. Stefanini, L. Sorini, and M. L. Guerra, "Fuzzy Numbers and Fuzzy Arithmetic," In: Handbook of Granular Computing, W. Pedrycz, A. Skowron, V. Kreinovich (Eds.), John Wiley and Sons, 2008, pp. 249-284.

[35] P. Grzegorzewski, "On the Interval Approximation of Fuzzy Numbers," In: Greco, S., Bouchon-Meunier, B., Coletti, G., Fedrizzi, M., Matarazzo, B., Yager, R.R. (Eds.): Advances in Computational Intelligence, IPMU 2012. Communications in Computer and Information Science, vol 299. Springer, Berlin, Heidelberg, 2012, pp. 59-68. DOI https://doi.org/10.1007/978-3-642-31718-7_7

[36] M. L. Guerra, and L. Stefanini, "Approximate fuzzy arithmetic operations using monotonic interpolations," Fuzzy Sets and Systems, vol. 150, iss. 1, pp. 40-55, 2005.

[37] L. Stefanini, "A generalization of Hukuhara difference and division for interval and fuzzy arithmetic," Fuzzy Sets and Systems, vol. 161, iss. 11, pp. 1564-1584, 2010.

[38] L. Coroianu, "Necessary and sufficient conditions for the equality of the interactive and non-interactive sums of two fuzzy numbers," Fuzzy Sets and Systems, vol. 283, pp. 40-55, 2016.

[39] S. N. Shahbazova, "Application of Fuzzy Sets for Control of Student Knowledge," An International Journal Applied and Computational Mathematics 10, no. 1, pp. 195-208, 2011.

[40] O. Chertov, and D.Tavrov, "Memetic algorithm for solving the task of providing group anonymity," In: Jamshidi M., Kreinovich V., Kacprzyk J. (Eds), Advance Trends in Soft Computing. Studies in Fuzziness and Soft Computing, vol. 312. Springer, Cham, 2014, pp. 281-292. DOI: https://doi.org/10.1007/978-3-319-03674-8_27

[41] M. Z. Solesvik, and S. Encheva, "Partner selection for interfirm collaboration in ship design," Industrial Management & Data Systems, 110(5), pp. 701-717, 2010.

[42] M. Solesvik, "Partner selection in green innovation projects," In: Berger-Vachon C., et al. (Eds.), Complex Systems: Solutions and Challenges in Economics, Management and Engineering, Studies in Systems, Decision and Control, vol. 125, Springer, Cham 2018,.pp. 471-480.

[43] A. N. Trunov, "An adequacy criterion in evaluating the effectiveness of a model design process," Eastern-European Journal of Enterprise Technologies 1, vol. 4 (73), pp. 36-41, 2015.

[44] A. V. Palagin, and V. N. Opanasenko, "Reconfigurable Computing Technology," J. Cybernetics and Systems Analysis, vol. 43(5), pp. 675-686, 2007.

[45] J. Drozd, and A. Drozd, "Models, methods and means as resources for solving challenges in co-design and testing of computer systems and their components," 9th Intern. Conf. on Digital Technologies 2013. Zhilina, Slovak Republic, pp. 225-230, 29-31 May, 2013.

[46] Y. P. Kondratenko, S. B. Encheva, and E. V. Sidenko, "Synthesis of Inelligent Decision Support Systems for Transport Logistic," 6th IEEE Intern. Conf. on Intelligent Data Acquisition and Advanced Computing Systems: Technology and Applications, Prague, Czech Republic, vol. 2, pp. 642-646, Sept. 15-17, 2011. DOI: 10.1109/IDAACS.2011.6072847

[47] C. C. Ragin, Redesigning Social Inquiry: Fuzzy Sets and Beyond. The University of Chicago Press, Chicago, London, 2008.

[48] A. P. Rotshtein, "Fuzzy-algorithmic reliability analysis of complex systems," Cybern. Syst. Anal., vol. 47 (6), pp. 919-931, 2011. DOI https://doi.org/10.1007/s10559-011-9371-x

[49] O. Pomorova, O. Savenko, S. Lysenko, and A. Kryshchuk, "Multi-Agent Based Approach for Botnet Detection in a Corporate Area Network Using Fuzzy Logic," In: Kwiecien, A., Gaj, P., Stera, P. (eds.) 20th Conference on Computer Networks, CN 2013, Lwówek 'Slaski, Poland. Communications in Computer and Information Science, vol. 370, pp. 146-156. Springer-Verlag Berlin Heidelberg, 2013.

[50] D. Dubois, "An application of fuzzy arithmetic to the optimization of industrial machining processes," Mathematical Modelling, vol. 9, no. 6, pp. 461-475, 1987.