

# Selection the “Saturated” Block from Interval System of Linear Algebraic Equations for Recurrent Laryngeal Nerve Identification

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**Abstract**—The task of design a “saturated” experiment for measuring the characteristics of tissues in surgical wound in order to identify the recurrent laryngeal nerve (RLN) during operation on the neck organs considered in this paper. In this task, the method of selection a “saturated” block from an interval system of linear algebraic equations (ISLAE) is used, which allows to reduce the duration of surgical operation by decreasing the number of points for stimulation the surgical wound tissues to detect the RLN location and reduce the risk of its damage.

**Keywords**—neck surgery, recurrent laryngeal nerve, design of experiment, interval analysis, interval model.

## I. INTRODUCTION

RLN monitoring is very important procedure during the neck surgery. For these purposes, special neuro monitors are used. They work based on the principle of surgical wound tissues stimulation and estimation of results of such stimulation [1-4]. However, these methods intended solely for the RLN monitoring.

The methods of RLN identification considered in papers [5, 6]. In particular, in the paper [5] the task of visualizing the RLN location based on evaluation the amplitude of signal as response to its stimulation by alternating current was considered. In paper [6] the method of constructing the difference schemes as a model for RLN location identifying based on interval analysis of response to stimulation the tissue in surgical wound by alternating current was considered.

It should be noted that the informative parameter in both methods used maximum amplitude of the signal as response to stimulation of tissues in surgical wounds, and as basis for determining the RLN damaging area assigned an interval model of distribution on surgical wound surface the maximum amplitudes of information signals as responses to stimulation the tissues in surgical wounds. However, both methods require creation the uniform mesh on surgical wound for tissues stimulation, which substantially increases the time of surgical operation.

However, in [7, 8], the methods of design of “saturated” experiments, aimed at providing guaranteed prognostic properties of models and requiring a minimum amount of

measurements, are considered. These methods based on selection a “saturated” block from the interval system of linear algebraic equations, which used for calculating the parameters of interval model of distribution on the surface of surgical wound of the maximum amplitudes of signals as responses on stimulation the surgical wound tissues. In our case, application of such methods will significantly shorten the surgical operation duration by reducing the time for RLN detection.

Therefore, the task of design of “saturated” experiments and interval data analysis for RLN identification is actual. Solving this task will accelerate rendering of RLN location, reduce the risk of its damage during operations on the neck organs and decrease the overall duration of the surgery.

## II. TASK STATEMENT

The stimulation of surgical wound tissues during the neck surgery based on electrophysiological method allows identifying the type of tissue with the purpose of RLN identification.

In the papers [9], a method for RLN identification among tissues of a surgical wound is given. This method is based on stimulation the surgical wound tissues by alternating current with an active intensity from 0.5 to 2 mA and on registering the results of stimulation by sound sensor located above the vocal cords.

In respiratory tube 1 that inserted into larynx 2, the sound sensor 3 implemented and positioned above vocal cords 4.

Probe 5 is connected to stimulation block 7. It functions as a current generator controlled by the single-board computer 8. Surgical wound tissues are stimulated by the block 7 via probe 5. As a result, vocal cords 4 are stretched.

Flow of air that passes through patient’s larynx, is modulated by stretched vocal cords. The result is registered by sound sensor 3. Obtained signal is amplified and processed by single-board computer 8.

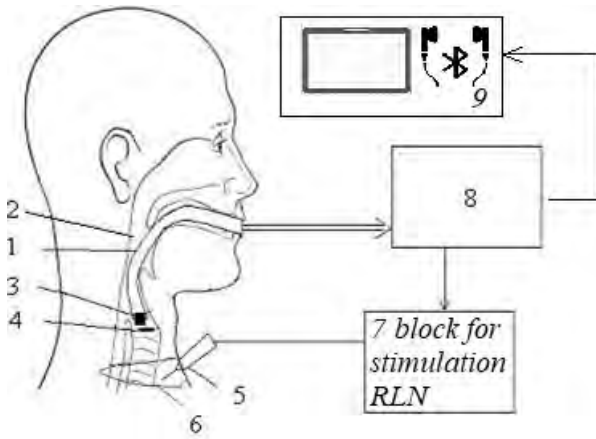
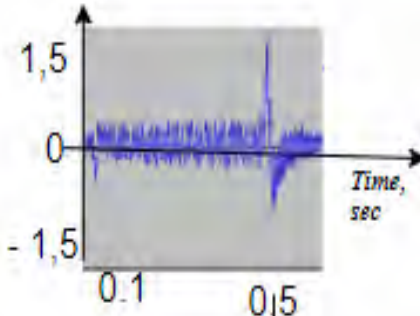


Fig. 1. Method of RLN identification among tissues in surgical wound.

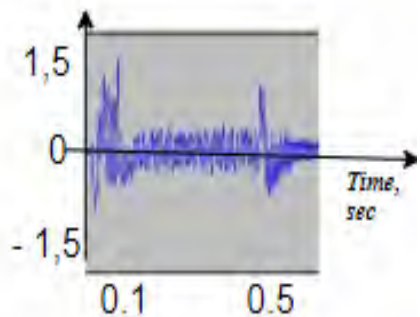
For processing of obtained signal, special software is installed on a single-board computer. The main functions of the software are:

- segmentation the information signal based on analysis of its amplitude;
- calculation the maximum amplitude of signal;
- classification the tissues in surgical wound at the points of stimulation [9].

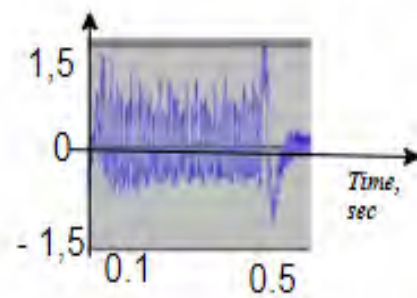
On Fig. 2 the fragments of amplified information signal which registered by sound sensor are shown.



a)



b)



c)

Fig. 2. Result of RLN stimulation by alternating current with frequency 300 Hz.

On Fig 2 a) it can be seen the result of stimulation the muscle tissue on distance more than 1 cm to RLN with minimal amplitude of information as reaction on stimulation. On Fig. 2 b) shown result of stimulation the muscle tissue on distance not more 3 mm. As we can see the signal as reaction on stimulation has higher amplitude then the signal as reaction on stimulation in previous case. Lastly on Fig. 2 c) shown result of RLN stimulation. As we can see the reaction has the highest amplitude.

Above mentioned research give possibility to assume that this characteristic can be used for RLN identification.

Let's represent the resulting set of points as follows:

$$[z(x_{1i}, x_{2i})], i=1, \dots, I, \quad (1)$$

where  $[z(x_{1i}, x_{2i})]$  is an interval estimation of maximal amplitude of information signal fragment;  $x_{1i}, x_{2i}$  are increments of coordinate values on  $x_1$  and  $x_2$  axes relative to some initially given point. Interval estimation of amplitude  $[z(x_{1i}, x_{2i})]$  caused by that for equal  $x_1, x_2$  can be obtained different values of maximum amplitude  $[z(x_{1i}, x_{2i})]$  of information signal.

Moreover some error of finding the point with coordinates  $x_1, x_2$  exist. Let's denote by  $i_o, o=1, \dots, O$  the points indexes in the vicinity of the point with  $x_1, x_2$  coordinates. Lower and upper values of interval of maximal amplitude estimations of information signal are obtaining by Eqs.:

$$z^-_{i_o} = \min\{z_{i_o}, o=1, \dots, O\}; z^+_{i_o} = \max\{z_{i_o}, o=1, \dots, O\}.$$

Let's consider the mathematical model for RLN identification in kind of algebraic equation described in [5]:

$$\hat{z}(\bar{x}) = \beta_1 \cdot \varphi_1(\bar{x}) + \dots + \beta_m \cdot \varphi_m(\bar{x}), \quad (2)$$

where  $\vec{\beta} = (\beta_1, \dots, \beta_m)^T$  is the vector of unknown parameters;  $\vec{\varphi}^T(\bar{x}) = (\varphi_1(\bar{x}), \dots, \varphi_m(\bar{x}))^T$  is the vector of known basic functions;  $\bar{x} = (x_1, x_2)$  is vector of stimulation point coordinates;  $\hat{z}(\bar{x})$  is predicted value of maximal amplitude of information signal in point with coordinates  $(x_1, x_2)$ . Further, the model (2) will be called an interval model (IM).

Based on the requirements ensuring accuracy of the model within the accuracy of the experiment, the setting of IM (1) will realize with the using of such criterion [10, 11]:

$$[\hat{z}_i^-; \hat{z}_i^+] \subset [z_i^-; z_i^+], \forall i = 1, \dots, I. \quad (3)$$

By substituting to the expression (3), the recurrent expression (1) instead of the interval estimates  $[\hat{z}_i^-; \hat{z}_i^+]$  together with the defined initial interval values of each we receive the following ISLAE [12]:

$$\begin{cases} z_1^- \leq b_1 \varphi_1(\bar{x}_1) + \dots + b_m \varphi_m(\bar{x}_1) \leq z_1^+; \\ \vdots \\ z_i^- \leq b_1 \varphi_1(\bar{x}_i) + \dots + b_m \varphi_m(\bar{x}_i) \leq z_i^+; \\ \vdots \\ z_N^- \leq b_1 \varphi_1(\bar{x}_N) + \dots + b_m \varphi_m(\bar{x}_N) \leq z_N^+; \end{cases} \quad (4)$$

where  $\bar{b} = (b_1, \dots, b_m)^T$  is the vector of parameters  $\bar{\beta} = (\beta_1, \dots, \beta_m)^T$  estimation.

### III. METHOD OF SELECTION THE "SATURATED" BLOCK FROM ISLAE

The method of directed selection the "saturated" block built in accordance with the procedure of  $I_G$ -optimal design of experiment. The essence of this method consist in selecting a "saturated" block; calculating the corridor of interval models; analyzing the prognostic properties of these models, which influence on designing the way of next "saturated" block selection.

If structure of mathematical model of a static system is determined by the Eq. (2) with unknown parameters and interval data are given then ISLAE is created in kind (4).

Let's select from ISLAE arbitrarily a "saturated" block, calculate its area of solutions and construct a prediction corridor of interval models:

$$[\hat{z}(\bar{x})] = [\bar{\varphi}^T(\bar{x}) \cdot \bar{b} - \frac{1}{2} \cdot \Delta_{z(\bar{x})}; \bar{\varphi}^T(\bar{x}) \cdot \bar{b} + \frac{1}{2} \cdot \Delta_{z(\bar{x})}]. \quad (5)$$

Then by analogy with procedure of sequential  $I_G$ -optimal design of experiment from  $\bar{x}_i, i = 1, \dots, N$  points for which ISLAE (4) created, will calculate the vector  $\bar{x}^{\max}$  which has maximal prediction error:

$$\begin{aligned} \bar{x}^{\max} &= \arg \max_{\bar{x}_i, i=1, \dots, N} \left\{ 2 \cdot \sum_{j=1}^m |\alpha_j(\bar{x}_i) \cdot \Delta_j|, \bar{x}_i, i = 1, \dots, N \right\}, \\ \bar{\alpha}^T(\bar{x}_i) &= \bar{\varphi}^T(\bar{x}_i) \cdot F_m^{-1} \end{aligned} \quad (6)$$

It is significant that procedure (6) is simple as executed on finite set of points  $\bar{x}_i, i = 1, \dots, N$ . The vector obtained by Eq. (6) is a vector of input variables. This vector defines a certain interval equation in ISLAE (4). In accordance with procedure of sequential  $I_G$ -optimal design of experiment at this point, it is necessary to carry out the next measurement.

In paper [7] it is proved that if vector  $\bar{x}^{\max}$  coincides with vector of values the input variables in one from interval equations in the "saturated" block in ISLAE, then it specifies a point with a minimum value of prediction error. Hence, it is advisable to replace one interval equation in current "saturated" block by interval equation from ISLAE with vector of values the input variables  $\bar{x}^{\max}$  determined by Eq. (6). Thus, by analogy with the procedure of sequential  $I_G$ -optimal design, we "simulate" the procedure of additional measurement at a point  $\bar{x}^{\max}$  with the maximum error of prediction by the interval model. We will perform this procedure for each interval equation in "saturated" block. We obtain  $p$  ( $p = 1, \dots, m$ ) new "saturated" blocks.

As a result, for each of the  $m$  "saturated" blocks we obtain  $m$  values of maximum errors for corresponding interval models:

$$\begin{aligned} \Delta_{\max}^p &= \max_{x_i, i=1, \dots, N} \left\{ 2 \cdot \sum_{j=1}^m |\alpha_{jp}(\bar{x}_i) \cdot \Delta_j| \right\}, \\ \bar{\alpha}_p^T(\bar{x}_i) &= \bar{\varphi}^T(\bar{x}_i) \cdot F_m^{-1}(p), p = 1, \dots, m, \end{aligned} \quad (7)$$

where  $p$  is index, which means number of "saturated" block,  $F_m(p)$  is matrix of base functions values for  $p$  block,  $\alpha_{jp}(\bar{x}_i)$  is  $i$ -th component of vector  $\bar{\alpha}$ , that is calculated for  $p$ -th "saturated" block. Obviously, in order to choose the optimal "saturated" block in this step, it is enough to choose from  $m$  "saturated" blocks the one that provides the lowest value of sequence (7):

$$F_m^{opt} = \arg \min_{p=1, \dots, m} \left\{ \Delta_{\max}^p, p = 1, \dots, m \right\}. \quad (8)$$

We get  $\bar{x}^{\max}$  that is the vector for which the maximum prediction error for the interval model is reached. The scope of the parameters of this model is calculated from the "saturated" block chosen in the above-described method. Then the iterations continue until such a "saturated" block is obtained, the replacement of which equations does not lead to a decrease in the maximum prediction error by interval models. Localization method of solutions ISLAE allows to obtain explicitly guaranteed ellipsoidal estimation of ISLAE solutions [13]:

$$Q_m(k+1) = \left\{ \bar{b} \in R^m \mid (\bar{b} - \bar{b}(k+1))^T \cdot F^T \cdot E^{-2}(k+1) \cdot F \cdot (\bar{b} - \bar{b}(k+1)) = 1 \right\} \quad (9)$$

where  $\bar{b}(k+1) = F_m^{-1} \cdot ((y_1^+(k+1) - y_1^-(k+1)), \dots, (y_m^+(k+1) - y_m^-(k+1)))^T$  is a vector that specifies the center of the ellipsoid;  $E(k+1) = \text{diag}(y_1^+(k+1) - y_1^-(k+1), \dots, y_1^+(k+1) - y_1^-(k+1), \dots, y_m^+(k+1) - y_m^-(k+1))$  is diagonal matrix of the differences of limits of intervals from the Eq. (9).

The corridor for interval models, which are defined by their predictive properties, in this case will look like this:

$$[\hat{y}(\bar{x})]_{\bar{b} \in Q_m} = [\bar{\varphi}^T(\bar{x}) \cdot \bar{b} - \frac{1}{2} \cdot \Delta_{\hat{y}(\bar{x})} \Big|_{\bar{b} \in Q_m}; \bar{\varphi}^T(\bar{x}) \cdot \bar{b} + \frac{1}{2} \cdot \Delta_{\hat{y}(\bar{x})} \Big|_{\bar{b} \in Q_m}] \quad (10)$$

where  $\Delta_{\hat{y}(\bar{x})} \Big|_{\bar{b} \in Q_m}$  – the error of prediction (the width of the corridor), which is calculated by expression:

$$\Delta_{y(\bar{x})} \Big|_{\bar{b} \in Q_m} = \sqrt{\bar{\varphi}^T(\bar{x}) \cdot (F_m^T \cdot E^{-2} \cdot F_m)^{-1} \cdot \bar{\varphi}(\bar{x})} \quad (11)$$

#### IV. EXAMPLE OF APPLICATION THE PROPOSED METHOD FOR RLN IDENTIFICATION

Consider the example of constructing a model of distribution on the surface of a surgical wound the maximum signal amplitudes as reaction to stimulation of surgical wound tissues. The structure of this mathematical model, obtained from the work [5], has the following form:

$$\bar{z}(\bar{x}) = b_0 + b_1 \cdot \sin^2(x_1 \cdot x_2 \cdot \frac{\pi}{36}) + b_2 \cdot x_2 + b_3 \cdot (x_2^2)$$

A fragment of data obtained during the surgical operation on thyroid gland is given in Table 1 in [5]. We apply the method of selection the “saturated” block from ISLAE to find the most informative points. Using the described above method, interval model was obtained in form:

$$\begin{aligned} [\bar{z}(\bar{x})] &= [\bar{b}_0 + \bar{b}_1 \cdot \sin^2(x_1 \cdot x_2 \cdot \frac{\pi}{36}) + \bar{b}_2 \cdot x_2 + \bar{b}_3 \cdot x_2^2 - \Delta_{z(\bar{x})} \Big|_{\bar{b} \in Q_m}; \\ &\bar{b}_0 + \bar{b}_1 \cdot \sin^2(x_1 \cdot x_2 \cdot \frac{\pi}{36}) + \bar{b}_2 \cdot x_2 + \bar{b}_3 \cdot x_2^2 + \Delta_{z(\bar{x})} \Big|_{\bar{b} \in Q_m}] \end{aligned}$$

where  $\bar{b} = (10.744; 36.81; 7.82; -1.06)^T$  is the vector of coordinates estimations of the ellipsoid  $Q_m$  center [13].

Prediction error  $\Delta_{y(\bar{x})} \Big|_{\bar{b} \in Q_m}$  is represented for this case in such form:

$$\begin{aligned} \Delta_{z(\bar{x})} \Big|_{\bar{b} \in Q_m} &= \\ &= \sqrt{\begin{pmatrix} 1 \\ \sin^2(x_1 x_2 \frac{\pi}{36}) \\ x_2 \\ x_2^2 \end{pmatrix} \cdot (F_m^T \cdot \tilde{E}^{-2} \cdot F_m)^{-1} \cdot \begin{pmatrix} 1 \\ \sin^2(x_1 x_2 \frac{\pi}{36}) \\ x_2 \\ x_2^2 \end{pmatrix}^T}, \end{aligned}$$

where

$$F_m = \begin{pmatrix} 1 & 1 & 6 & 36 \\ 1 & 0.25 & 5 & 25 \\ 1 & 0.179 & 1 & 1 \\ 1 & 1 & 3 & 9 \end{pmatrix}$$

is the matrix of the basic function values of “saturated” optimized block;

$$\tilde{E} = \begin{pmatrix} 9.84375 & 0 & 0 & 0 \\ 0 & 5.6875 & 0 & 0 \\ 0 & 0 & 4.2131 & 0 \\ 0 & 0 & 0 & 10.7546 \end{pmatrix}$$

is modified diagonal matrix.

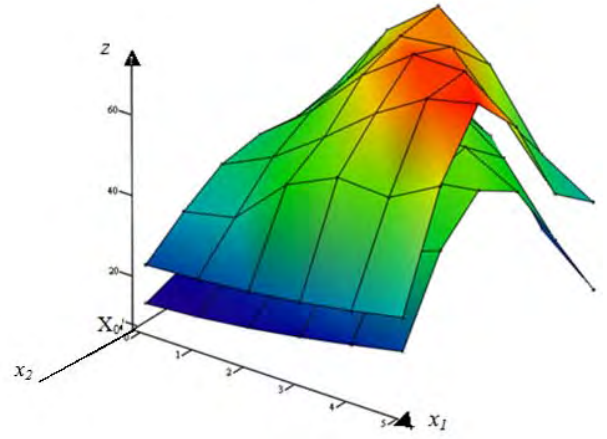


Fig. 3. Graph of distribution the maximal amplitude of information signal relatively to  $x_0$ .

As we see on Fig. 3, instead of applying 36 points of stimulation of surgical wound tissues, it is enough to select 4 most informative points with coordinates [5; 1], [6; 3], [6, 5], [3; 6], with greater accuracy of estimating the maximum amplitude of the information signal.

#### V. CONCLUSIONS

The method of design of experiment based on new procedure of “saturated” block selection and its application to the task of measuring the characteristics of tissues in surgical wound in order to RLN identification during operation on the neck organs considered in this paper.

As distinct from existing method, in which for constructing the model of distribution the maximum amplitude of information signal it is necessary to use a sterile mesh, in case of application the method with selection the “saturated” block it is enough to choose  $m$  base points with relative to some point  $x_0$  on surgical wound. The proposed method reduces the amount of stimulations of surgical wound tissues from  $m^2$  to  $m$ , which significantly reduces the time spent on RLN identification, does not require the use of a sterile mesh, and thus reduces the time spent on surgical operation integrally.

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