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Analysis of EEG using Multilayer Neural Network with Multi-Valued Neurons

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***Abstract*—There is a wealth of analysis techniques that can be used in analyzing data of such a nature as EEG (Electroencephalogram), yet there are still many more ways and possibilities of analysis techniques to consider in order to produce a method that far exceeds the capabilities of the prevalent method. Since a multilayer neural network with multi-valued neurons (MLMVN) was successfully used earlier to decode EEG signals in a brain/computer interface (BCI) by analysis of their Fourier transform, it seemed very attractive to use it as a tool for EEG analysis. This work aims to further investigate how a complex-valued machine learning tool can be used to analyze EEG in the frequency domain. Our goal was to check how Fourier transform and complex wavelet transform of EEG can be analyzed using MLMVN in order to diagnose epilepsy, its remission or absence. We worked with a commonly used benchmark data set of epilepsy-related EEGs. The analysis of the transformed data was performed to determine a set of relevant statistical characteristics of DTCWT and Fourier transform components, which were then used as inputs of the MLMVN. The obtained results show a very high efficiency of the proposed approach.**

Keywords*—*Complex-Valued Neural Networks, Multi-Valued Neuron, Multilayer Neural Network with Multi-Valued Neurons, MLMVN, EEG, Fourier transform

I. INTRODUCTION

We would like to use here MLMVN to analyze EEG in the frequency domain. MLMVN is a representative of complex-valued neural networks (CVNN) family. There is plenty of work done that states the use of CVNN, for example, a good observation is given in [1]-[3]. Traditionally CVNNs have been very successful in solving a number of real-world problems. We should mention such applications as detection of landmines [4], prediction of winds and their profiles [5], analysis of bio-medical images [6], prediction of oil production [7], frequency domain analysis of signals in EEG-based BCIs [8].

MLMVN is on the one hand a feedforward neural network, topologically identical to a multilayer perceptron (MLP). But on the other hand, MLVVN, being built from multi-valued neurons (MVNs) has its unique properties and important advantages over MLP. MLMVN was introduced in [9] as a 2-layer network. Then it was further developed [10] where MLMVN with an arbitrary number of hidden layers was introduced. Every particular property of MLMVN and its favorable distinctions over MLP are dictated by the utilization of the multi-valued neuron (MVN) as its essential unit. MVN was initially suggested in [11] as a *k*-valued threshold element and then re-introduced as a discrete MVN in [12].

MLMVN was effectively utilized in numerous applications. It was applied, for instance, for image deblurring through recognition of point-spread function and its specific parameters [13], long term time series prediction [7], analysis of signals in EEG-based BCIs [8], satellite information reversal for assurance of meteorological information profiles in the environment [15], solving various classification problems [3], [10], [16], and system identification [17]. MLMVN generalization capability in solving problems with discrete output, particularly classification and pattern recognition problems, was improved by a modified learning algorithm with soft margins [16]. To speed up a learning process and maintain simultaneously big learning sets and a high generalization capability, a batch learning algorithm was proposed in [17] and further developed in [18]. This algorithm as it is described in [18] was used in all experiments described in this paper.

EEG is used to collect the data about brain electrical activity. Then analysis of these data can be used to discover a certain dysfunction of some groups of neurons in the brain. Particularly, EEG is used to diagnose epilepsy in its different stages and examine patients with this diagnosis in remission. In the context of computing, computer science and computer engineering, EEG is used in building brain/computer interfaces, which help people with disabilities caused by some brain dysfunctions to perform certain tasks. A seminal work on electrical activity in the brain was published in 1875 by Caton [19]. His ideas were significantly developed 50 years later by Berger [20], [21]. It was succeeded to him to detect electrical activity in the brain using special electrodes placed on the head. Corresponding signals were acquired and recorded using a galvanometer connected to these special electrodes. It was noticed that electrical activity of the brain may change, for example, when eyes are open or closed. With these developments, the presence of EEG signals was scientifically proven. EEG signals are used in diagnostics, controlling of the anesthesia stage during surgical procedures, studies of sleep disorders, sleep psychology, and investigation of migraine. These signals are measured using a BCI. It consists of special electrodes which are used for measuring the electrical activity of the brain from the head surface.

Evaluation of EEGs is a specific job. It can typically be performed only by medical doctors whose area of specialization is EEG analysis. It is important to mention that EEG signals are not stable and they change continuously. They change their phases, frequencies and magnitudes. This makes interpretation of EEGs a challenging task. Medical doctors, depending on how different is their practical experience, may interpret a same EEG differently. Hence it becomes quite important to use some intelligent computing tool, which should be able to analyze and interpret EEG signals. In [8] Manyakov et. al. suggested analyzing EEG by the analysis of its Fourier transform using MLMVN. This approach was pretty natural, since MLMVN works with complex-valued inputs and can be therefore used to analyze complex-valued information. This analysis was used in [8] to decode EEG signals in a brain/computer interface. In [22], [23] it was suggested to use another complex-valued neural network to analyze the dual-tree complex wavelet transform (DTWCT) [24] of EEG. This method was used to diagnose epilepsy. We would like to use here MLMVN as a sophisticated intelligent EEG signal classifier for epilepsy patients, in order to improve their treatment and distinguish EEGs of patients with epilepsy in remission and of those people who are healthy. This intelligent EEG signal classifier would also help treat those who are not yet fully have been the victims of epilepsy that is patients from a group of risk.

II. Data Gathering and Features Employed

A. EEG data set

To study EEG signals, we used a commonly employed benchmark dataset [25] containing data for five different classes. Class A consisting of all the EEG's of healthy patients, B, C, and D consists of EEG's of patients with remission, while Class E consist of EEG's of patents that are unhealthy in terms of epilepsy. Hence our dataset consists of five subsets. Each of these subsets contains EEGs of 100 volunteers recorded during a period of 23.6 seconds and sampled in 4097 samples. Subsets A and B contain a single channel EEGs recorded from healthy volunteers. Subset C contains EEGs recorded from hippocampal opposing hemisphere of sick patients before seizures [22]. Subset D consists of EEG recordings obtained from the epileptogenic region in the sick patients before seizures [22]. Subset E consists of EEGs containing seizures recorded from sick volunteers [22]. As it was mentioned above, the original data contain 4097 samples in each EEG. We did not use the last sample, thus we worked with EEG containing 4096 samples.

B. Features used for classification

To analyze a 4096-sampled EEG in the Fourier or DTCWT domain, it is necessary to have a relevant set of features using which it should be possible to perform this analysis. In fact, for example, a Fourier transform of such a 4096-sampled EEG contains 2048 frequencies, but EEGs from all five subsets A-E do not decisively differ from each other in medium and high frequencies. This means that the activity of a brain in those frequencies mostly is not different in different groups of sick and healthy people. However, this activity differs in EEGs of sick and healthy people in low frequency domain. Fig. 1 depicts magnitudes of the first 256 Fourier transform coefficients taken from representatives of each of A-E subsets.

Nevertheless, the analysis even of 256 spectral coefficients using MLMVN or any other machine learning tool should not be efficient. EEGs of different healthy and sick people are different from each other. For example, EEGs of sick people show some abnormal activities in some frequencies, but while shapes of these activities are similar, certain frequencies where these abnormal patterns can be found, are distinct, even if they are close to each other. The same properties are demonstrated by DTWCT transforms of EEG. This means that it is necessary to extract some specific statistical features from the frequency domain data. We should look for some targeting features, which are similar for representatives from the same class of EEGs, but different for representatives from other classes. It was proposed in [22] where classification of EEGs based on DTCWT was studied to use the following 5 statistical characteristics as features for classification. These characteristics include (see Table I) mean (complex), engineered complex "minimum", engineered complex "maximum", engineered complex "standard deviation", and engineered complex "median". We call the last four complex-valued characteristics "engineered" because they are complex numbers artificially synthesized by finding corresponding real-valued characteristics separately over real and imaginary parts of the corresponding complex numbers followed by creation complex numbers from them by their pairing.

TABLE I. Statistical Characteristics Used

Characteristic	**Mathematical expression**
Minimum (complex) .	$\min[\mathrm{Re}(x_n)] + i\min[\mathrm{Im}(X_n)]$
Maximum (complex)	$\max[\mathrm{Re}(x_n)] + i\max[\mathrm{Im}(X_n)]$
Expectation (mean) (complex)	$\frac{1}{N}\sum_{k=1}^{N} x_k$
Standard deviation (complex)	$\mathrm{Re}\left(\sqrt{\frac{\sum_{n=1}^{N}(X_n - AM)^2}{N-1}}\right) + i\,\mathrm{Im}\left(\sqrt{\frac{\sum_{n=1}^{N}(X_n - AM)^2}{N-1}}\right)$
Median (complex odd) $\left(\frac{N+1}{2}\right)^{th}$ Re value $+i\left(\frac{N+1}{2}\right)^{th}$ Im value	
Median (complex even)	$i((N/2)\uparrow th\,\mathrm{Im}value + (N/2+1)\uparrow th\,\mathrm{Im}value)/2)$

However, these characteristics except of mean are in fact artificial. They represent statistical characteristics of real and imaginary parts of complex numbers separately. Being merged into complex numbers they may not represent actual behavior of complex numbers or may represent it incompletely. A major issue here is that when we consider real and imaginary parts of complex numbers as separate abstract real numbers, we may completely lose circular nature of phase and very important information contained in phase. This issue becomes especially sensitive when we need to find characteristics relevant to some complex-valued random process or such a selection of complex numbers as Fourier transform or DTCWT. This problem is comprehensively studied in [26] where there is a very wide observation of first and second order moments suggested by different authors (particularly in [27], [28], [29]) relevant to complex-valued random processes and problems related complex-valued signal and data processing is presented. Let x and y are samples of the random variable. Then their *covariance* as the central moment is defined as follows [26]:

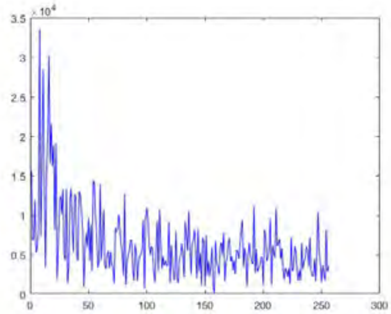
$$\mathrm{cov}(x,y) = E\left[\left(x - E(x)\right)\overline{\left(y - E(y)\right)}\right], \tag{1}$$

where E is an expectation and bar stands for complex conjugation.

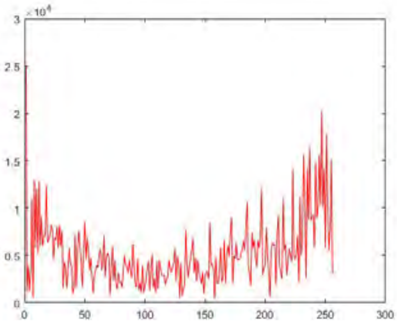
Based on (1) the *pseudo-variance* of x [26] should be defined as follows

$$\tilde{\sigma}_x^2 = \operatorname{cov}(x,\bar{x}) = E\left[\left(x - E(x)\right)\left(\overline{\bar{x} - \overline{E(y)}}\right)\right], \qquad (2)$$

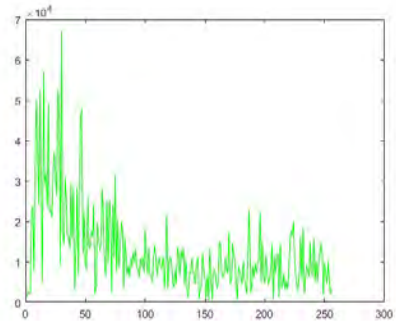
where bar stands for complex conjugation. Evidently $\tilde{\sigma}_x^2 \in \mathbb{C}$ that is it is in fact complex because $\tilde{\sigma}_x^2 = \operatorname{cov}(x,\bar{x}) = \sigma_{x_{\mathrm{Re}}}^2 - \sigma_{x_{\mathrm{Im}}}^2 + 2i\operatorname{cov}(x_{\mathrm{Re}}, x_{\mathrm{Im}}) \in \mathbb{C}$ (where i is an imaginary unit) whenever $x \in \mathbb{C}$.



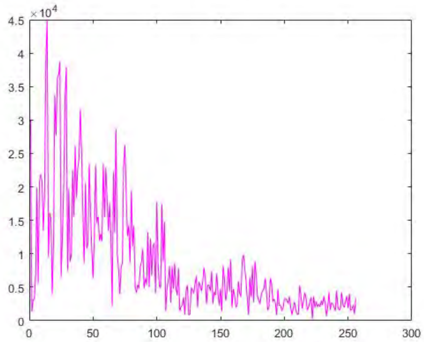
a) Magnitude of the Fourier transform of the representative from subset A (first 256 Fourier coefficients)



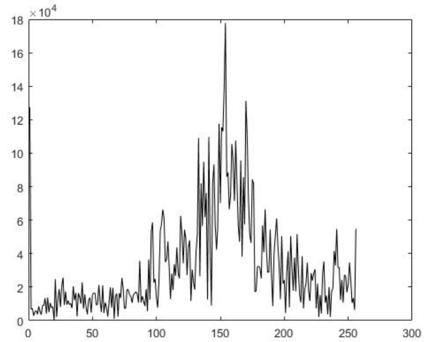
b) Magnitude of the Fourier transform of the representative from subset B (first 256 Fourier coefficients)



c) Magnitude of the Fourier transform of the representative from subset C (first 256 Fourier coefficients)



d) Magnitude of the Fourier transform of the representative from subset D (first 256 Fourier coefficients)



e) Magnitude of the Fourier transform of the representative from subset E (first 256 Fourier coefficients)

Fig. 1. Magnitudes of the Fourier transforms of EEGs

Then it follows from (2) that the *pseudo-variance* of N samples $x_1,\ldots,x_N$ of random variable X (the *pseudo mean square deviation*) should be defined as

$$\tilde{\sigma}_X^2 = \left(\sum_{k=1}^{N} \tilde{\sigma}_{x_k}^2\right) / N. \qquad (3)$$

The *complex correlation* coefficient is used as a measure for the degree of impropriety of x [26]. It is defined [29] as follows

$$\rho = \frac{\operatorname{cov}(x,\bar{x})}{\operatorname{cov}(x,x)}, \qquad (4)$$

where bar again stands for complex conjugation. Evidently $\rho \in \mathbb{C}$ because $\operatorname{cov}(x,\bar{x}) \in \mathbb{C}$ whenever $x \in \mathbb{C}$.

Let us now define the sample *complex correlation* coefficient ρ_X for N samples $x_1,\ldots,x_N$ of random variable X. Taking into account (4) we obtain the following

$$\rho_X = \frac{\sum_{k=1}^{N} \operatorname{cov}(x_k, \bar{x}_k)}{\sum_{k=1}^{N} \operatorname{cov}(x_k, x_k)}. \qquad (5)$$

Empirically it looks that the complex pseudo-variance (3) and the complex correlation (5) should better represent closeness and distinctions among samples of complex variables X and Y than synthesized ‘‘median’’, ‘‘min’’, ‘‘max’’, and ‘‘standard deviation’’ from Table I. It is very important that (3) and (5) are complex numbers and they in fact should not significantly differ from each other if X and Y are similar in probabilistic/statistical terms, but they should be quite different from each other when X and Y are probabilistically/statistically different from each other. This should also be true when X and Y represent Fourier or DTCWT spectra of EEGs taken from healthy people, patients with epilepsy and patients with epilepsy in remission. Hence we will use in our experiments the following three features to describe each EEG (its Fourier or DTCWT transform):

1) *Complex expectation (mean)* $E(X) = \left(\sum_{k=1}^{N} x_k\right) / N$

2) *Complex pseudo-variance (3)*

3) *Complex correlation coefficient (5)*

III. Experimental Testing

Our goal was to perform the same classification experiments, which were performed in [22], but using Fourier transform along with DTCWN for EEG representation, a set of three features, which was just determined instead of those features listed in Table I, and MLMVN as a machine learning tool instead of another complex-valued neural network used in [22]. In all our experiments, we used 10-fold cross-validation (as it was done in [22]).

A. Experiment 1

The goal in this experiment is to classify only healthy people and patients with epilepsy (clusters A and E from the dataset) based on their DTCWT and Fourier transform. Thus, here we have to solve a 2-class classification problem. We used 10-fold cross-validation in this experiment.

B. Experiment 2

This experiment extends Experiment 1 involving the cluster D (sick patients before seizures). We again used both DTWCT and Fourier transform to represent EEGs. Hence in this experiment we are dealing with a 3-class classification problem with the following three classes: 1) healthy class (all representatives from subset A were included); 2) patients with epilepsy having seizure-free intervals (all representatives from subset D were included); 3) patients with epilepsy having seizure (all representatives from subset E were included). We again used 10-fold cross-validation here.

C. Experiment 3

Experiment 3 was operated by involvement all five subsets from the dataset. In this experiment, we classify EEGs as belonging to the following three classes. The first class was formed from subsets A and B (all healthy people), the second one was formed from subsets C (sick patients before seizures) and D (patients with epilepsy having seizure-free intervals), and the third one was formed from the representatives of class E (patients with seizures). We used 10-fold cross-validation applied to the data extracted from the corresponding EEGs using both DTWCT and Fourier transform.

IV. Results

Our experimental results are excellent. All of them are summarized below in Table II.

TABLE II. Experimental Results

Experi-ment	Fourier Transform			Dual-tree complex wavelet transform (DTCWT)					
				Level 1			Level 2		
	MLMVN topology	# of learning iterations (mean over 10 experiments)	Classification accuracy (mean over 10 experiments)	MLMVN topology	# of learning iterations (mean over 10 experiments)	Classification accuracy (mean over 10 experiments)	MLMVN topology	# of learning iterations (mean over 10 experiments)	Classification accuracy (mean over 10 experiments)
1) A-E	3-2-1	71	100%	3-1	12	100%	3-3-1	6	100%
2) A-D-E	3-2-3	83	100%	2-3	423	100%	3-2-3	27	100%
3) AB-CD-E	3-4-3	84	100%	4-3	210	100%	3-4-3	26	100%

We used MLMVN-SM-LLS as it is presented in [18] with a very slight modification in the learning rule described in [30], which makes it possible to use arbitrary complex-valued input in MVN. We employed MLMVN with a single hidden layer and the output layer. A network topology is represented everywhere as *n-N-M* where *n* is the number of network inputs, *N* is the number of hidden neurons and *M* is the number of output neurons.

In Experiments 2 and 3 where we worked on solving 3-class classification problems, we used 3 output neurons performing binary classification. The output of the network was determined in such a case using the winner takes it all technique suggested with regard to MVN in [31] and employed for MLMVN-SM-LLS in [18]. The global threshold of 0.78 radian was used for soft margins in MLMVN. After publication of this paper, software and data used here will be available online[1]. Hence, the use of the features, which we suggested in this paper, and MLMVN as a machine learning tool improves the results presented in [22]. We got a stable 100% classification rate in all our experiments for both DTWCT and Fourier transform using only three features. Very small networks were enough to use to get the 100% classification accuracy. In fact, the largest network, which we employed in Experiment 3, contains only 7 neurons (4 hidden neurons and 3 output neurons). The learning process converges very quickly. Actually, this is basically a real time immediate convergence. It is important that while the authors of [22] were skeptical on the use of Fourier transform as a source of features for EEG analysis, we have shown here that it also can be used (while the learning process for DTWCT requires less iterations for its convergence).

V. Discussion and Conclusions

We suggested to use complex expectation (mean), complex pseudo-variance and complex correlation coefficient as three features to classify EEGs in the frequency domain. It was justified why these features characterize complex-valued data better than those applied separately to real and imaginary parts. We have also shown that along with DTWCT, which was earlier proven as a very good space for EEG representation, Fourier transform can also be used for the same purposes. It was also shown that MLMVN can successfully be used as an intelligent EEG classifier. The proposed approach should be further developed and tested using other appropriate EEG data. It should be attractive to use it not only for epilepsy diagnostics

[1] Software and data are available here https://www.freewebs.com/igora/Downloads.htm

and epilepsy-related EEG analysis, but for classification of EEGs related to other health issues. This work also confirms high importance of the frequency domain representation of data, which are related to brain activity (since biological neurons exchange information with each other in terms of frequencies of spikes generated). It also shows how important it is to use a proper tool (a complex-valued neural network that is MLMVN in our particular case) for frequency domain data analysis.

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