

# A New Approach for Forming a Probabilistic Risk Assessment Model of Innovative Project Implementation Under Risk

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**Abstract**— In this paper we propose a new approach for constructing a probabilistic risk assessment model of innovative project. The method is based on the task of comparison and ranking of fuzzy numbers that has an important role in more applications related to the decision analysis. In the literature there are many approaches to compare fuzzy numbers. The majority of these approaches are based on the quantitative measurements. For that, we propose a new method to calculate the risk level that we can accord to equality  $X < Y$  when  $X$  and  $Y$  are two generalized fuzzy numbers.

**Keywords**—Innovative project selection; fuzzy logic; Risk; Uncertainty.

## I. INTRODUCTION

The risk assessment of innovative project is related to the optimization task under uncertainty and risk [5]. To invest in innovative projects, companies must have a various strategies; these strategies must reach an effective level of coherence through a variety of decisions [4, 7]. Among the various models of innovative project assessment, we can distinguish those based on linear, nonlinear, dynamic, stochastic, multicriteria decision support system [2], and fuzzy programming [9]. The fuzzy sets theory is used to represent uncertain information in multiple systems [8], such as planning support systems and decision support in the innovative project assessment [6, 7]. Risk assessment is a main element in project success and should be integrated in all innovative projects. Furthermore, exist a great link between project risk assessment and a project's success [1, 3]. To deal with the risk rates of innovative projects, decision makers must use specific methods and techniques that will allow them to assess and manage these risks effectively. In this paper we build a fuzzy probabilistic approach to assess a risk related to innovative project task, after that we provide a numerical example to describe the results of the proposed approach.

## II. THE TASK OF BUILDING A PROBABILISTIC RISK ASSESSMENT MODEL OF INNOVATIVE PROJECT

As a risk assessment of innovative project implementation it is advisable to take the estimated parameter of project's profitability  $P_{Prof}$  and their rate of return value  $Q_{RR}$ .

In the case when  $P_{Prof} < Q_{RR}$ , the implementation of the innovative project is considered inappropriate. The values of

$P_{Prof}$  and  $Q_{RR}$  are given in the triangular fuzzy numbers form  $P_{Prof} = [P_{min}, P_0, P_{max}]$  and  $Q_{RR} = [Q_{min}, Q_0, Q_{max}]$ . Their membership functions are respectively represented as follow: [1-3]

$$\mu_{P_{Prof}}(x) = \begin{cases} \frac{1}{P_0 - P_{min}}x + \frac{P_{min}}{P_{min} - P_0}, & P_{min} < x < P_0; \\ \frac{1}{P_0 - P_{max}}x + \frac{P_{max}}{P_{max} - P_0}, & P_0 < x < P_{max}; \\ 0, & (x < P_{min}) \vee (x > P_{max}). \end{cases} \quad (1)$$

$$\mu_{Q_{RR}}(x) = \begin{cases} \frac{1}{Q_0 - Q_{min}}x + \frac{Q_{min}}{Q_{min} - Q_0}, & Q_{min} < x < Q_0; \\ \frac{1}{Q_0 - Q_{max}}x + \frac{Q_{max}}{Q_{max} - Q_0}, & Q_0 < x < Q_{max}; \\ 0, & (x < Q_{min}) \vee (x > Q_{max}). \end{cases} \quad (2)$$

When we build the graphs of  $\mu_{P_{Prof}}(x)$  and  $\mu_{Q_{RR}}(x)$  in one coordinate system, depending on the current values  $[Q_1(\alpha), Q_2(\alpha)]$  and  $[P_1(\alpha), P_2(\alpha)]$ , we have various possible arrangements of graphs prescribed functions in relation to each other. The general scheme of reasoning used in the present method does not depend on the location of triangular numbers  $\overline{P_{Prof}}$ ,  $\overline{Q_{RR}}$ , therefore, we will consider in more detail one of the variants, shown in Figure 1.

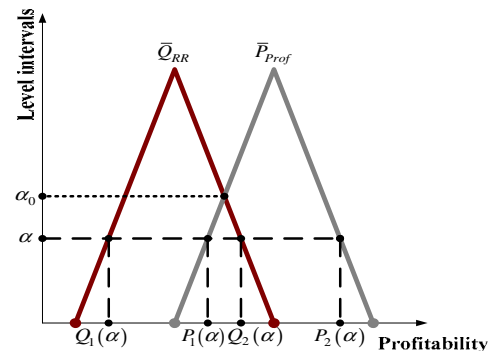


Fig. 1. The dependence between the values  $\overline{P_{Prof}}$  and  $\overline{Q_{RR}}$  in  $\alpha$ -level intervals

These two membership functions intersect at the point with the ordinate  $\alpha_0$ . When  $\alpha \geq \alpha_0$  and  $P_{prof} > Q_{RR}$ , the  $\alpha$ -level sets do not intersect, and the risky zone is absent. When  $\alpha < \alpha_0$  there is a risk that the value of  $P_{prof}$ , included in the intersection of the intervals  $[Q_1(\alpha), Q_2(\alpha)]$  and  $[P_1(\alpha), P_2(\alpha)]$ , may be less than the value of  $Q_{RR}$ , that is to say the  $[P_1(\alpha), Q_2(\alpha)]$  interval is the risk zone. We conclude that if  $0 \leq \alpha \leq \alpha_0$  the  $\alpha$ -level sets intersect. By shifting each selected  $\alpha$  level in the  $(P, Q)$ -plane, we obtain the results shown in the Figure 2. The shaded area of inefficient investments is limited by the straight lines  $P_{prof} = P_{prof_1}$ ,  $P_{prof} = P_{prof_2}$ ,  $Q_{RR} = Q_{RR_1}$ , and the bisector of the quadrangle angle  $P_{prof} = Q_{RR}$  as shown on Figure 2.

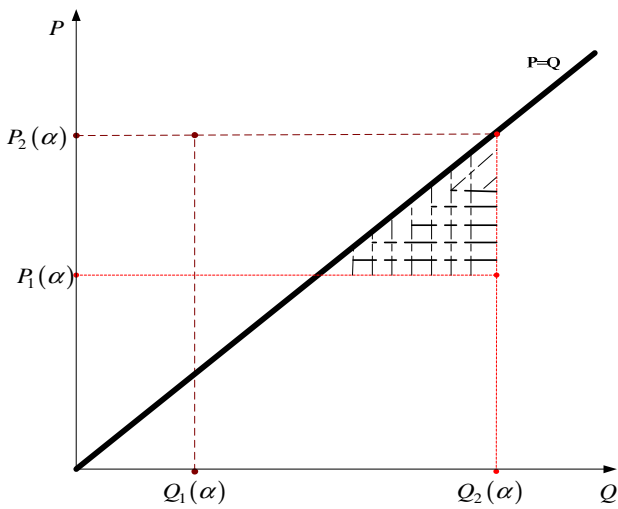


Fig. 2. The result of the transition from  $\alpha$ -leveled sets to the  $(P, Q)$ -plane for one selected criterion.

In Figure 2 the shaded area indicates the risk area, and the entire rectangle is the range of possible implementations of the selected parameter. For a selected  $\alpha$ -level, the probability of hitting the point with the current coordinates  $(P, Q)$  in the shaded area represents the probability of an insufficient level of profitability for a given pair of values.

We denote this probability by  $P(\alpha)$ . Then  $P(\alpha)$  is determined according to the expression (3) and the graph of the function shown in Figure 3.

$$P(\alpha) = \frac{S_1(\alpha)}{S_2(\alpha)}$$

where:

$S_1(\alpha)$  is the shaded area;

$S_2(\alpha)$  is the rectangular area.

If we express the area  $S_1(\alpha)$  through in explicit form, after elementary transformations we obtain the following expressions:

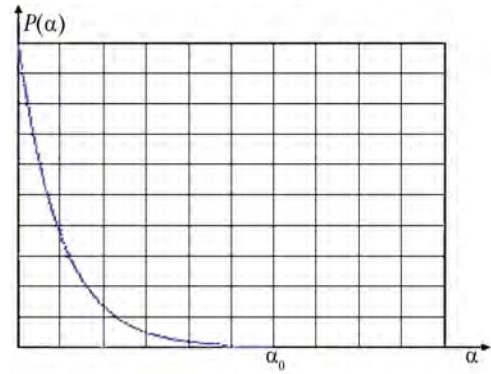


Fig. 3. The probability of hitting the point with the current coordinates  $(P, Q)$  in the shaded area of the selected  $\alpha$  level

$$S_1(\alpha) = \begin{cases} 0, & \alpha \in I_1; \\ \frac{(P_2(\alpha) - Q_1(\alpha))^2}{2}, & \alpha \in I_2; \\ \frac{(P_1(\alpha) - Q_1(\alpha) + (P_2(\alpha) - Q_1(\alpha))) \times (P_2(\alpha) - P_1(\alpha))}{2}, & \alpha \in I_3; \\ \frac{(P_2(\alpha) - Q_2(\alpha) + (P_2(\alpha) - P_1(\alpha))) \times (Q_2(\alpha) - Q_1(\alpha))}{2}, & \alpha \in I_4; \\ \frac{(P_2(\alpha) - P_1(\alpha))(Q_2(\alpha) - Q_1(\alpha)) - \frac{(Q_2(\alpha) - P_1(\alpha))^2}{2}}{2}, & \alpha \in I_5; \\ (P_2(\alpha) - P_1(\alpha))(Q_2(\alpha) - Q_1(\alpha)), & \alpha \in I_6; \end{cases} \quad (3)$$

$$I_1 = \{\alpha \mid Q_1(\alpha) > P_2(\alpha)\} = [\alpha_4, 1];$$

$$I_2 = \{\alpha \mid P_1(\alpha) < Q_1(\alpha) < P_2(\alpha) \leq Q_2(\alpha)\} = [\alpha_3, \alpha_4];$$

$$I_3 = \{\alpha \mid Q_1(\alpha) < P_1(\alpha) < P_2(\alpha) \leq Q_2(\alpha)\} = [\alpha_2, \alpha_3];$$

$$I_4 = \{\alpha \mid P_1(\alpha) < Q_1(\alpha) < Q_2(\alpha) \leq P_2(\alpha)\} = [\alpha_1, \alpha_2];$$

$$I_5 = \{\alpha \mid Q_1(\alpha) \leq P_1(\alpha) \leq Q_2(\alpha) \leq P_2(\alpha)\} = [\alpha_0, \alpha_1];$$

$$I_6 = \{\alpha \mid Q_2(\alpha) \leq P_1(\alpha)\} = [0, \alpha_0].$$

It is understood that the area depends on the relative position of intervals  $[Q_1(\alpha), Q_2(\alpha)]$  and  $[P_1(\alpha), P_2(\alpha)]$ .

For each point with the coordinates  $(P, Q)$  belongs to the shaded area, it represents the probability of an insufficient profitability level for a pair of values.

Since all variants  $(P, Q)$  are equally possible at the set level of belonging  $\alpha$ , the extent of the risk of project inefficiency  $P(\alpha)$  is a geometrical probability of finding a point  $(P, Q)$  in the zone of inefficient investments:

$$P(\alpha) = \frac{S_1(\alpha)}{(P_2(\alpha) - P_1(\alpha))(Q_2(\alpha) - Q_1(\alpha))} \quad (4)$$

In the proposed method, as a risk assessment we take the risk probability value  $P(\alpha)$  in a point of interest. Thus, for each value of  $\alpha$  has its own risk.

However, the risk for a specific values of  $\alpha$  cannot describe the value of a lack of profitability in general, because  $P(\alpha)$  have a local characteristic.

Therefore, it is appropriate to introduce the value of maximum risk of profitability, ie.

$$R_{\max} = \max_{0 < \alpha < 1} P(\alpha) = \frac{(Q_{\max} - P_{\min})^2}{2(Q_{\min} - Q_{\max})(P_{\min} - P_{\max})}. \quad (5)$$

Maximal risk does not depend on the values of  $\alpha$ , but depends exclusively on the parameters  $P_{\min}, P_{\max}, Q_{\min}, Q_{\max}$ . We deduce that, the degree of risk is determined by formulas (3) and (4) for each  $\alpha$  level can be represented as follow:

$$Risk = \int_0^1 p(\alpha) d\alpha.$$

$$\text{Where: } P(\alpha) = \frac{S_1(\alpha)}{(P_2(\alpha) - P_1(\alpha))(Q_2(\alpha) - Q_1(\alpha))}. \quad (5.1)$$

Since  $S_1(\alpha)$  as we see in (3) is based on five conditions, and then  $P(\alpha)$  also takes the following values:

$$P(\alpha) = \begin{cases} 0, & \text{for } P_{\alpha}^2 \leq Q_{\alpha}^1; \\ P_1 = \frac{(P_2(\alpha) - Q_1(\alpha))^2}{2(P_2(\alpha) - P_1(\alpha))(Q_2(\alpha) - Q_1(\alpha))}, & \text{for } P_1(\alpha) < Q_1(\alpha) < P_2(\alpha) \leq Q_2(\alpha); \\ P_2 = \frac{(P_1(\alpha) - Q_1(\alpha)) + (P_2(\alpha) - Q_1(\alpha))}{2(Q_2(\alpha) - Q_1(\alpha))}, & \text{for } Q_1(\alpha) \leq P_1(\alpha) < P_2(\alpha) \leq Q_2(\alpha); \\ P_3 = \frac{(P_2(\alpha) - Q_2(\alpha)) + (P_2(\alpha) - P_1(\alpha))}{2(P_2(\alpha) - P_1(\alpha))}, & \text{for } P_1(\alpha) \leq Q_1(\alpha) < Q_2(\alpha) \leq P_2(\alpha); \\ P_4 = 1 - \frac{(Q_2(\alpha) - P_1(\alpha))^2}{2(P_2(\alpha) - P_1(\alpha))(Q_2(\alpha) - Q_1(\alpha))}, & \text{for } Q_1(\alpha) \leq P_1(\alpha) \leq Q_2(\alpha) \leq P_2(\alpha); \\ P_5 = 1, & \text{for } Q_2(\alpha) \leq P_1(\alpha). \end{cases} \quad (6)$$

Should be noted that with a triangular fuzzy numbers  $P$  and  $Q$ , the function  $P$  cannot exist simultaneously in all intervals, and the integral will take the following form:

$$\int_0^1 P(\alpha) d\alpha = \int_{I_1} P_1(\alpha) d\alpha + \int_{I_2} P_2(\alpha) d\alpha + \int_{I_3} P_3(\alpha) d\alpha + \int_{I_4} P_4(\alpha) d\alpha + \int_{I_5} P_5(\alpha) d\alpha \quad (7)$$

In addition, we must formally express the function  $P(\alpha)$  and find the  $Q_1(\alpha)$ ,  $Q_2(\alpha)$ ,  $P_1(\alpha)$  and  $P_2(\alpha)$  values depending on  $\alpha$  as follow:  $\alpha = a.Q_{\alpha}^1 + b$ , and using the points  $(Q_{\min}, 0)$  and  $(Q_0, 1)$  of the line we can determine the coefficients  $a$  and  $b$  and therefore the value of  $Q_{\alpha}^1$  be as follow:

$$Q_{\alpha}^1 = \alpha(Q_0 - Q_{\min}) + Q_{\min}. \quad (8.1)$$

Similarly, we obtain the relation for  $Q_{\alpha}^2$ ,  $P_{\alpha}^1$  and  $P_{\alpha}^2$ .

$$Q_{\alpha}^2 = Q_{\max} - \alpha(Q_{\max} - Q_0) \quad (8.2)$$

$$P_{\alpha}^1 = \alpha(P_0 - P_{\min}) + P_{\min} \quad (8.3)$$

$$P_{\alpha}^2 = P_{\max} - \alpha(P_{\max} - P_0) \quad (8.4)$$

By using the formulas (8.1, 8.2, 8.3, 8.4) producing the corresponding changes in the function (6) we can write the resulting expressions as follow:

$$\int_{\alpha_{j-1}}^{\alpha_j} P_j d\alpha, \quad j = \overline{1, 5}.$$

### III. NUMERICAL EXAMPLE

We present an example of this model in the case of the project evaluation by one criterion shown in Fig. 4. Suppose that  $\overline{P_{prof}} = [-800, 700, 1300]$  and  $\overline{Q_{RR}} = [-220, 0, 280]$ . Their membership functions and graphical presentation are represented as follow:

$$\mu_{P_{prof}}(x) = \begin{cases} \frac{x+800}{1500}, & -800 < x < 700; \\ \frac{-x+1300}{600}, & 700 < x < 1300; \\ 0, & (x < -800) \vee (x > 1300). \end{cases}$$

$$\mu_{Q_{RR}}(x) = \begin{cases} \frac{-x+280}{280}, & 0 < x < 280; \\ \frac{x+220}{220}, & -220 < x < 0; \\ 0, & (x < -220) \vee (x > 280). \end{cases}$$

For given fuzzy numbers  $\mu_P$  and  $\mu_R$ , the function  $P(\alpha)$  exists only on three intervals. The first one is:  $P_\alpha^1 < Q_\alpha^1 < Q_\alpha^2 < P_\alpha^2$  when  $\alpha \in [0, \alpha_0]$ , the second one is:  $Q_\alpha^1 < P_\alpha^1 < Q_\alpha^2 < P_\alpha^2$  when  $\alpha \in [\alpha_0, \alpha_1]$ , and the last one is:  $Q_\alpha^2 < P_\alpha^1$  when  $\alpha \in [\alpha_1, 1]$ . We must find the values of  $\alpha_0$  and  $\alpha_1$ . Equating the functions  $\mu_P$  and  $\mu_R$  at the corresponding intervals, we obtain the following result:

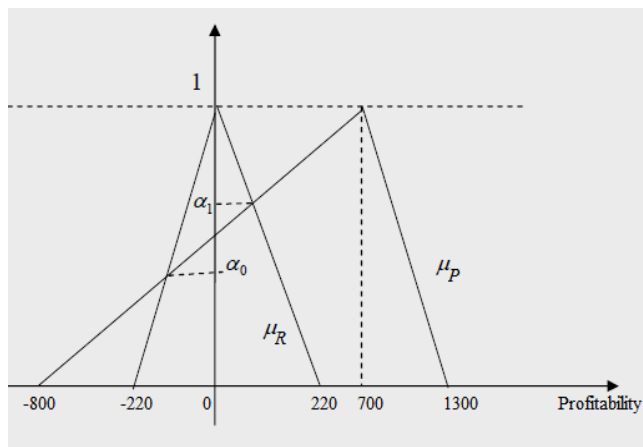


Fig. 4. Example of the values  $\overline{P_{prof}}$  and  $\overline{Q_{RR}}$  in  $\alpha$ -level intervals

$$\alpha_0 = 0.45 \text{ then } P = Q = -115.$$

$$\alpha_1 = 0.65 \text{ then } P = Q = 120.$$

Based on these data, we calculate the risk degree by implementing the project as follow:

$$Risk = \int_0^{0.65} P(\alpha) d\alpha = \int_0^{0.45} P_2 d\alpha + \int_{0.45}^{0.65} P_1 d\alpha.$$

The risk degree of this project is: Risk = 0.144.

If you determine the risk of the project by an approximate method according (5.1) and (6) we obtained the following results:

TABLE I. RISK ASSESSMENT USING APPROXIMATE METHOD

Alpha	$P_1(\alpha)$	$P_2(\alpha)$	$Q_1(\alpha)$	$Q_2(\alpha)$	$P(\alpha)$
0	-800	1300	-220	280	0.401
0.1	-660	1250	-190	265	0.352
0.2	-506	1200	-162	243	0.310
0.3	-350	1121	-135	205	0.253
0.4	-205	1068	-119	178	0.174
0.5	-53	1005	-110	140	0.075
0.6	100	941	-75	115	0.018
0.7	240	880	-58	87	0.000
0.8	380	805	-37	55	0.000
0.9	538	743	-23	28	0.000

Based on these results we obtain the risk value:

$$Risk = 0.159.$$

The approximate method gives us a 10% higher risk rating than that used in our method.

#### IV. CONCLUSION

The application of the fuzzy set theory provides a new method for the risk assessment of innovative project. In this paper, we have developed a fuzzy approach to deal with risk by introducing an innovative project. As a result, the decision makers have now a better possibility for describing the information uncertainty in the project, by applying the fuzzy set theory. Consequently, the fuzzy sets allow the users to determine the project qualitative characteristics, and to transform them into a mathematical model. In conclusion, our proposed approach can describe the risk level in an uncertain environment.

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