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About Kernel Structure Construction of the Generalized Neural Functions

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***Abstract*— The paper introduces concept of a modified kernel of the Boolean functions. Applying such a concept, the criteria for the implementation of the Boolean functions by one generalized neural element are obtained. The effective and necessary conditions to check whether the algebra of logic functions belong to the class of the generalized neural functions are given. A sufficient condition for the implementation of the Boolean functions is obtained by one generalized neural element on the basis of which it is possible to develop effective methods for the synthesis of the generalized integer neural elements with a large number of inputs.**

Keywords*— spectrum of function, generalized neural element, structure vector, kernel of function, convex hull, character of group, synthesis, metric, matrix of tolerance.

I. Introduction

In recent years, there has been an increased interest in neural-like structures, which are widely used in image recognition, compression of discrete signals and images, forecasting, business, medicine and engineering.

Widespread usage of neural networks to effectively solve applied problems will become possible if effective methods of synthesis (training) of neural elements with different functions of activation and synthesis of logical circuits of them are developed.

It is required to synthesize reliable (integer) neural elements with a large number of inputs concerning such issues as image recognition, compression and transmission of discrete signals. Classical methods of approximation and various iterative methods for the synthesis of neural elements are almost not suitable for finding the vectors of neural element structures to implement discrete functions with a large number of inputs (several hundreds, thousands).

Artificial neural networks and neural-like structures are effectively used for the classification and recognition of images [1] and for improving their quality [2]. Intelligent blocks of various systems for controlling chemical processes [3], for prediction of economic [4], biological [5] processes are developed on their basis. Neural network methods have been successfully applied to compress signals and images [6-9], in the banking sector to assess credit risk [9,11], in automated control systems of technological processes [12], in the field of intellectual data processing [13] and for constructing the logical blocks of various safety systems [14].

When selecting mathematical models of neural elements for the construction of neural-like structures, the functionality of these elements is of importance. It is crucial for optimizing the number of elements in the corresponding logical structures. The generalized neural elements allowing to extend the functionality of ordinary neural elements with threshold activation functions are considered. The properties of the kernels of functions of the algebra of logic that are implemented on these elements are discovered.

II. The Kernel Properties of the Generalized Neural Functions

Let $H_2 = \{-1,1\}$ – a cyclic group of the $2nd$ order, $G_n = H_2 \otimes \ldots \otimes H_2$ – a direct product n of the cyclic groups H_2 i $\chi(G_n)$ – a group of characters [15] of the group G_n over the field of R. Define on the set $R \setminus \{0\}$ a function as follows:

$$\text{Rsign}\, x = \begin{cases} 1, & if\ \ x > 0, \\ -1, & if\ \ x < 0. \end{cases} \quad (1)$$

Let $Z_2 = \{0,1\}$, $i \in \{0,1,2,\ldots,2^n-1\}$ and $(i_1,\ldots,i_n)$ – binary code of i , i.e. $i = i_1 2^{n-1} + i_2 2^{n-2} + \ldots + i_n$, $i_j \in \{0,1\}$. The value of the character χ_i on the element $\mathbf{g} = ((-1)^{\alpha_1},\ldots,(-1)^{\alpha_n}) \in G_n$ $((\alpha_1,\ldots,\alpha_n) \in Z_2^n$ – n-fold Cartesian product $Z_2 = \{0,1\})$ is determined as follows:

$$\chi_i(\mathbf{g}) = (-1)^{\alpha_1 i_1 + \alpha_2 i_2 + \ldots + \alpha_n i_n}. \quad (2)$$

Considering the orthogonality of the characters [15] the group of characters $X(G_n)$ forms an orthogonal basis of the space $V_R = \{\phi \mid \phi : G_n \to R\}$. Since the Boolean function of n variables in $\{-1,1\}$ sets unambiguous mapping of the

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form $f: G_n \to H_2$, то $f \in V_R$ and it means that an arbitrary Boolean function f can be written unambiguously:

$$f(\mathbf{g}) = s_0\chi_0(\mathbf{g}) + s_1\chi_1(\mathbf{g}) + \ldots + s_{2^n-1}\chi_{2^n-1}(\mathbf{g}). \qquad (3)$$

A vector $\mathbf{s}_f = (s_0, s_1, \ldots, s_{2^n-1})$ is called the spectrum of the Boolean function f in the system of characters $\chi(G_n)$ (in the system of Walsh-Hadamard basic functions [16]).

With different characters $\chi(G_n)$, in addition to the main one, make up m -element set $\chi = \{\chi_{i_1}, \ldots, \chi_{i_m}\}$ and concerning the chosen system of characters, according to[17], consider the mathematical model of a neural element with a generalized threshold activation function (of a generalized neural element):

$$f(x_1(\mathbf{g}), \ldots, x_n(\mathbf{g})) = \mathrm{Rsign}(\sum_{j=1}^{m} \omega_j \chi_{i_j}(\mathbf{g}) + \omega_0), \qquad (4)$$

where $\mathbf{w} = (\omega_1, \ldots, \omega_m; \omega_0)$ is called a vector of the structure of the generalized neural element (GNE) regarding the system of characters $\chi \in \chi(G_n)$ і $\mathbf{g} \in G_n$.

Let $\mathrm{w}(\mathbf{g}) = \omega_1\chi_{i_1}(\mathbf{g}) + \ldots + \omega_m\chi_{i_m}(\mathbf{g}) + \omega_0$. If $\mathbf{w} = (\omega_1, \ldots, \omega_m; \omega_0)$ is a vector of the GNE structure regarding the system of characters $\chi = \{\chi_{i_1}, \ldots \chi_{i_m}\}$ of the group $\chi(G_n)$, that realizes the Boolean function $f: G_n \to H_2$, from (1) and (4) we have the following result:

$$\forall \mathbf{g} \in G_n \quad \mathrm{w}(\mathbf{g}) \neq 0. \qquad (5)$$

Further we only consider such neural elements whose vectors of structure satisfy the condition (5). A set of all such $m+1$-dimensional real vectors satisfying the condition (5) regarding the system χ we will denote as $\mathrm{W}_{m+1}(\chi) = \mathrm{W}_{m+1}(\chi_{i_1}, \ldots \chi_{i_m})$.

Definition 1. The Boolean function $f: G_n \to H_2$ is called a generalized neural function concerning the system of characters $\chi = \{\chi_{i_1}, \ldots, \chi_{i_m}\} \subset \chi(G_n)$ if $\mathbf{w} = (\omega_1, \ldots, \omega_m; \omega_0) \in \mathrm{W}_{m+1}(\chi)$ that is applied to as an equation (4)

To introduce the concept of kernels of the generalized neural functions and to study their basic properties, the Boolean functions will be considered as in $H_2 = \{-1, 1\}$, as in $\mathrm{Z}_2 = \{0, 1\}$. Let $f(x_1, \ldots, x_n)$ the Boolean function in $\{-1, 1\}$, i.e. $f: G_n \to H_2$. We will consider the problem on the implementation of the Boolean function $f(x_1, \ldots, x_n)$ by one GNE regarding the system of characters $\chi = \{\chi_{i_1}, \chi_{i_2}, \ldots, \chi_{i_m}\} \subset X(G_n)$ in $\{0, 1\}$. Using the transformation $\mathbf{x}' = \frac{1}{2}(\mathbf{x} + 1)$ we define the mapping of the form $\{-1, 1\} \to \{0, 1\}$ and build the system $\chi' = \left\{\chi'_{i_1} = \frac{1}{2}(\chi_{i_1} + 1), \chi'_{i_2} = \frac{1}{2}(\chi_{i_2} + 1), \ldots, \chi'_{i_m} = \frac{1}{2}(\chi_{i_m} + 1)\right\}$

Let $f^{-1}(-1) = \{\mathbf{g} \in G_n \mid f(\mathbf{g}) = -1\}$ and $f^{-1}(1) = \{\mathbf{g} \in G_n \mid f(\mathbf{g}) = 1\}$. Using the system χ' we will determine:

$$f_\chi^{-1}(0) = \bigcup_{\mathbf{g} \in f^{-1}(-1)} \left\{\left(\chi'_{i_1}(\mathbf{g}), \ldots, \chi'_{i_m}(\mathbf{g})\right)\right\},$$

$$f_\chi^{-1}(1) = \bigcup_{\mathbf{g} \in f^{-1}(1)} \left\{\left(\chi'_{i_1}(\mathbf{g}), \ldots, \chi'_{i_m}(\mathbf{g})\right)\right\}.$$

Definition 2. The kernel of the Boolean function $f: G_n \to H_2$ regarding the system of characters $\chi = \{\chi_{i_1}, \chi_{i_2}, \ldots, \chi_{i_m}\} \subset X(G_n)$ in $\{0, 1\}$ is called a set $K(f_\chi)$, which is determined as follows:

$$K(f_\chi) = \begin{cases} f_\chi^{-1}(1), & if \ \left|f_\chi^{-1}(1)\right| \le \left|f_\chi^{-1}(0)\right|, \\ f_\chi^{-1}(0), & if \ \left|f_\chi^{-1}(1)\right| > \left|f_\chi^{-1}(0)\right|, \end{cases}$$

if $f_\chi^{-1}(1) \cap f_\chi^{-1}(0) = \varnothing$, where $\left|f_\chi^{-1}(i)\right|$ – a number of set elements $f_\chi^{-1}(i)$ $(i \in \{0, 1\})$.

If $f_\chi^{-1}(1) \cap f_\chi^{-1}(0) \neq \varnothing$, the kernel $K(f_\chi)$ does not exist and it means that a function f is not realized by one GNE regarding the sytem χ.

Let Z_2^m – m*th* Cartesian degree of the set $\mathrm{Z}_2 = \{0, 1\}$. Assume, that the function $f: G_n \to H_2$ has the kernel $K(f_\chi)$ regarding the system $\chi = \{\chi_{i_1}, \ldots, \chi_{i_m}\}$, i.e. $f_\chi^{-1}(1) \cap f_\chi^{-1}(0) = \varnothing$. The sets $f_\chi^{-1}(1)$, $f_\chi^{-1}(0)$ satisfy one of the conditions:

1) $Z_2^m = f_\chi^{-1}(1) \cup f_\chi^{-1}(0)$ – a function f_χ completely defined on the set Z_2^m;

2) $Z_2^m \neq f_\chi^{-1}(1) \cup f_\chi^{-1}(0)$ – a function f_χ partially defined on the set Z_2^m.

In the first case, the kernel $K(f_\chi)$ is defined unambiguously and $K(f_\chi, \varnothing) = K(f_\chi)$.

In the second case, we introduce the concept of the generalized kernel concerning the system of characters χ.

Let $K(f_\chi) = \{\mathbf{a}_1, \ldots, \mathbf{a}_q\}$ – a kernel of the Boolean function f_χ regarding the system $\chi = \{\chi_{i_1}, \ldots, \chi_{i_m}\}$ and

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$f_\chi^{-1}(*)$ is a set of those combinations with Z_2^m on which the function is not defined, then under the extended kernel function f_χ regarding the system χ we imply $K(f_\chi, A) = \{\mathbf{a}_1,\dots,\mathbf{a}_q,\mathbf{a}_{q+1},\dots,\mathbf{a}_{q+s}\}$, where $\mathbf{a}_{q+1},\dots,\mathbf{a}_{q+s}$ – various arbitrary elements of the set from $f_\chi^{-1}(*)$, $q+s \le 2^{m-1}$ and $A = \{\mathbf{a}_{q+1},\dots,\mathbf{a}_{q+s}\}$. Note, that a set A may be empty.

We introduce a concept of the modified kernel $K(f_\chi, M)$ of the Boolean function $f: G_n \to H_2$ concerning the system of characters $\chi = \{\chi_{i_1},\chi_{i_2},\dots,\chi_{i_m}\} \subset X(G_n)$ as follows:

$$K(f_\chi, M) = \begin{cases} K(f_\chi), & if \quad A = \varnothing; \\ K(f_\chi, A), & if \quad A \ne \varnothing. \end{cases}$$

Definition 3. The Boolean function $f: G_n \to H_2$ is realized by one generalized neural element with the structure vector $\mathbf{w} = (\omega_1,\dots,\omega_m;\omega_0)$ over R concerning the system of characters $\chi = \{\chi_{i_1},\chi_{i_2},\dots,\chi_{i_m}\} \subset X(G_n)$ in $\{0,1\}$, if there exists such a modified kernel $K(f_\chi, M)$. Let $f: G_n \to H_2$, the system $\chi = \{\chi_{i_1},\chi_{i_2},\dots,\chi_{i_m}\} \subset X(G_n)$ and the modified kernel $K(f_\chi, M)$. We will make for a function f concerning the system χ and the modified kernel $K(f_\chi, M)$ the Boolean function $f_\chi^*: Z_2^m \to Z_2$ as follows:

$$\forall \mathbf{a} \in K(f_\chi, M) \quad f_\chi^*(\mathbf{a}) = f_\chi(\mathbf{a}),$$

$$\forall \mathbf{a} \in Z_2^m \setminus K(f_\chi, M) \quad f_\chi^*(\mathbf{a}) = \overline{f_\chi}(\mathbf{a}),$$

where the vinculum means a logical denial operation.

By definition, function f_χ^* and $K(f_\chi) \subset K(f_\chi, M)$ we obtain

$$f_\chi^{-1}(\alpha) \subset f_\chi^{*-1}(\alpha),$$

where $\alpha \in \{0,1\}$.

We define the kernel of function f_χ^* as a modified kernel of function f_χ, i.e.

$$K(f_\chi^*) = K(f_\chi, M).$$

We will determine a convex linear hull $\mathrm{conv}K(f_\chi^*)$ of the kernel elements $K(f_\chi^*)$ as follows:

$$\mathrm{conv}K(f_\chi^*) = \Big\{\mathbf{x} \in [0,1]^m \,\Big|\, \mathbf{x} = \sum_{i=1}^{t} \lambda_i \mathbf{a}_i, \sum_{i=1}^{t} \lambda_i = 1, \lambda_1 \ge 0,\dots,\lambda_t \ge 0;\ \mathbf{a}_1,\dots,\mathbf{a}_t \in K(f_\chi^*)\Big\}.$$

Similarly, we define $\mathrm{conv}K(f_\chi^*)^*$ for $K(f_\chi^*)^* = Z_2^m \setminus K(f_\chi^*)$.

Theorem 1. The Boolean function $f: G_n \to H_2$ ($f \ne const$) is realized by one generalized neural element regarding the system of characters $\chi = \{\chi_{i_1},\chi_{i_2},\dots,\chi_{i_m}\} \subset X(G_n)$ only when there exists such a modified kernel $K(f_\chi, M)$, that

$$\mathrm{conv}K(f_\chi^*) \cap \mathrm{conv}K(f_\chi^*)^* = \varnothing.$$

Let $\mathbf{a},\mathbf{b}$ – arbitrary kernel elements $K(f_\chi)$ $(\mathbf{a} \ne \mathbf{b})$ of the Boolean function $f: G_n \to H_2$ regarding the system of characters $\chi = \{\chi_{i_1},\chi_{i_2},\dots,\chi_{i_m}\} \subset X(G_n)$ i $O(\mathbf{a},\mathbf{b})$ – a set of such ortho-vectors $\mathbf{e}_{i_1},\dots,\mathbf{e}_{i_s}$, that $\mathbf{a} \oplus \mathbf{b} = \mathbf{e}_{i_1} + \mathbf{e}_{i_2} + \dots + \mathbf{e}_{i_s}$, where $\oplus$ – a coordinate-wise sum of vectors by module 2, $i_r \ne i_k$, if $r \ne k$. Denote $H(\mathbf{a},\mathbf{b})$ to be the subset of the group Z_2^m (Z_2^m forms a group regarding the operation $\oplus$), which is generated by the elements $O(\mathbf{a},\mathbf{b})$, i.e.

$$\mathrm{H}(\mathbf{a},\mathbf{b}) = \left\langle \mathbf{e}_{i_1},\dots,\mathbf{e}_{i_s} \mid \mathbf{e}_{i_1},\dots,\mathbf{e}_{i_s} \in O(\mathbf{a},\mathbf{b}) \right\rangle.$$

Let $\mathbf{a} = (\alpha_1,\dots,\alpha_m), \mathbf{b} = (\beta_1,\dots,\beta_m) \in Z_2^m$. A coordinate-wise conjunction of vectors $\mathbf{a}$ and $\mathbf{b}$ we will define as $\mathbf{a} \,\&\, \mathbf{b} = (\alpha_1 \,\&\, \beta_1,\dots,\alpha_m \,\&\, \beta_m)$ as well as $H(\mathbf{a} \,\&\, \mathbf{b})$ we will define an adjacent group class Z_2^m by the subset $H(\mathbf{a},\mathbf{b})$, that is defined by the element $\mathbf{a} \,\&\, \mathbf{b}$, i.e. $H(\mathbf{a} \,\&\, \mathbf{b}) = \mathbf{a} \,\&\, \mathbf{b} \oplus H(\mathbf{a},\mathbf{b})$.

Set the metric $\rho(\mathbf{a},\mathbf{b})$ on Z_2^m as follows:

$$\rho(\mathbf{a},\mathbf{b}) = \sum_{i=1}^{m} (\alpha_i \oplus \beta_i),$$

where $\mathbf{a} = (\alpha_1,\dots,\alpha_m), \mathbf{b} = (\beta_1,\dots,\beta_m) \in Z_2^m$.

Theorem 2. If the Boolean function $f: G_n \to H_2$ ($f \ne const$) is realized by one generalized neural element regarding the system of characters $\chi = \{\chi_{i_1},\chi_{i_2},\dots,\chi_{i_m}\} \subset X(G_n)$ with the modified kernel $K(f_\chi, M)$, for any two different elements $\mathbf{a},\mathbf{b}$ with $K(f_\chi^*)$, for which $|H(\mathbf{a} \,\&\, \mathbf{b}) \cap K(f_\chi^*)^*| \ge 2$ and for any two different elements $\mathbf{g},\mathbf{h}$ with $H(\mathbf{a} \,\&\, \mathbf{b}) \cap K(f_\chi^*)^*$, inequality is realized $\rho(\mathbf{g},\mathbf{h}) < \rho(\mathbf{a},\mathbf{b})$.

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Let $K(f_\chi^*)=\{\mathbf{a}_1,...,\mathbf{a}_t\}$ – a kernel of the Boolean function $f:G_n\to H_2$ ($f\neq const$) regarding the system of characters $\chi=\{\chi_{i_1},\chi_{i_2},\dots,\chi_{i_m}\}\subset X(G_n)$ with the modified kernel $K(f_\chi,M)$ and $K(f_\chi^*)_i=\{\mathbf{a}_i\oplus\mathbf{a}_1,...,\mathbf{a}_i\oplus\mathbf{a}_t\}$ – the consolidated kernel [18] of function f_χ^* concerning the element $\mathbf{a}_i\in K(f_\chi^*)$. A set of all consolidated kernels of the Boolean function f_χ^* we will defined as $T(f_\chi^*)=\{K(f_\chi^*)_i=\mathbf{a}_iK(f_\chi^*)\,|\,i=1,2,...,t\}$.

It is believed that the vector $\mathbf{a}=(\alpha_1,...,\alpha_m)\in Z_2^m$ precedes the vector $\mathbf{b}=(\beta_1,...,\beta_m)\in Z_2^m$ $\mathbf{a}\prec\mathbf{b}$, if $\alpha_i\le\beta_i\,(i=1,2,...,m)$. We will denote $N_\mathbf{a}$ to be a set of all such vectors with Z_2^m, which precedes the vector $\mathbf{a}$.

Theorem 3. If the Boolean function $f:G_n\to H_2$ is realized by one generalized neural element regarding the system of characters $\chi=\{\chi_{i_1},\chi_{i_2},\dots,\chi_{i_m}\}\subset X(G_n)$ with the modified kernel $K(f_\chi,M)$, the kernel $K(f_\chi^*)$ of function f_χ^* satisfies the condition

$$\mathbf{a}=(\alpha_1,\dots,\alpha_m)\in K(f_\chi^*)\Rightarrow\overline{\mathbf{a}}=(\overline{\alpha}_1,\dots,\overline{\alpha}_m)\notin K(f_\chi^*),$$

where $\overline{\alpha}_i$ – an inverted value α_i.

Theorem 4. If the Boolean function $f:G_n\to H_2$ is realized by one generalized neural element regarding the system of characters in $\chi=\{\chi_{i_1},\chi_{i_2},\dots,\chi_{i_m}\}\subset X(G_n)$ with the modified kernel $K(f_\chi,M)$, a set of the consolidated kernel $T(f_\chi^*)$ contains the element $K(f_\chi^*)_i$, that

$$\forall\mathbf{a}\in K(f_\chi^*)_i\Rightarrow N_\mathbf{a}\subset K(f_\chi^*)_i\ .$$

Consider a set of matrices of tolerance [17]

$$F_m=\left\{L_1=(0_1),\ L_2=\begin{pmatrix}L_1 & 0_1\\ L_1^* & 0_1\end{pmatrix},\ ...,\ L_m=\begin{pmatrix}L_{m-1} & 0_{m-1}\\ L_{m-1}^* & 0_{m-1}\end{pmatrix}\right\},$$

where 0_r – zero column of $2^{r-1}\times1$.

Using the kernel elements $K(f_\chi^*)$ we will construct the matrix $K_\xi(f_\chi^*)$ as follows: the first line of the matrix $K_\xi(f_\chi^*)$ will be $\mathbf{a}_{\xi(1)}=(\alpha_{\xi(1)1},...,\alpha_{\xi(1)m})$ out of $K(f_\chi)$, the second line of the matrix will be $\mathbf{a}_{\xi(2)}=(\alpha_{\xi(2)1},...,\alpha_{\xi(2)m})$, the last line $K_\xi(f_\chi^*)$ will be $\mathbf{a}_{\xi(q)}=(\alpha_{\xi(q)1},...,\alpha_{\xi(q)m})$, where $\xi(i)$ – an effect of substitution $\xi\in S_q$ for i (a symmetric group of degree q) . Let , then (a symmetric group of degree *m*).

Theorem 5. If a set of the consolidated kernels $T(f_\chi^*)$ of the Boolean function $f:G_n\to H_2$ ($f\neq const$) regarding the system of characters $\chi=\{\chi_{i_1},\dots,\chi_{i_m}\}\subset G_n$ with the modified kernel $K(f_\chi,M)$ contains the element $K(f_\chi^*)_i$ for which there exist the elements $\xi\in S_t$, $\sigma\in S_m$ and such a matrix of tolerance $L_{j_i}\in F_m$, that

$$K_\xi^\sigma(f_\chi^*)_i=(L_{j_i}\underbrace{0...0}_{m-j_i})\nabla(L_{j_i}^*(q_0^i)\underbrace{0...0}_{m-j_i}))\nabla(L_{j_i+1}^*(q_1^i)\underbrace{0...0}_{m-j_i-1})\nabla...$$
$$...\nabla(L_{j_i+r}^*(q_r^i)\underbrace{0...0}_{m-j_i-r})$$

where $q_0^i\ge q_1^i\ge...\ge q_r^i$, then the function $f:G_n\to H_2$ isrealized by one generalized neural element regarding the system of characters χ. Based on the above mentioned theorems, an effective algorithm for synthesizing optimal generalized integer neural elements with a large number of inputs can be constructed.

III. THE EXAMPLES OF PRACTICAL APPLICATION OF RESEARCH FINDINGS

We will demonstrate the effectiveness and practical feasibility of using the research results obtained for the synthesis of the generalized neural elements with a large number of inputs (several hundred, several thousand) in the following examples.

1.Let $\mathbf{e}_0=(0,0,...,0)$, $\mathbf{e}_1=(1,0,...,0)$, $\mathbf{e}_2=(0,1,...,0),...,$ $\mathbf{e}_m=(0,0,...,1)$ – *m*- dimensional Boolean vectors and a complete prototype of Boolean function units $f:G_n\to H_2$ regarding the system of characters $\chi=\{\chi_{i_1},\dots,\chi_{i_m}\}\subset\chi(G_n)(2\le m\le n)$ is set as follows:

$$f_\chi^{-1}(1)=\{\mathbf{e}_1,\mathbf{e}_2,...,\mathbf{e}_m\}\ \text{i}\ \ f_\chi^{-1}(1)\cup f_\chi^{-1}(0)=Z_2^m.$$

Then by definition $K(f_\chi^*)=K(f_\chi)=f_\chi^{-1}(1)$. We will construct a set of the consolidated kernels

$$T(f_\chi^*)=\{K(f_\chi^*)_1=\mathbf{e}_1K(f_\chi^*),K(f_\chi^*)_2=\mathbf{e}_2K(f_\chi^*),...$$
$$...,K(f_\chi^*)_m=\mathbf{e}_mK(f_\chi^*)\},$$

where

$$K(f_\chi^*)_1=\{\mathbf{e}_0,\mathbf{e}_1\oplus\mathbf{e}_2,\mathbf{e}_1\oplus\mathbf{e}_3,...,\mathbf{e}_1\oplus\mathbf{e}_m\},$$

...

$$K(f_\chi^*)_m=\{\mathbf{e}_m\oplus\mathbf{e}_1,\mathbf{e}_m\oplus\mathbf{e}_2,\mathbf{e}_m\oplus\mathbf{e}_3,...,\mathbf{e}_0\}$$

Each consolidated kernel $K(f_\chi^*)_k$ has an element $\mathbf{e}_k\oplus\mathbf{e}_i\,(k\neq i;i\neq0)$, but does not contain the elements $\mathbf{e}_k$ i $\mathbf{e}_i$, which precede this element. So, none of the consolidated kernels out of $T(f_\chi^*)$ does not satisfy the conditions of Theorem 4 and it means that the function f is not realized by one generalized neural element concerning the system χ.

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2. Let $\mathbf{e}_0=(0,0,...,0)$, $\mathbf{e}_1=(1,0,...,0)$, $\mathbf{e}_2=(0,1,...,0),...,$ $\mathbf{e}_m=(0,0,...,1)$ – m- dimensional Boolean vectors. We will consider the problem of the Boolean function implementation $f:G_n\to H_2$ regarding the system of characters $\chi=\{\chi_{i_1},\dots,\chi_{i_m}\}\subset\chi(G_n)(3\le t<m\le n)$, if

$$f_\chi^{-1}(1)=\{\mathbf{e}_1,\mathbf{e}_2,...,\mathbf{e}_t,\mathbf{e}_0,\mathbf{e}_1\oplus\mathbf{e}_2,...,\mathbf{e}_1\oplus\mathbf{e}_m\} \text{ and } f_\chi^{-1}(1)\cup f_\chi^{-1}(0)=Z_2^m.$$

The restriction $t\ge 3$ is imposed in terms of unambiguousness of the definition $K(f_\chi^*)$. If $t\ge 3$, then any $m(t<m\le n)$ $m+t\le 2^{m-1}$ and in this case $K(f_\chi^*)=K(f_\chi)=f_\chi^{-1}(1)$, and in the opposite case $(t<3)$ $K(f_\chi^*)=f_\chi^{-1}(0)$.

In this case $K(f_\chi^*)=f_\chi^{-1}(1)$. We will construct a set of the consolidated kernels

$$T(f_\chi^*)=\left\{\begin{matrix}K(f_\chi^*)_1=\mathbf{e}_1K(f_\chi^*),K(f_\chi^*)_2=\mathbf{e}_2K(f_\chi^*),...\\ ...,K(f_\chi^*)_{t+m}=(\mathbf{e}_1\oplus\mathbf{e}_m)K(f_\chi^*)\end{matrix}\right\}.$$

We will write out the elements of the consolidated kernels $K(f_\chi^*)_i$:

$$K(f_\chi^*)_1=\{\mathbf{e}_0,\mathbf{e}_1\oplus\mathbf{e}_2,...,\mathbf{e}_1\oplus\mathbf{e}_t,\mathbf{e}_1,\mathbf{e}_2,...,\mathbf{e}_m\},$$

$$K(f_\chi^*)_2=\left\{\begin{matrix}\mathbf{e}_1\oplus\mathbf{e}_2,\mathbf{e}_0,...,\mathbf{e}_2\oplus\mathbf{e}_t,\mathbf{e}_2,\mathbf{e}_1,\mathbf{e}_1\oplus\mathbf{e}_2\oplus\mathbf{e}_3...\\ ...,\mathbf{e}_1\oplus\mathbf{e}_2\oplus\mathbf{e}_m\end{matrix}\right\},$$

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$$K(f_\chi^*)_{t+m}=\left\{\begin{matrix}\mathbf{e}_m,\mathbf{e}_1\oplus\mathbf{e}_2\oplus\mathbf{e}_m,...,\mathbf{e}_1\oplus\mathbf{e}_t\oplus\mathbf{e}_m,\mathbf{e}_1\oplus\mathbf{e}_m,\\ \mathbf{e}_2,...,\mathbf{e}_{m-1},\mathbf{e}_0\end{matrix}\right\}.$$

Out of the elements $K(f_\chi^*)_1$ we can construct a matrix

$$\begin{aligned}K_\xi(f_\chi^*)_1=(L_3\underbrace{0...0}_{m-3})\nabla(L_3^*(2)\underbrace{0...0}_{m-3})\nabla...\\ \nabla(L_t^*(2)\underbrace{0...0}_{m-t})\nabla(L_{t+1}^*(1)\underbrace{0...0}_{m-t-1})\nabla...\nabla(L_m^*(1))\end{aligned}, \qquad (6)$$

where the element $\xi\in S_{t+m}$ defines the order of the elements 3 $K(f_\chi^*)_1$ within the matrix.

On the basis of Theorem 5 and equality (6) we will construct a vector $\mathbf{w}=(\omega_1,\omega_2,...,\omega_m)$:

$$\omega_1=-1,\omega_2=\omega_1-1=-2,\omega_3=\omega_4=...=\omega_t=-4,\omega_{t+1}=...\\ ...,=\omega_m=-5.$$

Then

$$\mathbf{w}_1=\mathbf{e}_1\mathbf{w}^{\sigma^{-1}} \text{ i } \omega_0^{(1*)}=\min\{(\mathbf{w}_1,\mathbf{x})\,|\,\mathbf{x}\in f_\chi^{-1}(1)\}-\varepsilon,$$

where $\varepsilon\in\left(0,\frac{1}{2}\right)$.

Taking into account that σ is a single element of the group S_m and setting $\varepsilon=\frac{1}{4}$ we will obtain the following vector structure $\left(\mathbf{w}_1=\left(\omega_1^{(1)},...,\omega_m^{(1)}\right);\omega_0^{(1*)}\right)$ GNE regarding the system $\chi=\{\chi_{i_1},\dots,\chi_{i_m}\}\subset\chi(G_n)$:

$$\omega_1^{(1)}=1,\omega_2^{(1)}=-2,\omega_3^{(1)}=\omega_4^{(1)}=...=\omega_t^{(1)}=-4,\omega_{t+1}^{(1)}=...\\ =\omega_m^{(1)}=-5 \text{ i } \omega_0^{(1*)}=-\frac{17}{4},$$

that realizes the function f_χ in $\{0,1\}$.

$$\omega_0^{(1)}=\sum_{j=1}^m\omega_j^{(1)}-2\omega_0^{(1*)}=1-2+(-4)(t-2)+\\ +(-5)(m-t)-2\left(-\frac{17}{4}\right)=t-5m+15{,}5$$

We reveal and a generalized neural element with vector structure $\left(\mathbf{w}_1;\omega_0^{(1)}\right)$ realizes the function $f:G_n\to H_2$ in $\{-1,1\}$.

3. We will consider the case where a number of characters m within the system of characters $\chi=\{\chi_{i_1},\dots,\chi_{i_m}\}\subset\chi(G_n)$ satisfies inequality $n<m<2^n$. Let $f:G_n\to H_2$ and $f_\chi^{-1}(1)=\{\mathbf{e}_1,\mathbf{e}_2,...,\mathbf{e}_m\}$, $f_\chi^{-1}(0)=\{\bar{\mathbf{e}}_1,\bar{\mathbf{e}}_2,...,\bar{\mathbf{e}}_m\}$, $f_\chi^{-1}(*)=Z_2^m\setminus\left(f_\chi^{-1}(1)\cup f_\chi^{-1}(0)\right)$, where the vinculum signifies a logical denial operation of the Boolean vector coordinates. In case of $m>n$ we always have $Z_2^m\ne f_\chi^{-1}(1)\cup f_\chi^{-1}(0)$. Since $f_\chi^{-1}(1)\cap f_\chi^{-1}(0)=\varnothing$, the kernel $K(f_\chi)$ the following exists $K(f_\chi)=f_\chi^{-1}(1)$. It was shown above (point 1) that when $K(f_\chi)=f_\chi^{-1}(1)=\{\mathbf{e}_1,\mathbf{e}_2,...,\mathbf{e}_m\}$, $Z_2^m=f_\chi^{-1}(1)\cup f_\chi^{-1}(0)$, the function f is not realized by one GNE regarding the system χ . In this case $Z_2^m\ne f_\chi^{-1}(1)\cup f_\chi^{-1}(0)$ and we will demonstrate, that it is possible to construct such an extended kernel that $K(f_\chi,A)$, that a function corresponding to this kernel f_χ^* will be realized by one generalized neural element regarding the system χ , which means that the function f is also realized by one GNE concerning χ . Actually, if one of the elements $f_\chi^{-1}(*)$ we will construct a set $A=\{\mathbf{e}_0,\mathbf{e}_1\oplus\mathbf{e}_2,...,\mathbf{e}_1\oplus\mathbf{e}_m\}$, then

$$K(f_\chi,A)=\{\mathbf{e}_1,\mathbf{e}_2,...,\mathbf{e}_t,\mathbf{e}_0,\mathbf{e}_1\oplus\mathbf{e}_2,...,\mathbf{e}_1\oplus\mathbf{e}_m\} \text{ and } K(f_\chi^*)=K(f_\chi,A).$$

Out of the elements of the consolidated kernel

$$K(f_\chi^*)_1=\mathbf{e}_1K(f_\chi^*)=\{\mathbf{e}_0,\mathbf{e}_1\oplus\mathbf{e}_2,...,\mathbf{e}_1\oplus\mathbf{e}_m,\mathbf{e}_1,\mathbf{e}_2,...,\mathbf{e}_m\}$$

The following matrix can be constructed

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$$K_{\xi}(f_{\chi}^{*})_1 = (L_3\underbrace{0...0}_{m-3})\nabla(L_3^{*}(2)\underbrace{0...0}_{m-3})\nabla(L_4^{*}(2)\underbrace{0...0}_{m-4})\nabla... \quad (7)$$
$$...\nabla(L_m^{*}(2))$$

in which the order of the elements of $K(f_{\chi}^{*})_1$ into the matrix lines is determined by the element $\xi \in S_{2m}$.

Then, by Theorem 5 and equation (7), the coordinates of the vector $\mathbf{w} = (\omega_1, \omega_2, ..., \omega_m)$ are determined as follows: $\omega_1 = -1, \omega_2 = -2, \omega_3 = \omega_4 = \omega_m = -4.$

Similar to the previous case we obtain

$$\omega_1^{(1)} = 1, \omega_2^{(1)} = -2, \omega_3^{(1)} = \omega_4^{(1)} = ... = \omega_m^{(1)} = -4 \quad \text{i} \quad \omega_0^{(1*)} = -\frac{17}{4}.$$

Hence GNE with the structure vector $\left(\mathbf{w}_1 = (1,-2,-4,...,-4); \omega_0^{(1*)} = -\frac{17}{4}\right)$ implements a function f_{χ} in $\{0,1\}$. We will calculate

$$\omega_0^{(1)} = \sum_{j=1}^{m} \omega_j^{(1)} - 2\omega_0^{(1*)} = 1 - 2 + (-4)(\mathrm{m}-2) - 2\left(-\frac{17}{4}\right) =$$
$$= -4m + 15{,}5.$$

To conclude, a generalized neural element with vector structure $(\mathbf{w}_1; \omega_0^{(1)})$ implement a function $f: G_n \to H_2$ concerning the system $\chi = \{\chi_{i_1}, \ldots, \chi_{i_m}\} \subset \chi(G_n)$.

IV. Conclusion

Concerning the research findings, the effective methods of synthesis of neural networks on the basis of the generalized neural elements for the processing, compression, classification and recognition of discrete signals are developed, as well as when the approximation and iteration methods of synthesis of neural elements can not practically be applied due to a large number of inputs of neural elements.

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