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Mathematical Model for the Probabilistic Minutia Distribution in Biometric Fingerprint Images

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***Abstract*—The research involves development of a mathematical model for the probabilistic minutia distribution in biometric fingerprint images. The suggested model is based on heuristic analysis of the fingerprint scanning results with account for the nature of the potential errors.**

Keywords—biometric authentication, fingerprint images, minutiaes

I. INTRODUCTION

Dactyloscopy occupies a special place among the known methods of biometric authentication [1-8]. Biometric fingerprint image minutiae processing which underlines the above method, enables robust and efficient identification of individuals.

At the same time, minutiae distribution of certain implementations may be described with rather complex dependencies. This is explained by the significant differences in the number of minutiae and their placement [9-12]. Error types and their distribution functions are also rather ambiguous due to the multiple nature of possible causes.

The choice between simplicity and adequacy of the models, describing minutiae placement and errors, is a compromise option. However, the closed nature of existing fingerprint recognition algorithms makes it impossible to collect an amount of statistics enough for solving the problem in a straightforward way. Therefore, the research and development of mathematical models for the probabilistic minutiae distribution in biometric fingerprint images is an important and relevant scientific problem. The models represented in this paper were obtained through heuristic analysis of the fingerprint scanning results with account for the nature of the potential errors.

II. THE ANALYSIS OF BIOMETRIC FINGERPRINT IMAGES

Let us use database DB1_1 [13] for the analysis of characteristic and error distributions which may occur during fingerprint image processing. This database contains 8 images (files 101_1.tif – 101_8.tif) of the same fingerprint. The goal of the analysis is to make a preliminary conclusion about the nature of the errors typical for minutia recognition.

The original fingerprint images are shown on figure 1. Figure 2 shows the processing results of the given samples using SourceAFIS.FingerprintAnalysis [14-15].

Fig. 1. Biometric images of a single finger

The results represent detected minutiae which correspond to the endings and bifurcations of the ridges. The arrows represent the angles of given minutiae.

Variation of fingerprint orientations and their displacement, as well as the changes in contrast and brightness, cause the significant differences of processing results which translates to the variation of the number of minutiae and their positioning. The following figure shows the circles that correspond to the same area of the fingerprint, but displaced and rotated during the scanning process.

Fig. 2. Fingerprint minutiae extraction results

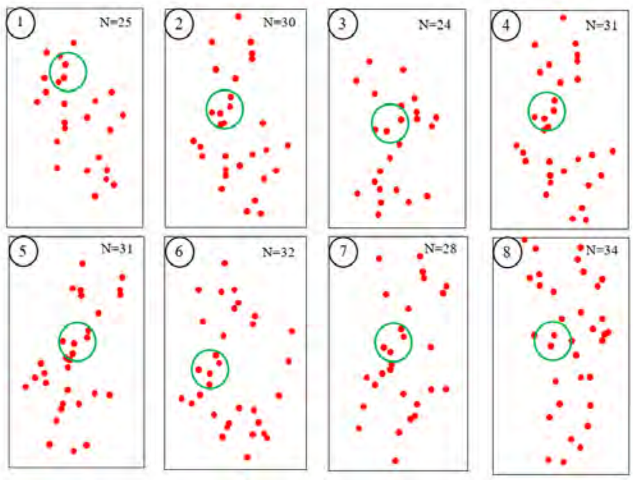

Fig. 3. Plain portraits of the extracted minutiae distribution

Figure 3 represents plane portraits of minutiae placement for the given fingerprint images. As we can see, the degree of similarity of the given portraits is rather low. Visual similarity takes place only in case of similar scanning conditions (on Fig. 3 it is the pairs 2 and 4 or 3 and 7). Apparently, the cause of the problem mentioned above is not only the complexity of the procedure itself, but also the imperfection of the used recognition algorithm implemented in SourceAFIS.FingerprintAnalysis [14-15].

III. Mathematical Model For The Probabilistic Minutiae Distribution

The analysis of the portraits mentioned above indicates the following *features* that we can base our empiric choice of the type of minutiae distribution on.

- the density of distribution of points along the horizontal (X) and vertical (Y) axes is roughly uniform in the central part of the frame and slightly decreases to its edges;
- linear displacements of the center of the fingerprint horizontally and vertically do not imply the appearance of zones free of minutiae at the edges of the frame (new points may enter the scanning area);
- The distribution of minutiae angles is approximately uniform in the range of $[0,\ 2\pi]$.

Let us use the following assumptions to construct a model of minutiae distribution according to the features mentioned above:

- the portrait coordinates of the fingerprint X, Y, as well as the minutiae angles values are normalized in the range $[-0.5;+0.5]$, while the geometric center of the image has zero coordinates on the plane $[0;0]$, and the portrait itself is placed in a unit square area covering all 4 quadrants of the image plane;
- for the primary generation of random numbers necessary to obtain the distribution of minutiae coordinates on the fingerprint image portraits, a uniformly distributed (continuous) random number generator in range $[0;1]$: $f(x_i, y_i):unif[0,1]$, $i \in 1 \ldots N$, where N - the number of minutiae in the portrait, a random value that does not go out of range $[15;60]$ with a mathematical expectation $m_N = 25 \div 35$ and unimodal distribution.

The analysis of features which were discussed before, as well as taking into account the assumptions made above, allow us to use the dependency shown on Figure 4 to describe the probability density function (PDF) $f(x)$ and $f(y)$ Cartesian coordinates of the minutiae on the plain portraits This type of PDF provides a uniform points distribution in the central part of the unit square and the decreasing probability of point appearance at the edges of the square area of the portrait. The area of non-zero PDF values $[-0.75;+0.75]$ is 0.25 in both directions beyond the unit square, which provides a non-zero probability of the point appearance in the border areas of the portrait. The errors appear in the form of possible geometric center drifting. The choice of this PDF is, of course, not the only one possible, however, in our opinion, is acceptable, considering the tradeoff between the simplicity and features mentioned above.

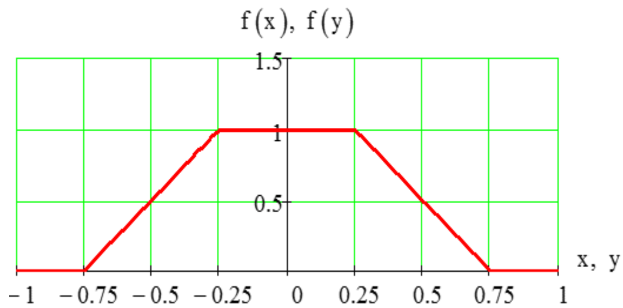

Fig. 4. Probability density function of the minutiae coordinates

To obtain test samples of minutia placement portraits the generator of random numbers, distributed according to PDF $f(x)$, $f(y)$ (Fig. 4), is required. Considering the identity of the distributions along the coordinates of the plane when using the normalized unit square of the portrait, in the following we shall consider only the function $f(x)$:

$$f(x) = \begin{cases} 2x+1.5 & if \ \ -0.75 \le x < -0.25; \\ 1 & if \ \ -0.25 \le x \le 0.25; \\ -2x+1.5 & if \ \ 0.25 < x \le 0.75; \\ 0 & if \ \ |x| > 0.75. \end{cases} \qquad (1)$$

To generate a random variable subject to distribution (1), one can use the functional result transformation of the standard for most programming systems of a random number generator located continuously uniformly in the range $[0,1]$. We use the inverse function method: if $z{:}unif[0,1]$ then the random variable x obtained by a functional transformation z in the form of

$$x = \begin{cases} \sqrt{z} - 0.75 & \text{if } 0 \le z < 0.25; \\ z - 0.5 & \text{if } 0.25 \le z \le 0.75; \\ -\sqrt{1-z} + 0.75 & \text{if } 0.75 < z \le 1; \end{cases} \quad (2)$$

will have a PDF (1).

Figure 5 shows the histogram of the statistical tests of the functional transformation (2) from $unif[0,1]$ the number of trials equal to 30,000 and dividing the interval $[-0.75, +0.75]$ into 100 equal subintervals. The dashed line in Fig. 5 shows the envelope (1).

The resulting algorithm for random number generation will be used later to obtain the coordinates of the characteristic points of the normalized square fingerprint portraits.

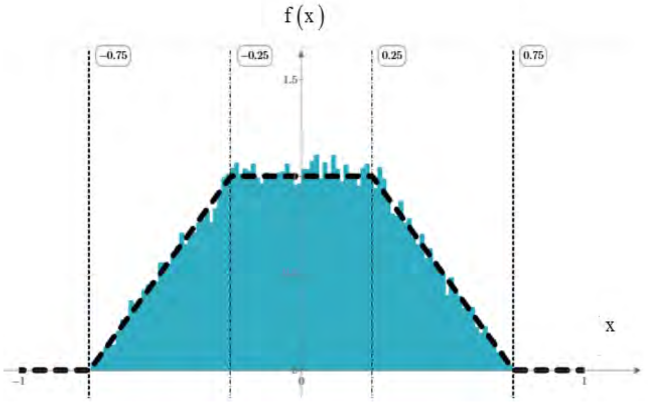

Fig. 5. Result of statistical tests of the random coordinate sensor

Let us choose the use of a discrete (integer) random value N from the range of integers $[15, 45]$ with a discrete normal truncated distribution and the following numerical characteristics to generate a random variable – number of minutiae in a fingerprint portrait sample:

- mathematical expectation $m_N \approx 30$;
- standard deviation $\sigma \approx 2 \div 5$.

Let us again use the functional transformation data of the generator $unif[0,1]$ to obtain a random number of minutiae on a fingerprint portrait sample . We simulate the samples of a random variable N based on the central limit theorem. We proceed to the discrete form of uniformly distributed numbers using the operation of integer rounding and centering:

$$z' = round(z) - 0.5 \text{ , where } z{:}unif[0,1]. \quad (3)$$

Then, limiting the number of terms to $m_N = 30$, the random number of minutiae in the portrait can be determined as the sum

$$N = \sum_{i=1}^{30} z' + 30. \quad (4)$$

A discrete random variable N can take integer values from a range $[15, 45]$. The truncated normal function of the PDF of this random variable is approximated by weighted binomial coefficients:

$$Q(N_i) = \binom{i}{30} \cdot \left(\frac{1}{2}\right)^{30}, i \in [0,30], N_i \in [15,45], \quad (5)$$

where $Q(N_i)$ is the probability that the number of minutiae on a portrait (taking into account points masked outside the unit square) will be a value N_i.

The form and numerical characteristics of the distribution (5) are shown in Fig. 6.

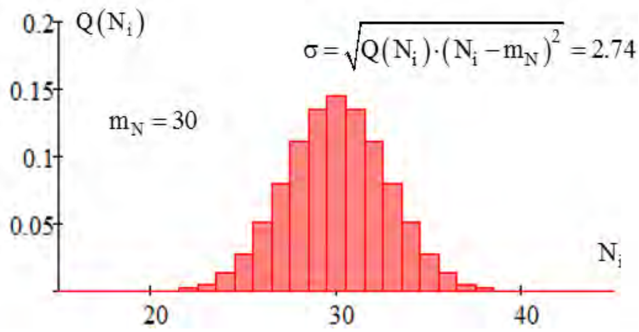

Fig. 6. PDF of the number of minutiae on a normalized fingerprint portrait

To simulate the random values of the minutiae angles normalized in the unit square, it is expedient to use a random variable $z{:}unif[0,1]$ uniformly distributed over a unit interval:

$$\varphi = z. \quad (6)$$

The true minutia angle is determined on the basis of the normalized value (6): $\Phi = 2\pi \cdot \varphi$.

Table I presents the results of modeling a normalized portrait based on the distributions (1), (4), and (6).

The highlighted rows in Table I correspond to points that did not fall into a unit square. Therefore, in spite of the fact that during the experiment we obtained $N = 28$, only 21 points were found in the unit square (Fig. 7). "Masked" points can appear in case the shifts along the axes X and Y or the image rotation occur.

In case of necessity, it is possible to consider a three-dimensional space for point placing by adding a third coordinate for the normalized angle φ in the corresponding processing algorithm.

IV. CONCLUSION

The analysis of the fingerprint scanning results shows an extremely small degree of similarity among the obtained images. Considering the nature of the possible errors, visual similarity takes place only in case of similar scanning conditions. The cause of the problem mentioned above is not only the complexity of the procedure itself, but also the imperfection of the used recognition algorithms. The research involves development of a mathematical model for the probabilistic minutiae distribution in biometric fingerprint images. The suggested model is based on heuristic analysis of the fingerprint scanning results with account for the nature of the potential errors. As the result, we were able to model a typical minutiae behavior in the biometric fingerprint images. This research might be useful for the improvement of various biometric methods of information security, as well as other practical use [16-54].

TABLE I. PORTRAIT MATRIX

№	X	Y	φ
1	0.21	-0.07	0.6
2	0.28	0.18	0.58
3	0.12	-0	0.49
4	0.19	0.31	0.74
5	0.07	-0.68	0.62
6	0.05	-0.12	0.8
7	-0.25	0.33	0.58
8	-0.01	0.42	0.91
9	0.17	0.15	0.73
10	0.12	0.23	0.67
11	0.3	0.54	0.32
12	0.64	-0.14	0.31
13	0.13	0.44	0.11
14	0.09	-0.05	0.85
15	0.01	0.39	0.15
16	-0.05	-0.55	0.08
17	0.51	0.5	0.64
18	-0.25	-0.34	0.55
19	0.23	-0.41	0.41
20	0.13	-0.41	0.47
21	0.14	0.08	0.15
22	0.06	0.23	0.74
23	-0.05	0.17	0.83
24	0.69	0.31	0.87
25	0.1	0.7	0.3
26	-0.33	-0.3	0.13
27	-0.17	-0.11	0.78
28	0.15	0.08	0.61

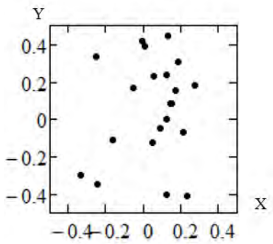

Fig. 7. Random points distribution sample

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