BEARING ESTIMATION OF A SOURCE WITH COMPLICATED SPECTRUM USING A MOVING SPARSE ANTENNA ARRAY

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Abstract

Estimation of bearing of a narrowband source with the use of a sparse equidistant array usually turns out to be ambiguous. For a wideband source with smooth spectrum, this ambiguity can be overcome at the expense of decreasing the spatial resolution. At the same time, the ambiguity of bearing estimate of a narrowband source can be eliminated if the array is moving in respect to the source, and the well-known technique used in radars and sonars with synthetic aperture is used (in this case, a single receiver is enough). In the present paper, a scenario is analyzed when the source spectrum has a smooth wideband component and several narrowband components; such a spectrum is typical, for example, for underwater noise of ships. It is shown that a procedure of aperture synthesis applied to such a signal gives, under some conditions, the same result as conventional beamforming using a motionless array. This fact allows for effective separation of discrete components of the spectrum from its continuous part. Estimates of aperture synthesis duration allowing one to perform such a separation, as well as output signal-to-noise ratios, are presented.

Keywords: Acoustic array systems and processing, aperture synthesis and source localization, sparse antenna array.

1. INTRODUCTION

One of the major problems of passive location is direction finding for a narrowband source [1]. This problem arises in radars and, especially, in sonars when the spectrum of noise (of a ship) has narrowband components [2]. Because of technical restrictions, sparse antenna arrays are often used at high frequencies; it is known that, in this case, bearing estimation becomes ambiguous. This ambiguousness can be overcome by means of aperture synthesis which can be performed if the array is moving. Under real conditions, along with narrowband components of the received signal spectrum, there is also a continuous component. The purpose of the present work is to analyze the ability of aperture synthesis to estimate the bearing of a source possessing such a combined spectrum. The relationship between time required for the aperture synthesis and bandwidth of the narrowband component, as well as the achievable signal-to-noise ratio, are investigated in the paper.

2. PROBLEM FORMULATION AND THE METHOD

Consider the case when a linear equidistant antenna array is uniformly moving along its axis at a great distance (in the far zone of the synthetic aperture) from a motionless source with the spectrum $Q(\omega)$ where ω is the cyclic frequency. Fig. 1 illustrates the scheme of the experiment.

We will suppose that the output at the n-th element at the time t can be expressed as

$$y_n(t) = u_n(t) + \xi_n(t),$$

$$u_n(t) = \int_{\omega_1}^{\omega_2} q(\omega) \exp\left\{-i\frac{\omega}{c}(x_n - Vt)s_0 - i\omega t\right\} d\omega, \quad (1)$$

$$t \in [-T/2, T/2],$$

where T is the time of synthesis, c is the sound speed in water, $x_n = (n-(N+1)/2)d$ is the coordinate of the n-th array element along the array axis, N is the number

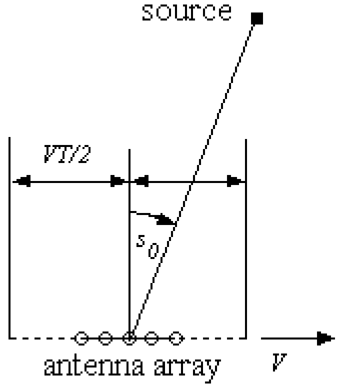


Fig. 1. Scheme of the experiment (top view).

of array elements, d is the array spacing, $u_n(t)$ is the signal from the source, $q(\omega)$ is the instant spectrum of the source ($Q(\omega)=\left\langle |q(\omega)|^2 \right\rangle$), $\xi_n(t)$ is the white isotropic zero-mean Gaussian noise with the variance $\left\langle |\xi_n(t)|^2 \right\rangle = \sigma^2$, s_0 is the sine of bearing counted from the center of the synthetic aperture. In this case, the aperture synthesis represents a construction of frequency-angular distribution of the correlation function of the received signal and the model of the signal from a source:

$$\Phi(\omega,s)=\sum_{n=1}^{N}\int_{-T/2}^{T/2} y_n(t)\exp\left\{i\frac{\omega}{c}(x_n-Vt)s+i\omega t\right\}dt, \quad (2)$$

where ω, s is the frequency and the sine of bearing, respectively. For estimation of bearing of the sources of narrowband components, the maximum of the function $|\Phi(\omega,s)|^2 \to \max\limits_{\omega,s}$ must be determined. The purpose of the present work is in investigation of properties of the presented technique of aperture synthesis. For this purpose, in the following section an analysis of average theoretical correlation function (the ambiguity function) in the absence of noise

$$F(\omega,s)=\left\langle |\Phi(\omega,s)|^2 \right\rangle \quad (3)$$

is performed.

3. Analysis of the Method

After some transformations, the function $F(\omega,s)$ can be rewritten as

$$F(\omega,s)= T^2\int_{\omega_1}^{\omega_2} Q(\omega')\, f_N^2(\omega,\omega',s,s_0)\, f_T^2(\omega,\omega',s,s_0)d\omega', \quad (4)$$

$$f_N\left(\omega,\omega',s,s_0\right)=\frac{\sin\left(\frac{D}{2}\Delta(\omega,\omega',s,s_0)\right)}{\sin\left(\frac{D}{2N}\Delta(\omega,\omega',s,s_0)\right)},$$

$$f_T\left(\omega,\omega',s,s_0\right)=\operatorname{sinc}\left(\frac{T}{2}\left[\omega-\omega'-V\Delta(\omega,\omega',s,s_0)\right]\right),$$

$$\Delta(\omega,\omega',s,s_0)=\frac{1}{c}(\omega s-\omega' s_0),\ \operatorname{sinc}(z)=\frac{\sin(z)}{z},$$

where $D = Nd$ is the array length, f_N^2 is the array beam pattern with the conventional width $\Delta s_N = c/(f_0 D)$, f_T^2 is the synthetic antenna beam pattern for one element ($N = 1$); its width is $\Delta s_T = c/(f_0 VT)$. Consider a situation when the source spectrum contains the only component with the finite bandwidth at the frequency ω_0. In this case, when the synthesis time unlimitedly increases ($T\to\infty$), the function $\pi^{-1}Tf_T^2$ tends, as it is known, to the Dirac delta function $\delta\left(\omega'-\omega\frac{1-\beta s}{1-\beta s_0}\right)$, where $\beta = V/c$, and, accordingly, Eq. (4) turns into

$$F_0(\omega,s)=2\pi TQ\left(\omega\frac{1-\beta s}{1-\beta s_0}\right) f_N^2\left(D\frac{\omega}{c}\frac{s-s_0}{1-\beta s_0}\right). \quad (5)$$

The width of the first product in (5), when β is small, is about $\Delta s_Q = \beta^{-1}\Delta f_0/f_0$, where $\Delta f_0 = \Delta\omega_0/(2\pi)$ is the source bandwidth at the frequency $f_0 = \omega_0/(2\pi)$. The parameter Δs_Q represents an estimate of the achievable width of the synthetic array beam pattern. It is clear that in the case when the spectrum width is great ($\Delta f_0/f_0 > 2V/c$) the synthesis will be ineffective for any time T, and the width of the resultant beam pattern depends on the factor f_N related to the array. In the case when the source spectrum is narrow and the synthesis time is limited, the situation is different. The case when the source bandwidth on frequency ω_0 is close to zero: $Q(\omega)\approx Q(\omega_0)\delta(\omega-\omega_0)$ is illustrative here: in this case, the resultant beam pattern will simply represent a product of the array beam pattern and the synthetic aperture beam pattern. The width of the latter is approximately equal to $c/(f_0 VT)$. In the intermediate case, the increase of the synthesis time will lead, at first, to decreasing of the width of the resultant beam pattern until the values $T > (\Delta f_0)^{-1}$ when $\Delta s_T > \Delta s_Q$; for these values, the decreasing will stop. The signal-to-noise ratio when increasing the synthesis time will increase as $T\,Q(\omega)/\sigma^2$ up to the value of $(\Delta f_0)^{-1}Q(\omega)/\sigma^2$. In Fig. 2, time dependences of width of the synthetic aperture beam pattern for the case of the only array element ($N = 1$) are shown. Each curve corresponds to a certain bandwidth of the spectrum of the source in the range from 0.06 to 0.3 Hz. The frequencies of narrowband components were in the range 1450–1550 Hz. The values $1.2\Delta s_Q$ (coefficient 1.2 was determined empirically) are shown for each value of

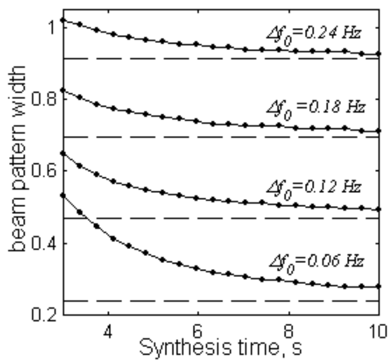


Fig. 2. Synthetic aperture beam pattern width versus synthesis time.

Δf_0 by dashed lines. It follows from Fig. 2 that for each value of the source spectrum bandwidth, the time of effective synthesis is limited by a value which is approximately in inverse proportion to this width.

Now consider the case when a sparse antenna array is used and the signal spectrum consists of a continuous part and narrowband components. Such situation is typical for a spectrum of acoustic radiation of a moving ship. The example of part of such a spectrum is shown in Fig. 3.

As was noted before, the resultant beam pattern at the frequencies corresponding to the continuous part of the spectrum will be ambiguous. This ambiguousness can be eliminated by means of averaging of the beam pattern over the frequency; then the grating lobes will be summed with a shift that will lead to their weakening. For the narrowband components, elimination of this ambiguity can be reached by means of aperture synthesis. To illustrate the described effects, a synthesis of the aperture for a signal whose spectrum consisted of several narrowband components of different width and a continuous component with the constant level (supposed to be equal to 0 dB) was considered (see Fig. 4a). The bandwidth of the narrowband component increased from 0.08 Hz to 0.25 Hz when increasing the frequency. The number of elements in the array was $N = 10$; the spacing was $d = c/f_0$ ($f_0 = 1500$ Hz); the time of synthesis was $T = 64$ s; the sine of bearing of the source was $s_0 = 0.3$. In Fig. 4b, the ambiguity function $F(\omega, s)$ corresponding to the case in point is shown. It follows from Fig. 4b that there is no ambiguousness in bearing for narrowband components, whereas the height of the grating and main lobes of the beam pattern are equal for the continuous part of the spectrum. However, the inclination of the grating lobe is clearly visible in Fig. 4b; this confirms a possibility to eliminate the ambiguousness by means of averaging over frequency [3].

4. Conclusion

A method of aperture synthesis by means of a moving sparse array applied to bearing estimation of a source with a complicated spectral structure is considered. The form of the resultant beam pattern in dependence on the time of synthesis and the signal bandwidth is analyzed. It is shown that the effective time of synthesis, as well as the signal-to-noise ratio, depends solely on the source bandwidth. It is shown that the aperture synthesis technique for a narrowband component allows for elimination the ambiguousness of source bearing estimation. At the same time, for a continuous part of the spectrum, the aperture synthesis technique does not change the form of the beam pattern.

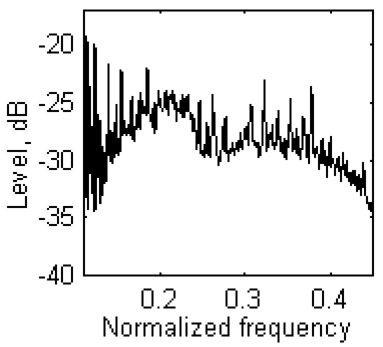
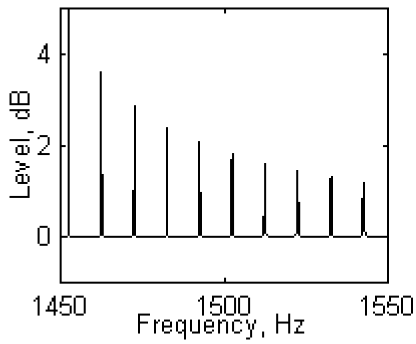
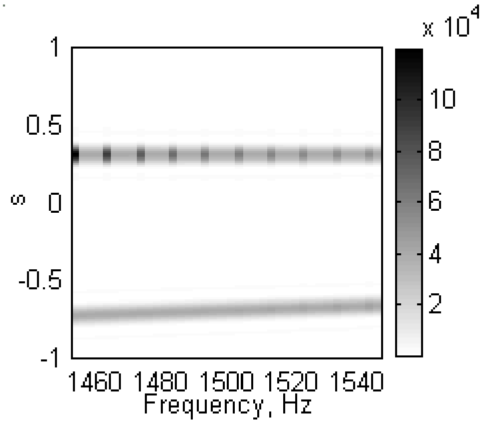


Fig. 3. An example of acoustical spectrum of a moving ship.



(a)



(b)

Fig. 4. (a): spectrum of the model signal, (b): ambiguity function $F(\omega, s)$ for the model signal.

Acknowledgment

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