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NONLINEAR SYNTHESIS PROBLEM OF FLAT EQUIDISTANT ANTENNA ARRAYS ACCORDING TO THE PRESCRIBED AMPLITUDE DIRECTIVITY PATTERN

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Abstract

Within the context of elementary mathematical model a nonlinear synthesis problem of flat equidistant antenna array at the given requirements to the amplitude directivity pattern is investigated. It is shown, that for the problems of this class the non-uniqueness and branching of solutions, dependent on physical parameters of the array is characteristic. The justified algorithms of finding the branching lines of solutions and optimum synthesized amplitude directivity pattern are stated. A numerical example of synthesis of two-beam amplitude directivity pattern is given.

Keywords: Flat antenna arrays, synthesis, amplitude directivity pattern, non-uniqueness and branching of solutions, lines of branching.

1. INTRODUCTION

In the work a variational synthesis problem of flat rectangular antenna array according to the prescribed directivity pattern (DP) is considered. The non-uniqueness and branching of solutions is characteristic for these class of problems. The method of finding the branching lines of solutions is stated. The justified algorithms to find the optimum solutions of synthesis problem are presented. It is shown that finding the optimal phase DP considerably improves the quality of approximation the synthesized DP to the given one in comparison with synthesis in a class of synphase DP. In particular, it allows also to miniaturize array nearly by 30 % at identical efficiency of synthesis in comparison with synthesis in the class of synphase DP. Besides from the practical point of view, presence of several types of various solutions creating the same or similar amplitude DP, enables to choose that solution, which has more simple realization.

2. FORMULATION OF PROBLEM, BASIC EQUATIONS OF SYNTHESIS

It is supposed, that the plane of a rectangular antenna array consisting of $N_2 \times M_2 = = (2N+1)\times(2M+1)$ elements coincides with a plane XOY of the Cartesian system of coordinates, and its multiplier is determined by the formula [1]

$$f(s_1,s_2) = A\boldsymbol{I} \equiv \sum_{n=-N}^{N}\sum_{m=-M}^{M} I_{nm} e^{i(c_1 n s_1 + c_2 m s_2)}, \qquad (1)$$

where $s_1 = \sin\vartheta\cos\varphi/\sin\alpha_1$, $s_2 = \sin\vartheta\sin\varphi/\sin\alpha_2$ are the generalized angular coordinates; $c_1 = kd_1\sin\alpha_1$, $c_2 = kd_2\sin\alpha_2$ are the parameters of an array belonging to the area

$$\Lambda_c = \{(c_1,c_2): 0 < c_1 \le a,\ 0 < c_2 \le b\};$$

$k = 2\pi/\lambda$ is a wave number; d_1, d_2 are distances between the adjacent elements on the axes OX, OY, respectively; α_1, α_2 are the corners describing the area in which the required amplitude DP $F(s_1,s_2)$ is given. The domain of variables s_1, s_2, belonging to one period of an array, we shall designate as

$$\Omega = \{(s_1,s_2): |s_1| \le \pi/c_1,\ |s_2| \le \pi/c_2\}.$$

Suppose, that distinct from identical zero required amplitude DP $F(s_1,s_2)$, is given in the area $\bar{G} \subseteq \Omega$, and on the set $\Omega \setminus \bar{G}$ it is identically equal to zero. We shall formulate the synthesis problem as a minimization problem of the functional [2]

$$\min_{\boldsymbol{I}\in H_I} \sigma(\boldsymbol{I}) = \left\| F - |A\boldsymbol{I}| \right\|^2_{C^{(2)}_{(\Omega)}} \equiv \left\| F - |f| \right\|^2_{C^{(2)}_{(\Omega)}}. \qquad (2)$$

On the basis of necessary condition of minimum functional, we obtain nonlinear system of equations with respect to the components of vector $\boldsymbol{I}$

$$I_{nm} = \frac{c_1 c_2}{4\pi^2}\iint_{\Omega} F_1(s_1,s_2)\exp\left(i\arg\left(\sum_{n=-N}^{N}\sum_{m=-M}^{M} I_{nm} e^{i(c_1 n s_1 + c_2 m s_2)}\right)\right)\times$$
$$\times\exp\left(-i(c_1 n s_1 + c_2 m s_2)\right) ds_1 ds_2,$$

$$(n=-N \div N,\ m=-M \div M) \quad (3)$$

describing the stationary points $\sigma(\boldsymbol{I})$.

Acting on both parts of equation (3) by operator A, we obtain equivalent to (3) nonlinear integral equations of Hammerstein type concerning function f

$$f(Q)=\boldsymbol{B}f \equiv \iint_{\Omega} K(Q,Q',\boldsymbol{c})\,F(Q')e^{i\arg f(Q')}dQ', \quad (4)$$

where $Q=(s_1,s_2)$, $dQ=ds_1ds_2$; $\boldsymbol{c}=(c_1,c_2)$. The kernel of equations (4) is degenerate and real.

Biunique correspondence is between the solutions of equations (3) and (4). Owing to equivalence of the equations (3) and (4) more simple of them, equation (4) is investigated. The existence theorem of solutions for a equation (4) is proved.

We shall formulate important properties of the equation (4), which are validated directly.

1°. If function $f(Q)$ is the solution of equation (4), then complex conjugate function $\overline{f(Q)}$ is also solution of equation (4).

2°. If function $f(Q)$ is the solution of equation (4), $e^{i\beta}f(Q)$ is also the solution of (4), where β is an arbitrary real constant.

3°. For even functions $F(s_1,s_2)$ with respect to two arguments (or with respect to one argument) the nonlinear operator $\boldsymbol{B}$, taking place in the right part of equation (4), is invariant concerning the type of evenness of function $\arg f(s_1,s_2)$ by two arguments (or by that argument on which $F(s_1,s_2)$ is even function).

One of the solutions of equation (4) in the class of real functions is

$$f_0(Q,\boldsymbol{c})=\iint_{\bar{G}} F(Q')K(Q,Q',\boldsymbol{c})\,dQ', \quad (5)$$

what later on we shall name as initial.

The problem of finding the branching lines [3] and complex solutions of equation (4), that branch off from the initial solution $f_0(Q,\boldsymbol{c})$ is investigated. The problem is reduced to finding such parameters set $\boldsymbol{c}^{(0)}=\left(c_1^{(0)},c_2^{(0)}\right)$ on which a homogeneous integral equation

$$\varphi(Q)=\iint_{\bar{G}} \frac{F(Q')}{f_0(Q',c_1,c_2)} K(Q,Q',c_1,c_2)\,\varphi(Q')dQ' \quad (6)$$

has different from zero solutions.

Using the property of degeneration of kernel, the equation (6) is reduced to the corresponding system of linear algebraic equations

$$x_{kl}=\sum_{n=-N}^{N}\sum_{m=-M}^{M} a_{nm}^{(kl)}(c_1,c_2)\,x_{nm} \quad \begin{pmatrix} k=-N\div N, \\ l=-M\div M \end{pmatrix}, \quad (7)$$

which coefficients analytically depend on parameters c_1, c_2. In the operator form the problem (7) reads

$$\mathsf{A}_M(c_1,c_2)\boldsymbol{x} \equiv \left(\boldsymbol{E}_M-\boldsymbol{A}_M(c_1,c_2)\right)\boldsymbol{x}=0.$$

In order that the system (7) should have different from zero solutions, it is necessary

$$\Psi(c_1,c_2)=\det\left(\boldsymbol{E}_M-\boldsymbol{A}_M(c_1,c_2)\right)=0.$$

Considering the equation $\Psi(c_1,c_2)=0$, as a problem on finding the implicitly given function $c_2=c_2(c_1)$ (or $c_1=c_1(c_2)$) in the vicinity of the appropriate point $c_1^{(i)}$, we come to the Caushy problem

$$\frac{dc_2}{dc_1}=-\frac{\Psi'_{c_1}(c_1,c_2)}{\Psi'_{c_2}(c_1,c_2)}, \quad c_2\left(c_1^{(i)}\right)=c_2^{(i)}$$

on finding the i coherent component of spectrum, i.e. a line, in which from the initial solution $f_0(s_1,s_2)$ the complex solutions branch off.

For numerical finding the initial and branching off solutions of the system (4), we use the iterative process, the base of which is the method of successive approximations [2]:

$$f_{n+1}(Q)=\boldsymbol{B}f \equiv \iint_{\Omega} K(Q,Q',\boldsymbol{c})\,F(Q')e^{i\arg f_n(Q')}dQ'$$

$$(n=0,1,2,...).$$

3. Numerical Example of Solution of Synthesis Problem

An example of synthesis of flat antenna array from 11×11 radiators for the values of parameters of the array $c_1=1{,}6$, $c_2=1{,}2$, being on the beam $c_2=0{,}75c_1$, belonging to the area Λ_c is considered. The required two-beam amplitude DP is given as even function $F(s_1,s_2)=\cos(\pi s_1/2)\left|\sin(\pi s_2)\right|$ with respect to both arguments in the area $\bar{G}=\{(s_1,s_2):|s_1|\le 1,\ |s_2|\le 1\}\subset\Omega$. The first branching lines of solutions are found.

The values of functional σ which it takes on the beam $c_2=0{,}75c_1$ on solutions of two types: the curve 1 corresponds to initial (real) solution $f_0(Q)$; the curve 2 - to solution with odd with respect to s_2 phase diagram $\arg f(s_1,s_2)$, are shown in Fig. 1. As it is seen from the figure, at point $c_1\approx 0{,}77$ from real solution branch off more effective complex solutions, on which functional takes smaller value, than on the real solution.

The synthesized amplitude DP for $c_1=1{,}6$, $c_2=1{,}2$ is presented in Fig. 2.

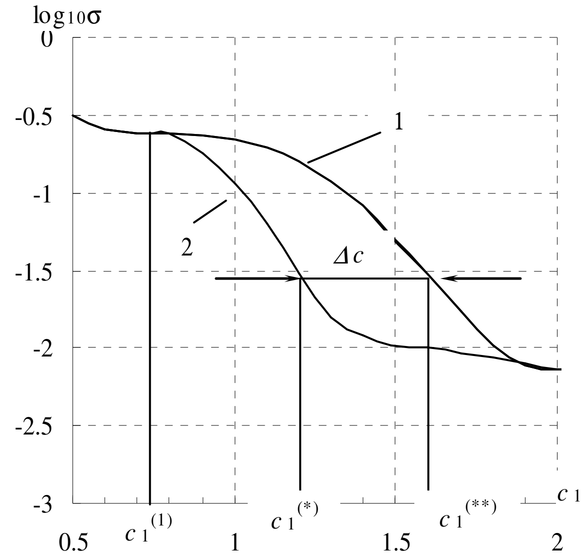

Fig. 1. The values of functional σ .

The optimum distribution of currents in the array corresponding to this DP is given in Fig. 3. As we see, the distribution of stimulating currents is asymmetrical concerning the plane YOZ , though created by it amplitude DP (Fig. 2) is symmetrical to YOZ .

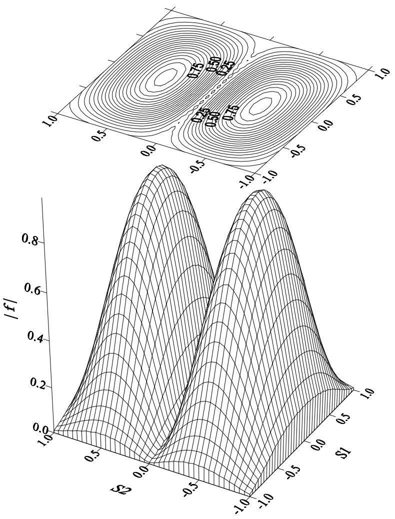

Fig. 2. The synthesized amplitude DP for $c_1 = 1,6$, $c_2 = 1,2$.

4. Conclusions

Let's note the basic features and problems, which arise at investigations of considered in the work class of problems:

- It follows from analysis of numerical results that the synthesized amplitude DP, corresponding to branching off solutions, allow one increase efficiency of synthesis in comparison with synthesis in a class of synphase DP. In particular, the identical efficiency of synthesis (equal values of functional σ on the initial and branching off solutions) can be obtained on branching off solution at decreased sizes of the array.

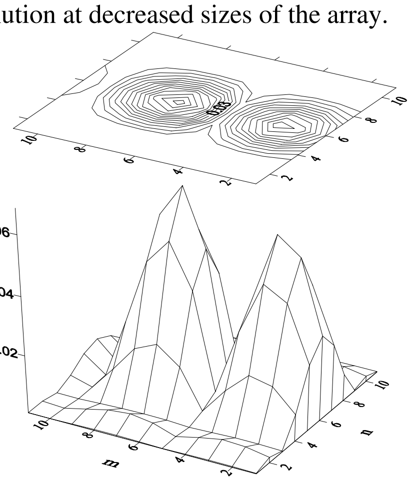

Fig. 3. The optimum distribution of currents in the array.

- The basic difficulty is study of non-uniqueness and branching of solutions dependent on physical parameters c_1 , c_2 . As follows from investigations for a special case, when $F(s_1, s_2) = F_1(s_1) \cdot F_2(s_2)$ [2, 4, 5], the quantity of existing solutions with increase of parameters c_1, c_2 considerably increase. However, in the synthesis problems of antenna arrays, it is important to find to the best approximation to the given amplitude DP $F(s_1, s_2)$ at rather small values of parameters c_1, c_2 that allows one to limit by researches of several first points (lines) of branching.
- Getting of irrefragable answer concerning the exact quantity of existing solutions of equation (4) at certain values of parameters c_1, c_2 is the subject of separate investigations.

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