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MATHEMATICAL MODEL OF A LOADED CONICAL ANTENNA EXCITATION

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Abstract

Electromagnetic wave scattering problems for the imperfectly conducting cones are considered. Boundary impedance conditions including the cone curvature parameters are used. Analytical solutions for some partial cases are obtained. The surface curvature parameters effects on the problems solutions are studied.

Keywords: semi-transparent cone, impedance surface, initial boundary problem, rigorous methods, boundary conditions

1. . INTRODUCTION

Model problems of the electromagnetic waves scattering on imperfectly conducting surfaces investigation causes significant interest for the practical applications (e.g. microwave technique). One of the approaches allowing the surface features considering is the Approximate Boundary Conditions (ABC) usage [1]. The most widespread are the Leontovich-Shchukin boundary conditions. The ABC usage for the model electromagnetic problems caused the investigation of the wave diffraction problems on the wire gratings, semitransparent and impedance surfaces, superconducting structures etc.

The main task of this work is to investigate a model problem of a loaded conical antenna excitation. One-sided or two-sided impedance boundary conditions are used and the cone curvature with its surface feat is taken into consideration.

2. SEMITRANSPARENT CONE. THE PROBLEM STATEMENT AND ANALYTICAL SOLUTION

A semitransparent circular cone (partial case of the thick conical metal gratings) with the aperture angle of 2γ placed into the $\vec{E}_0$, $\vec{H}_0$ field of an electrical radial dipole with the moment p_r is considered (Fig.1).

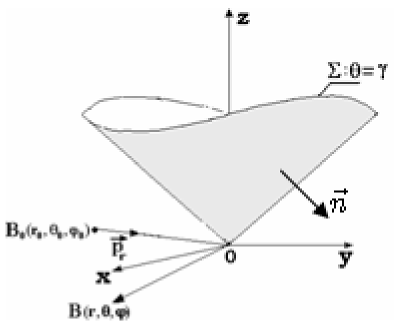

Fig. 1. The conical structure

The cone and the dipole are placed into a homogeneous and isotropic medium. The medium permittivity and permeability are ε , μ respectively.

The source is placed into the $B(\vec{r}_0)$ point. A time dependence is assumed to be $e^{ia\omega t}$, $a = \pm 1$. The conical surface Σ is defined by the $\theta = \gamma$ equation in the given spherical coordinate system r , θ , φ with the origin at the cone tip. The $\vec{E}$, $\vec{H}$ fields are found in the presence of the cone and the source . The fields satisfy the Maxwell equations, the finite stored energy condition and the condition on the infinite distance. On the cone the field satisfies the impedance boundary conditions containing the cone curvature factor and the high order derivatives.

$$\begin{cases} \vec{n}\times\left\{\vec{n}\times\left[\vec{E}^{+}+\vec{E}^{-}\right]\right\}=2\tilde{P}^{(1)}\vec{n}\times L\left[\vec{H}^{+}-\vec{H}^{-}\right], \\ \vec{n}\times\vec{E}^{+}=\vec{n}\times\vec{E}^{-}; \end{cases}$$

$$\vec{F}^{\pm} = \vec{F}|_{\Sigma^{\pm}} , \Sigma^{\pm} : \theta = \gamma \pm 0 ,$$

L is a linear differential operator, $\tilde{P}^{(1)} = \dfrac{w}{q} W_1 \sin\gamma$, w is the medium impedance, W_1 is the cone transparency parameter, ($W_1 \geq 0$), $q = iak$, k is a wave number. The boundary conditions in the coordinate form look as follows:

$$E_r|_{\Sigma} = -\tilde{P}^{(1)}\left(\frac{\partial^2}{\partial r^2} - q^2\right)(r\tilde{H}_{\varphi}) ,$$

$$\tilde{H}_{\varphi} = H_{\varphi}^{+} - H_{\varphi}^{-} , \; E_r^{+} = E_r^{-} .$$

In order to solve the boundary problem the Debye potentials υ allowing to derive the electromagnetic field components are involved. The Kontorovich-Lebedev integral transforms are applied to the Debye potentials [2]. The Debye potential corresponding to the scattered field has the following form:

$$\upsilon_{smtr.}(\vec{r}) = \upsilon_{istr.\Sigma}(\vec{r}) + f^{*}_{smtr.\Sigma}(\vec{r}) ,$$

$$\gamma < \theta_0 < \pi \,, \; \gamma < \theta < \pi \,, \qquad (1)$$

$\upsilon_{istr.\Sigma}(\vec{r})$ is the potential corresponding to the isotropic cone excitation and $f^{*}_{smtr.\Sigma}(\vec{r})$ causes anisotropic features of the semitransparent cone Σ where

$$f^{*(\chi)}_{n.np.\Sigma_2}(\vec{r}) = \frac{2}{\pi^2}\int_0^{+\infty} \tau sh\pi\tau \hat{f}^{*(\chi)}_{n.np.\Sigma_2} \frac{K_{i\tau}(qr)}{\sqrt{r}} d\tau \,,$$

$$f^{*}_{smtr.\Sigma}(\vec{r}) = \sum_{m=-\infty}^{+\infty} a_{m\tau} f^{*}_{m\tau,smtr.\Sigma} \frac{P^{m}_{-1/2+i\tau}(\cos\gamma_2)}{P^{m}_{-1/2+i\tau}(-\cos\gamma_2)} \times$$
$$\times P^{m}_{-1/2+i\tau}(-\cos\theta_0) P^{m}_{-1/2+i\tau}(-\cos\theta) e^{im\varphi},$$

$$f^{*}_{m\tau,smtr.\Sigma} = \frac{2W_1}{A^{m}_{i\tau} + 2W_1},$$

$$A^{m}_{i\tau} = (-1)^m \frac{\pi}{ch\pi\tau} \frac{\Gamma(1/2+i\tau-m)}{\Gamma(1/2+i\tau+m)} \times$$
$$\times P^{m}_{-1/2+i\tau}(\cos\gamma) P^{m}_{-1/2+i\tau}(-\cos\gamma).$$

$K_{i\tau}(qr)$ is the Macdonald function, $P^{m}_{-1/2+i\tau}(\cos\gamma_2)$ is the Legendre function, $\Gamma(z)$ is the gamma function, $a_{m\tau}$ are known coefficients. The spectrum of the boundary problem for the semi-transparent cone is defined by the roots of the following equation:

$$A^{m}_{\xi} + 2W_1 = 0 \,, \; \xi \in \Re \,.$$

3. PERFECTLY CONDUCTING CONE WITH A THIN COVER

The problem statement for the perfectly conducting cone with the thin cover is the same as for the semi-transparent one. The field satisfies the following boundary conditions on the conical surface:

$$\vec{n} \times \left\{ \vec{n} \times \left[\vec{E}^{+} + \vec{E}^{-} \right] \right\} = -w\tilde{R}_0 \vec{n} \times \vec{H}^{+}, \quad (2)$$

$\tilde{R}_0 = \zeta_0 \cdot (qr)^{-1}$, ζ_0 is a constant complex value. In case of axial symmetric excitation of cone with the thin cover excitation ($\theta_0 = \pi$, $m = 0$) the potential corresponding to the scattered field is written according to the formula

$$\upsilon_1^{(1)} = -\frac{2}{\pi^2}\int_0^{+\infty} \tau sh\pi\tau a^{*}_{0\tau} b^{*}_{0\tau} \breve{y}_0^{(1)} \times$$
$$\times \frac{P_{-1/2+i\tau}(\pm\cos\theta)}{P_{-1/2+i\tau}(\pm\cos\gamma)} \frac{K_{i\tau}(qr)}{\sqrt{r}} d\tau, \qquad (3)$$

$$\tilde{g}^{(1)}_{i\tau} = \frac{2}{\pi \sin\gamma P_{-1/2+i\tau}(\cos\gamma)} \times$$
$$\times \frac{ch\pi\tau}{\left(\tau^2 + \frac{1}{4}\right) P_{-1/2+i\tau}(-\cos\gamma) - \zeta_0 \frac{d}{d\gamma} P_{-1/2+i\tau}(-\cos\gamma)},$$

$$\breve{y}_0 = 1 - \zeta_0 \breve{g}_{i\tau}(\zeta_0, \gamma).$$

The top signs of the formula (3) correspond to the $0 < \theta < \gamma$ domain and the bottom ones to the $\gamma < \theta < \pi$ domain. The formula (3) can be rewritten as follows:

$$\upsilon_1^{(1)} = \upsilon^{(1)}_{1,per.c} + \upsilon^{(1)}_{1,\mathrm{cov.}}$$

$\upsilon^{(1)}_{1,per.c}$ is the Debye potential for the perfectly conducting cone excitation by an electric radial dipole, $\upsilon^{(1)}_{1,\mathrm{cov.}}$ is the correcting summand involving the thin cover effects.

$$\upsilon^{(1)}_{1,\mathrm{cov.}} = Z_0 \frac{4}{\pi^3}\int_0^{+\infty} \frac{\tau sh2\pi\tau a^{*(1)}_{0\tau}}{\Delta_{i\tau}} \times$$
$$\times \frac{P_{-1/2+i\tau}(-\cos\theta)}{P_{-1/2+i\tau}(-\cos\gamma)} \frac{K_{i\tau}(qr)}{\sqrt{r}} d\tau,$$
$$\gamma < \theta < \pi \,,$$

$$\Delta_{i\tau} = \left(\tau^2 + 1/4\right) P_{-1/2+i\tau}(-\cos\gamma) -$$
$$-\zeta_0 \sin\gamma \frac{d}{d\gamma} P_{-1/2+i\tau}(-\cos\gamma).$$

Taking into account the cone curvature in the boundary condition (2), we obtain

$$\vec{n} \times \left\{ \vec{n} \times \left[\vec{E}^{+} + \vec{E}^{-} \right] \right\} = -wR_0 \sin\gamma \vec{n} \times \vec{H}^{+},$$

$$y_0 = 1 - \zeta_0 g_{i\tau}(\zeta_0, \gamma),$$

$$g_{i\tau} = \frac{2}{\pi P_{-1/2+i\tau}(\cos\gamma)} \times$$
$$\times \frac{ch\pi\tau}{\left(\tau^2 + 1/4\right) P_{-1/2+i\tau}(-\cos\gamma) - \zeta_0 \sin\gamma \frac{d}{d\gamma} P_{-1/2+i\tau}(-\cos\gamma)}.$$

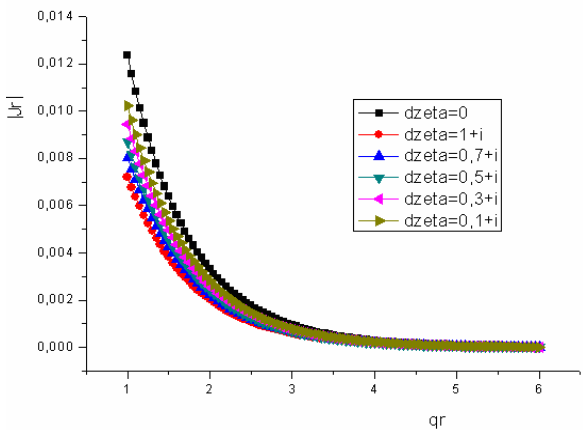

Fig. 2. The surface current absolute values for the perfectly conducting cone with the cover, $\left(\gamma = \frac{\pi}{8}, |qr_0| = 1\right)$

In order to outline the cone curvature influence in the boundary condition we assume that $0 \leq Z_0$ therefore $0 \leq \breve{y}_0 \leq y_0 \leq 1$. The absolute value of the surface current radial component dependencies on qr ($qr \in \mathbb{R}$) are shown in the Fig.2 (the impedance parameter is considered to be constant, $\theta_0 = \pi$, the cone curvature is taken into account.

As far as it can be seen, the provided graphs present that the obtained results for the semi-infinite cone can be used for the surface current density covering the finite cone analyzing provided that the distance from the cone apex is less than three wave lengths and that the excitation is axial symmetric.

4. CONCLUSIONS

The rigorous statement problems for the semi-infinite semitransparent circular cone or the perfectly conducting cone with thin cover excitation by the harmonic point source is investigated. The thin cover is simulated by means of the one-sided boundary conditions of the ABC type averaging provided that the conical surface curvature is taken into account. A special type of the impedance parameter that depends on the distance to the cone apex is considered. The analytical solutions of the cone excitation by the radial dipole problem are derived. The analytical solution that doesn't include the boundary condition curvature coefficient is shown for the curvature effects analyzing. The solutions are compared and the boundary condition curvature coefficient effect is studied. The losses caused by the thin cover presence are investigated.

REFERENCES

1. Senior T.B.A., Volakis J.L. Approximate boundary conditions in electromagnetics // The institution of Electrical Engineers, London, United Kingdom, 1995, 353 p.
2. Doroshenko V.A., Kravchenko V.F., Pustovoit V.I. Diffraction of electromagnetic waves on an imperfectly conducting conical structure of a partial shape// Doklady Physics. - 2006. - Vol.51, №10. - P.529-533.