

## NONLINEAR MODEL OF ROBUST-OPTIMAL STABILIZATION PROCESS OF QUADROPTER

**Annotation.** Solution, based on variable structure of robust-optimal system, that improves quadcopter's automated control process taking into account dynamic model of quadcopter and environment's uncertainty, is considered in the article.

**Keywords – quadcopter; optimal trajectories; robust circuit; variable structure of feedback**

Increasing intensity of ship exploitation in navigable channels and limited water areas requires high safety level while performing various tasks in technological regimes. Usage of unmanned aerial vehicles (e.g., quadcopter) for navigational safety monitoring and ecological control of marine environment would increase maneuvering and position's informative estimation of movable objects. This could help optimize control decisions made by boat masters, pilots and operators of maritime traffic when extreme situations appear. Quadcopter functions under influence of marine environment and should provide maneuvering with specified control quality indicators determined by the requirements of visual camera monitoring.

Development of control systems for movable objects is based on robust-optimal methods [1]. Quadcopter's control methods including ship service and monitoring of marine environment are based on design of optimal control systems and effective PID regulators. Problem of quadcopter stabilization in technological regimes or during movement along predetermined path involves creation of efficient and physically feasible control algorithms. Quadcopter control during monitoring of marine environment requires optimal (in terms of energy consumption) control algorithms because of limited battery capacity. At the same time, the control problem is complicated by the need of taking into account stochastic nature of uncontrolled wind disturbance which is typical for agitated sea surface. Thus, in order to form optimal stabilization trajectories  $\mathbf{X}_{opt}(t)$  over all quadcopter's controllable coordinates, an optimal criterion is set for minimum energy consumption

$$J = \int_0^T Q(\mathbf{X}, \mathbf{U}) dt = \min \quad (1)$$

where  $Q(\mathbf{X}, \mathbf{U})$  is functional of energy;  $\mathbf{X}$  is coordinates vector;  $\mathbf{U}$  is control vector;  $T$  is time of control.

Requirements for vector of control errors  $\mathbf{E}(t)$ , which arise from uncertain operating conditions of the quadcopter, are set as

$$\dot{\mathbf{E}}(t) + \mathbf{G}_1 \mathbf{E}(t) + \mathbf{G}_2 \mathbf{E}(t) = 0, \quad (2)$$

where  $\mathbf{G}_1, \mathbf{G}_2$  are matrices of weighting coefficients.

The quadcopter's dynamics is described by the system with six differential equations of second order [2] and is considered in the field of wind disturbance. Taking into account the wind disturbance, motion of the quadcopter can be described by the system of the following differential equations [2,3]

$$\dot{\mathbf{X}}(t) = \mathbf{A}_X \mathbf{X}(t) + \mathbf{B}_X \mathbf{U}(t) + \mathbf{C} \mathbf{f}(t) - \mathbf{g}, \quad (3)$$

where  $\mathbf{X}$  is state coordinates vector (12×1);  $\mathbf{A}_X$  is coordinate-dependent matrix (12×12) of quadcopter's parameters;  $\mathbf{B}_X$  is coordinate-dependent matrix (12×4) of control;  $\mathbf{U}$  is the control vector (4×1);  $\mathbf{C}$  is coefficient matrix (12×6);  $\mathbf{f}(t)$  is vector (6×1) of wind disturbance;  $\mathbf{g}$  is vector (12×1) with gravity acceleration component.

Given approach (when optimal control criterion is set as (1)) is based on usage of system with variable structure of feedbacks [3] and includes the following main stages: determination of optimal trajectories; determination of switch moments of control functions in the object's feedback loops; synthesis of control functions in the relevant feedback loops.

To provide motion along given segments of optimal trajectories, relevant control functions are determined using the differential transformation [3] of (3) with respect for example to the zero second derivative  $\ddot{\mathbf{X}}(t) = 0$  of quadcopter's coordinates vector and taking into account the requirements for

physical feasibility of control forces. It allows to form equations of force's (moment's) balance for second derivative as

$$\mathbf{A}_X \ddot{\mathbf{X}}(t) + \dot{\mathbf{A}}_X \dot{\mathbf{X}}(t) + \mathbf{B}_X \ddot{\mathbf{U}}(t) + \dot{\mathbf{B}}_X \dot{\mathbf{U}}(t) = 0. \quad (4)$$

After the vector-matrix transformations of (4) it can be written in the form that determines control vector  $\mathbf{U}$  and provides the movement of the quadcopter along the optimal trajectories as

$$\mathbf{B}_X \ddot{\mathbf{U}}(t) + (\mathbf{A}_X \mathbf{B}_X + \dot{\mathbf{B}}_X) \dot{\mathbf{U}}(t) = -(\mathbf{A}_X^2 + \dot{\mathbf{A}}_X) \dot{\mathbf{X}}(t) + \mathbf{A}_X \mathbf{g} \quad (5)$$

Switch moments  $t_i^s$  of control functions (5) are static points for which final values of the state variable on the  $i$ -th segment of the trajectory determine, jointly with the new value of the higher derivative of the coordinate, its initial values on the  $(i+1)$ -th segment of the trajectory.

Robust control problem (2) is solved by: formation of robust corrective circuit and quadcopter's reference model which contains system with variable structure of feedback. Control signal from the reference model sums with signal from the robust corrective circuit and goes to the input of physical quadcopter.

Synthesis procedure of quadcopter's control is proposed based on the system with variable structure of feedbacks and optimal criterion for minimum energy consumption. The required level of control invariance under conditions of incomplete information due to insufficient a priori data of quadcopter's parameters, parametric noises and action of uncontrolled disturbances is provided by robust corrective control that holds stabilization trajectories of quadcopter within acceptable vicinity. The modelling examples (Fig.1 and 2) demonstrate effectiveness of the approach in terms of minimum values of control errors.

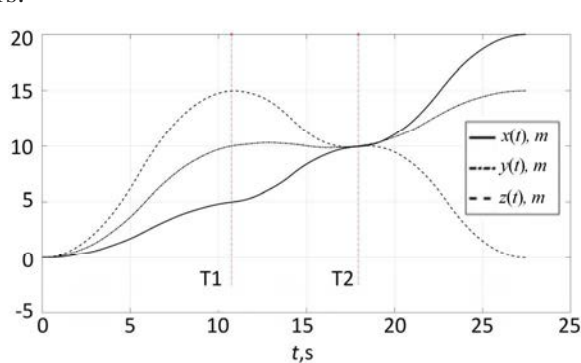


Fig. 1. Trajectories of main coordinates

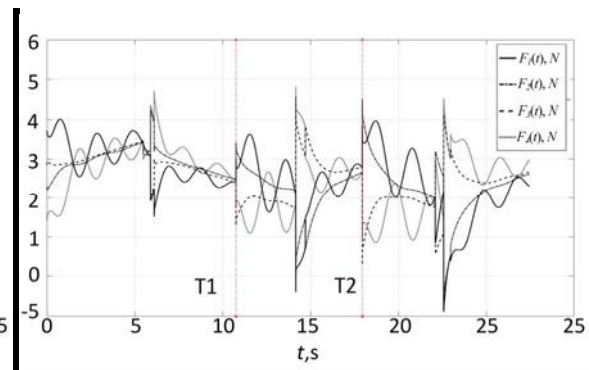


Fig. 2. Lifting forces of rotors

## References

1. Kuncевич V.M. Synthesis of robust – optimal control systems of non-stationary objects in case of bounded disturbances. Problems of control and informatics. 2004. №2. P. 19-31.
2. Altug E., Ostrowski J.P. and Taylor C.J. Control of a quadrotor helicopter using dual cameravisual feedback. Journal of Robotics Research. 2005. №24. P. 329-341.
3. Timchenko V.L., Lebedev D.O., Kuklina E.A and Timchenko I.V. Robust-optimal control system of quadcopter for maritime traffic's monitoring. Proceeding of IEEE 4th Intern. Conference «Actual Problems Of Unmanned Aerial Vehicles Development». K. 2017. P. 192-196.