

## MODELING AND IDENTIFICATION OF THE DYNAMICS OF THE PROCESS OF DRILLING WELLS ON THE BASIS OF THE CATASTROPH THEORY

**Annotation.** It is proposed to determine the catastrophic deviations of the intensity of the change in drilling time of 1 m rocks in the depth of the well using the model of the catastrophe of the "COLLECTION" of R.Toma or the model Vapor Pressure. Adjustment of the axial force on the bit according to the developed methodology allows to provide a stable dynamic drilling mode.

**Key words:** well-drilling, dynamic stability, chaotic regime, modeling, experimental analysis.

### 1 Formulation of the problem

The process of drilling oil and gas wells is a nonlinear dynamic chaotic process. The mode of its operation is difficult to predict due to changes in its temporal, functional and spatial structures. Accordingly, the problem of improving the management of well drilling [1, 2, 3, etc.] is updated.

However, a more detailed analysis of this optimization problem has shown that the drilling process is a nonlinear dynamic chaotic process. Therefore, it is not always possible accurately to describe and predict the mode of its operation. One of the problems is the recognition of the dynamic modes of wells (disasters) deepening and this process management. Solving this problem requires meeting the following challenges:

- to analyze the meter cost elements of the well drill and the influence of time  $T$ , which is spent on drilling each meter of rocks in depth, at cost;
- to compare the elemental catastrophe R.Toma of the "COLLECTION" type with the model Vapor Pressure on the example of an experimental analysis of the index  $T$  change on the depth of the well.

### 2 Results of the research

To recognize the dynamic modes of the drilling process and to control them in real time, we use a change in the drilling time of 1 m of rocks at the depth of the well  $T = \Delta t_d / \Delta h$ , h/m. The bases for  $T$ -control is the on-line analysis.

A prerequisite for changing management actions is not only the desire to provide a process with a minimal cost, but also a leap-like indicator change  $T = \Delta t_d / \Delta h$ .

We analyzed the data of more than 20 wells, drilled in the Carpathian region. For a qualitative assessment of the dynamic regime of deepening wells, we use the elemental catastrophe of R. Thomas, in particular the catastrophe "COLLECTION" and its first derivative:

$$T = \frac{dt_d}{dh} = t_d^3 + at_d + b = 0. \quad (1)$$

where  $dt_d/dh$  – intensity of change of drilling time of 1 m rocks along the well depth;  $a, b$  – coefficients characterizing mining-geological conditions and technological parameters of drilling; they can be determined by the method of OLS empirical data.

If the system behavior can be described by the differential equation (2), then the set of disasters, ie, leap-like changes in well drilling modes, is determined on the basis of the conditions:

$$\frac{dt_d}{dh} = \frac{d^2t_d}{dh^2} = 0 \quad (2)$$

$$\frac{d^2t_d}{dh^2} = 3t_d + a = 0 \quad (3)$$

Condition (2) is performed at critical points, condition (3) – twice degenerate critical points. The positions of critical points are found in the equation (1), which defines a two-dimensional manifold in a three-dimensional space with coordinate axes  $T - a - b$ . The roots of the cubic equation (1) can be defined in the canonical form using the Cardano formula.

For comparison, the mathematical model Vapor Pressure:

$$T = \exp\left(a + \frac{b}{h} + c \lg h\right) \quad (4)$$

where  $a, b, c$  – model parameters;  $h = 1, 2, 3, \dots, k$  – depth of well, m.

It is proved (Table 1) that with the help of model (4) it is also possible to simulate the dynamics of disasters in drilling: catastrophic wear of the bit, zones with abnormal reservoir pressure, sudden transition of the bit in the rock with other properties, changes in the drilling time of 1 m rocks along the depth of the well, etc.

**Table of Fits 1**

Borehole(wells) Depth	Fit type	SSE	R-square	DFE	Adj R-sq	RMSE	# Coeff
A 2020÷2050M	$x^3+a^*x...$	70,4731	0.9064	13	0.8992	2.3283	2
	$\exp(a+b...$	47.6311	0.9367	12	0.9262	1.9923	3
B 2050÷2100M	$x^3+a^*x...$	44.0374	0.9848	12	0.9835	1.9157	2
	$\exp(a+b...$	41.0841	0.9858	11	0.9832	1.9326	3
C 2104÷2116M	$x^3+a^*x...$	7.7907	0.8425	6	0.8162	1.1395	2
	$\exp(a+b...$	5.9872	0.8789	5	0.8305	1.0943	3
D 2116÷2124M	$x^3+a^*x...$	0.7106	0.9847	3	0.9796	0.4867	2
	$\exp(a+b...$	0.6001	0.9871	2	0.9742	0.5477	3
E 2125÷2165M	$x^3+a^*x...$	98.4660	0.9285	10	0.9214	3.1379	2
	$\exp(a+b...$	15.6818	0.9886	9	0.9861	1.3200	3
F 2165÷2195M	$x^3+a^*x...$	9.9319	0.9873	8	0.9857	1.1142	2
	$\exp(a+b...$	2.6521	0.9966	7	0.9956	0.6155	3

Model (4 fundamentally) differs from others by using it at every step of  $h$  (for example,  $h = 0.2$  m) in real time, when new information is received, all the model coefficients are adjusted according to the conditions of the well drilling conditions. That is, the coefficients  $a, b, c$  change together with the change in the main indicators of the well drilling process. The parameter  $a$  corresponds to the general amplitude of the model curve. Parameter  $b$  indicates the moment of "disaster". In our case, this is the extremum of the curve  $T = f(h)$ .

The model adequately reflects relationships with the correlation coefficients of 0.879 to 0.996 (Table 1), which enables to effectively solve the problem of identifying model parameters by a single algorithm, as well as reliably detect disasters.

### 3 Summary

3.1 On the basis of the analysis of meter cost elements of well drilling and the influence on this time factor  $T$ , which is spent on drilling each meter of rocks in the depth of the well, the expediency of using the mathematical apparatus of the catastrophe theory to describe the emergency situations and complications in the process of drilling wells has been substantiated.

3.2 Comparison of the elemental catastrophe R.Toma of the type "assembly" with the model of Vapor Pressure on the example of an experimental analysis of the change in the index  $T$  on the depth of the well showed that the behavior of the system can be described not only canonical cubic equation, whose solution is based on the Cardan formula, but also the equation Vapor Pressure, which describes the disasters with correlation coefficients of 0.897 to 0.996.

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