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ADAPTIVE PSEUDOINVERSE MODEL-BASED CONTROL OF SOME MEMORYLESS SIMO AND MIMO SYSTEMS

Abstract. This paper deals with the robust adaptive control of discrete-time multivariable memoryless systems having non-square gain matrices of full rank. The main feature of these systems is that the number of outputs exceeds the number of control inputs. To cope with the parameter uncertainty, an adaptive version of the so-called pseudoinverse model-based approach is advanced. Asymptotic properties of a new adaptive control algorithm first proposed are established. Simulation results are given to demonstrate these properties.

Keywords: adaptive control, multivariable system, discrete time, pseudoinversion, robust stability.

In practice, there is a class of multivariable uncertain systems where the number of control inputs is less than the number of their outputs. This class includes, in particular, the memoryless SIMO (single-input multi-output) and MIMO (multi-input multi-output) systems with parameter uncertainty. In order to ensure the boundedness of the control and outputs signals in the closed-loop control system, the nonadaptive controller containing a linear pseudoinverse model with fixed parameters has recently been designed in [1 – 3]. Although this control can, in principle, guarantee the robust stability of such control system, however it is not applicable if the initial uncertainty is great enough.

It turns out that it is possible to design an adaptive robust controller using the pseudoinverse approach to deal with memoryless and nonsquare MIMO system in the presence of arbitrary unmeasurable but bounded disturbances.

The following problem is stated and solved. Consider the difference equation

$$y_n = Bu_{n-1} + v_{n-1} \tag{1}$$

describing a multivariable system to be controlled at the discrete time $n = 1, 2, \mathbf{K}$. In this equation, $y_n \in \mathbf{R}^m$ denotes the output vector, $u_n \in \mathbf{R}^r$ denotes the control input vector, $v_n \in \mathbf{R}^m$ is the disturbance vector, and

$$B = \begin{pmatrix} b^{(11)} & \mathbf{K} & b^{(1r)} \\ \cdot & \cdot & \cdot \\ b^{(m1)} & \mathbf{K} & b^{(mr)} \end{pmatrix}$$

represents the fixed and unknown nonsquare $m \times r$ matrix ($m > r$) whose rank satisfies

$$\text{rank}\, B = r.$$

We assume that there are known constants $\varepsilon^{(i)}$ $(i = 1, \mathbf{K}\, m)$ such that

$$| v_n^{(i)} | \le \varepsilon^{(i)} < \infty \tag{2}$$

for each ith component $v_n^{(i)}$ of v_n and $n \in [0, \infty)$.

Let $y^0 = [y^{0(1)}, \mathbf{K}, y^{0(m)}]^T$ be the desired output vector for given $y_n = [y_n^{(1)}, \mathbf{K}, y_n^{(m)}]^T$. The problem is to design feedback controller guaranteeing

$$\limsup_{n\to\infty} \| y^0 - y_n \| < \infty, \qquad \limsup_{n\to\infty} \| u_n \| < \infty. \tag{3}$$

To solve the problem above stated, we propose the adaptive pseudoinverse model-based control law

$$u_n = u_{n-1} + B_n^{+} (y^0 - y_n), \tag{4}$$

where B_n^{+} represents the matrix which is pseudoinverse to the estimate

$$B_n = \begin{pmatrix} b_n^{(11)} & \mathbf{K} & b_n^{(1r)} \\ \cdot & \cdot & \cdot \\ b_n^{(m1)} & \mathbf{K} & b_n^{(mr)} \end{pmatrix}$$

updated via the standard recursive dead-zone parameter adaptation rule with respect to each *i*th row $b_n^{(i)} = [b_n^{(i1)}, \mathbf{K}, b_n^{(ir)}]^{\mathrm{T}}$ of B_n.

The adaptation algorithm is specified by

$$b_n^{(i)} = b_{n-1}^{(i)} + \gamma_n^{(i)} f(\bar{\varepsilon}^{0(i)}, \bar{\varepsilon}^{(i)}, \tilde{e}_n^{(i)}) \nabla u_{n-1} / \| \nabla u_{n-1} \|^2 \quad (i = 1, \mathbf{K}, m), \tag{5}$$

where $\nabla u_{n-1} := u_{n-1} - u_{n-2}$,

$$f(\bar{\varepsilon}^{0(i)}, \bar{\varepsilon}^{(i)}, \tilde{e}_n^{(i)}) = \begin{cases} 0, & \text{if } |\tilde{e}_n^{(i)}| \le \bar{\varepsilon}^{0(i)}, \\ \tilde{e}_n^{(i)} - \bar{\varepsilon}^{(i)} \operatorname{sign} \tilde{e}_n^{(i)} & \text{otherwise} \end{cases}$$

represents the dead-zone function depending on numbers $\bar{\varepsilon}^{0(i)} > \bar{\varepsilon}^{(i)}$ with $\bar{\varepsilon}^{(i)} = 2\varepsilon^{(i)}$ and on the *i*th estimation error determined as

$$\tilde{e}_n^{(i)} = e_n^{(i)} - e_{n-1}^{(i)} - b_{n-1}^{(i)\mathrm{T}} \nabla u_{n-1} \tag{6}$$

with $e_n^{(i)} = y^{0(i)} - y_n^{(i)}$, and $\gamma_n^{(i)}$ is the scalar coefficient satisfying

$$0 < \gamma' \le \gamma_n^{(i)} \le \gamma'' < 2. \tag{7}$$

By suitable choosing the coefficients $\gamma_n^{(i)}$ from $[\gamma', \gamma'']$, the condition

$$\operatorname{rank} B_n = r \quad \forall n$$

can be shown to be satisfied. This condition makes it possible to calculate B_n^+ as

$$B_n^+ = (B_n^T B_n)^{-1} B_n^T .$$

It is theoretically established that if the assumption (2) takes place, then the ultimate property (3) of the adaptive closed-loop control system containing the plant (1) and the control law (4) together with the adaptive estimation procedure given in (5) – (7) is achieved (the main result).

The simulation experiments show that the proposed adaptive control algorithm is useful to deal with some nonsquare memoryless multivariable systems. Thus, we can conclude that the adaptive pseudoinverse model-based controller is able to robust stabilize the uncertain memoryless SIMO and MIMO systems with bounded unmeasured disturbances for the case where the number of outputs exceeds the number of control inputs.

References

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