UDK 519-7

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COMPUTATIONAL ANALYSIS OF METHOD OF DETECTING OF PERIODIC POINTS IN NONLINEAR DYNAMICAL SYSTEMS

Annotation. The problem of detecting of non-stationary periodic points of nonlinear dynamical systems is considered in this work. A new approach has been suggested for the constructing of delayed feedback. Effectiveness of this approach is shown on the basis of results of constructed examples.
Keywords: chaos, cycles' stabilization, nonlinear dynamical systems, periodic orbits

The phenomenon of chaos in nonlinear dynamical systems has been studied for many years, however, it is still interesting to study new systems and methods of chaos control, and problems related to chaos control, are still relevant. A lot of famous methods of detecting of periodic orbits exist [1, 2, 3]. In this work it is suggested to use the method of periodic orbits' stabilization, which got its development from method suggested in [4], with applying the prehistory depth, that equals 3. The main purpose is to detect cycles with the greater length than in previous received results.

Let us consider the vector nonlinear dynamical system

$$x_{n+1} = f(x_n),\ x_n \in R^m,\ n = 1,2,... \tag{1}$$

It is considered that there is one or few non-stable cycles with period T in this system, $(\eta_1, \mathbf{K}, \eta_T)$ is a cycle. Multipliers of cycle are defined as roots of characteristic equation:

$$\det\left(mI - \prod_{i=1}^{T} Df\left(x_i^*\right) \right) = 0\,, \tag{2}$$

where μ is a multiplier of a cycle. It is considered, that there exist roots beyond the unit circle. Therefore, the cycle of the system is unstable and it is problematic to detect it. One of the solutions of this problem is connected with insertion in (1) the control to make the system such as given system:

$$x_{n+1} = f(x_n) + u_n,\ n = 1,2,... \tag{3}$$

O. Morgul suggested to use control in form [4]:

$$u_n = \varepsilon(x_{n-T+1} - f(x_n)),\ |\varepsilon| < 1 \tag{4}$$

The depth of prehistory equals 1 in this approach. In [5] it is used the depth of prehistory N=2, that let us to increase localization area of multipliers and find more cycles with greater length. Let us use N=3 to find cycles with greater length and increase effectiveness of control, using the scheme:

$$x_{i+1} = (1-\gamma) f\left(\sum_{j=1}^{N} a_j x_{i-Tj+T} \right) + \gamma \sum b_j x_{i-Tj+1} \tag{5}$$

where $a_j,\ b_j$ are optimal coefficients of control, $\sum_{j=1}^{N} b_j = \sum_{j=1}^{N} a_j = 1,\ a_j > 0,\ b_j > 0,\ j = \overline{1, N}$.

Let us consider the algorithm of constructing optimal a_j. Define nodes:

$$\psi_j = \frac{\pi(\sigma + T(2j-1))}{\tau + (N-1)T},\quad j = 1,..., \frac{N-2}{2}\ (N - \text{ even}),\ (\frac{N-1}{2}\ (N - \text{ odd})).$$

Define the polynomials: $\eta_{2k+1}(z) = z \sum_{j=0}^{2k} c_j(k) z^{2k-j}$, in particular $\eta_1(z) = z$,

$\eta_3(z) = z(c_0(1)z^2 + c_1(1)z + c_2(1)) = z(z^2 - 2\cos\psi_1 + 1)$, that is $c_0(1) = 1,\ c_1(1) = -2\cos\psi_1,\ c_2(1) = 1$.

Define the coefficients for $\eta_{2(k+1)+1}(z) = \eta_{2k+3}(z)$, consider, that $\eta_{2k+1}(z)$ are known by formulas:

$$c_{-1}(k) = 0,\ c_0(k) = 0 \tag{6}$$

$$c_s(k+1) = c_s(k) - 2c_{s-1}(k)\cos\psi_{k+1} + c_{s-2}(k),\ s = 1,...,2k-1 \tag{7}$$

$$c_{2k}(k) = 1,\ c_{2k+1}(k) = c_{2k+2} = 0 \tag{8}$$

We can build the polynomials in case $N = 2k+1$ consequently using formulas (6)-(8). In case $N = 2k+2$, polynomials will be constructed in the next way:

$$\eta_{2k+2}(z) = z(z+1)\sum_{j=0}^{2k} c_j(k) z^{2k-j}$$

In the same way, we can build polynomials in case $N = 2k + 2$, consequently using formulas (6)-(8). Таким образом, в случае нечетных $N = 2k + 1$ определим коэффициенты b_j по правилам $b_{j+1} = c_{2k-j}(k),\ j = 1,....,2k$, и в случае четных $N = 2k + 2$: $b_{j+1} = c_{2k-j+1}(k) + c_{2k-j}(k),$ $c_{-1}(k) = c_{2k+1}(k) = 0,\ \ j = 1,....,2k + 2.$

Eventually, optimal coefficients a_j are defined by formula:

$$a_j = K\left(1 - \frac{1 + (j-1)T}{2 + (N-1)T}\right) b_j,\ \ j = 1,..., N, \tag{9}$$

where constant of normalization *K* is defined so that $\sum_{j=1}^{N} c_j = 1,\ K = \dfrac{1}{\sum_{j=1}^{N}\left(1 - \frac{1+(j-1)T}{2+(N-1)T}\right) b_j}$.

Numeric computations were performed with some known maps: HENON, LOZI, IKEDA, etc., for comparison of using *N*=2 and *N*=3 in the method of the cycles' stabilization suggested in [5].

Due to applying of new algorithm of constructing coefficients of optimal control with depth of prehistory *N*=3, it is succeeded to find cycles with such lengths as 2, 3, 4, 5, 6, 8 for HOLMES map. At the same time, it is succeeded to find cycles 2, 3, 4, 5, when we used *N*=2. HENON: it is succeeded to find cycles with such lengths as 2, 4, 6, 7, 8, using *N*=3, and when we used *N*=2, we found cycles 2, 4, 6; Lesli-Ferhulst: it is succeeded to find cycles with such lengths as 2, 3, 4, 8, 9, using *N*=3, and when we used *N*=2, we found cycles 3, 8.

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