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Belousov A.A., candidate of phys.- math. sciences, senior sci. res.,
Glushkov Institute of Cybernetics NAS of Ukraine

ON DIFFERENTIAL GAMES UNDER CONVEX INTEGRAL CONSTRAINS ON CONTROLS

Summary. The paper deals with the problem of the approach of a trajectory of a linear conflict-controlled process to a linear subspace in the case of convex integral constraints on the player's controls. Using the technique of set-valued mappings and convex analysis (epigraph of a function, recession cone), we obtain sufficient conditions for problem solvability in the class of measurable controls.

Key words: differential game, integral constrains, resolving function, set-valued mapping, recession cone.

Dynamics of a game is set by the linear differential equation

$$\dot z = Az + Bu - Cv, \quad z(0) = z^0, \quad z \in \mathrm{R}^n,\ u \in \mathrm{R}^m,\ v \in \mathrm{R}^l. \qquad (1)$$

The controls of the player-pursuer $u(\cdot)$ and the evader $v(\cdot)$ are Lebesgue measurable functions that satisfy the integral constraints:

$$\int_0^\infty j(u(t))\,dt \le 1, \quad \int_0^\infty y(v(t))\,dt \le 1. \qquad (2)$$

The function j, $j : \mathrm{R}^m \to \mathrm{R}$, is assumed to be nonnegative, convex, and lower semicontinuous [1]. Suppose $j(0) = 0$ and the level set $\Phi(g) = \{u \in \mathrm{R}^m : j(u) \le g\}$ is limited to at least one nonnegative number g. The function y, $y : V \to \mathrm{R}$, $V \subset \mathrm{R}^l$, is assumed to be nonnegative and upper semicontinuous on its domain of definition V. The terminal set M is a linear subspace R^n.

We denote by p the projection onto the orthogonal complement L to the subspace M in R^n.

Condition. *There exist a number* l, $0 \le l < 1$, *such that for all* t, $t \ge 0$, *and* v, $v \in V$, *holds the inclusion*:

$$pe^{At}Cv \in pe^{At}B\Phi(l \cdot y(v)). \qquad (3)$$

We consider an auxiliary function, which can be called the resolving function of a differential game under integral constraints:

$$g(t,t,v) = \sup\{g \ge 0 : a(t)g + b(t,v) \in H(t)\,\mathrm{epi}\,j\},$$

where $a(t) = \begin{pmatrix} -pe^{At}z^0 \\ 1-l \end{pmatrix}$ $b(t,v) = \begin{pmatrix} pe^{At}Cv \\ l\,y(v) \end{pmatrix}$, $H(t) = \begin{pmatrix} pe^{At}B & 0 \\ 0 & 1 \end{pmatrix} : \mathrm{R}^m \times \mathrm{R} \to L \times \mathrm{R}$,

$\mathrm{epi}\,j = \{(u,m) \in \mathrm{R}^m \times \mathrm{R} : m \ge j(u)\}$ – epigraph of j [1], $(t,t,v) \in \mathrm{R}_+ \times \mathrm{R}_+ \times V$, $\mathrm{R}_+ = [0,\infty)$.

We recall [1] that the recession cone 0^+W of a convex set W, $W \subset \mathrm{R}^k$, is the set $0^+W = \{a \in \mathrm{R}^k : a + W \subset W\}$. Define the set $\Delta = \{(t,t) \in \mathrm{R}_+ \times \mathrm{R}_+ : a(t) \notin H(t) \cdot 0^+\,\mathrm{epi}\,j\}$.

Theorem. *Suppose that condition (3) is satisfied for the parameters of the game (1)-(2). Let there exists a moment* $T = T(z^0)$ *such that* $\{T\} \times [0,T] \subset \Delta$ *and for all admissible controls* $v(\cdot)$ *holds the inequality* $\int_0^T g(T, T-t, v(t))\,dt \ge 1$. *Then the differential game can be completed at time* T, *i.e. for any admissible evader's control* $v(t)$ *there is an admissible pursuer's control* $u(t)$ *that guarantees the approach of the solution of equation (1)* $z(t)$ *corresponding to the controls* $(u(t), v(t))$ *and initial position* z^0 *to the terminal set at moment* T *:* $z(T) \in M$.

References

1. Rockafellar R.T. Convex Analysis. Princeton University Press, Princeton, N.J., 1970.