## ERRORS IN THE MEASUREMENT OF SMALL PHASE SHIFTS WITH BINARY SAMPLING

Abstract. In this report the errors of the measurement phase shift between the harmonic signals using binary sampling are analyzed. The main source of errors in such measurementi is the inequality of signal amplitudes. We determined four methods of measuring the phase shift whose we are considering here. The mathematical models of additive errors for various methods were determined and a their comparative analysis is carried out. The graphs of dependences of errors on amplitudes inequality are constructed. The method that provides the least error is determined

**Key words:** measuring phase shift, errors, harmonic signals, low frequency, binary sampling, inequality of the amplitudes, threshold sensitivity

Measurements of small phase shifts between harmonic signals are used in ground and aerial electromagnetic systems for detection in the underground, in the offshore zone of the sea and in the sea depths of conducting bodies with diamagnetic or magnetic properties. To increasing the depth at which such bodies can be detected, frequencies of no more than several units of Hz are used [1,2]. At such frequencies, the signal level is very low, so using the sum-difference method of measuring the phase shift as the most sensitive. To reduce the effect of flicker noise, binary sampling is used [3].

In particular, the method of binary sampling, as other sum-difference methods require equality of amplitudes of compared signals, which is hard to provide. Therefore, the goal of this article is a theoretical analysis of the measurement errors of phase shift caused by inequality of comparable signals amplitude when using binary sampling.

Consider the harmonic signals with unequal amplitudes and initial phases

$$u_1 = U_1 \sin wt$$
 and  $u_2 = U_2 \sin(wt + j_x)$  (1)

where  $U_1$  and  $U_2$  - respectively amplitude of the first and a second signals;  $\omega$  - circular frequency of signals;  $\phi_x$  - phase shift between the signals.

For binary sampling of these signals with circular frequency  $\Omega >> \omega$  using automatic switch with two inputs and one output, controlled by voltage

$$u_K(t) = U_K sign \sin(\Omega t + j), \qquad (2)$$

where  $\boldsymbol{\phi}$  - the initial phase of a switching signal.

On the output of switch we get the amplitude-modulated signal, which can be filed (3)

$$u_{VK}(t) = \frac{1}{2} \sqrt{U_1^2 + U_2^2 + 2U_1 U_2 \cos j_x} \sin\{wt + \frac{j_x}{2} + \arctan \frac{-U_1 \sin \frac{J_x}{2} + U_2 \sin \frac{J_x}{2}}{U_1 \cos \frac{j_x}{2} + U_2 \cos \frac{j_x}{2}}\} +$$
(3)  
$$-\frac{1}{2} \sqrt{U_1^2 + U_2^2 - 2U_1 U_2 \cos j_x} \sin\{wt - \arctan \frac{U_2 \sin j_x}{U_1 - U_2 \cos j_x}\} sign \sin(\Omega t + j)$$

and which is shown in Fig. 1.

On the basis of the analysis (3), four different expressions were obtained for phase shift:

$$j_x = 2 \arccos \frac{U_{mw1}}{K_{FNCH}U}$$
(4) 
$$j_x = 2 \arcsin \frac{U_{mw2}}{K_{FVCH}U}$$
(5)

$$j_{x} = 2 \operatorname{arcctg} \frac{K_{FVCH} U_{mw1}}{K_{FNCH} U_{mw2}} \quad (6) \qquad j_{x} = 2 \operatorname{arctg} \frac{K_{FNCH} U_{mw2}}{K_{FVCH} U_{mw1}} \quad (7)$$

For each of them, the expressions of the measurement error on the relative inequalities of the signal amplitudes are obtained.

On the basis of the obtained expressions, their sensitivity thresholds are calculated for certain values of the relative inequality of the amplitudes of the comparable signals. The results are shown on (Figure 2).

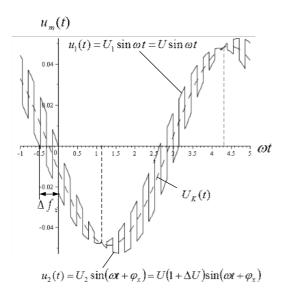


Fig. 1. Binary-sampled signal at the output of switch

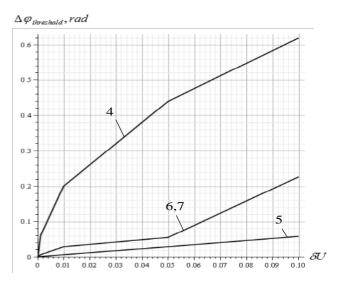


Figure 2. Dependence of phase shifts threshold from the amplitudes inequality for expressions (4-7) that determines phase shift  $\varphi_x$ 

According to these graphs it is evident that the smallest errors are obtained by using the expression (5) for measuring the phase shift.

Thus, the use of expression (5) to measure the phase shift between harmonic signals provides the lowest sensitivity threshold. Further reduction of the threshold of sensitivity can be achieved by preliminary alignment of the amplitudes of the compared signals.

## References

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