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ANALYTICAL METHOD OF DETERMINING THE ELECTROMAGNETIC FIELD OF THE STANDARD CURRENT PULSES FLOWING NEAR TO A CONDUCTIVE OBJECT

Yuriy Vasetsky

Institute of Electrodynamics, Ukrainian National Academy of Sciences, Kyiv, Ukraine

yuriy.vasetsky@gmail.com

Abstract: The mathematical model based on the developed theory of the analytical solving the quasi-stationary tasks of pulse current flowing near to a conductive object with flat surface is introduced. The applied mathematical model includes an approximate solution with the use of asymptotic expansion for computing the intensity of a magnetic and electrical field in the case of transient electro-magnetic processes. It is noted that the calculations by an approximation method are limited to a certain time period from the beginning of a pulse, but, as usual, just within this time period the field changes most rapidly and accesses maximum values. The electrical field is considered at the presence of the standard current pulses such as exponentially decaying pulse, pulse represented by the difference between two decaying exponents, exponentially decaying oscillating pulse. For them the main peculiarities of applying the approximate analytical method of field calculations have been analyzed. Integral indicators for taking into account limitations by frequency and time, depending on the pulse parameters, have been found. Time dependencies have been obtained with the use of special functions and their representation as series.

Key words: quasi-stationary three-dimensional pulsed electromagnetic field, analytical and asymptotic calculation methods, typical current pulses.

1. Introduction

Attention to studying pulse electromagnetic fields of devices whose elements demonstrate a strong skin effect is conditioned on the one hand by the wide range of the use of such devices, and on the other hand – by the necessity of taking into account the geometrical and electrophysical peculiarities at the modelling of physical processes. Installations for creating pulse voltages and currents at the electrotechnical and electrophysical laboratories [1-3], devices providing the electrodynamic influence of a pulse magnetic field on metal objects [4–6], systems for applying a pulse electromagnetic field and electric current to affecting the mechanical properties of metals, in particular, on weld joint [7, 8], can be given as the examples of these phenomena.

The determination of a three-dimensional electromagnetic field is a quite difficult problem because of a big amount of calculations, even when well-developed numerical computation techniques are applied. The difficulties with obtaining the results sufficiently increase while solving the inverse problems of a field theory and optimization of the geometry of electromagnetic systems. In these cases, analytical or numerical analytical approaches, which enable taking into account a limited number of the most sufficient system characteristics, remain effective. The example of applying the analytical accurate and approximate methods of computation of an alternating electromagnetic field is solving the inverse problems of determining the spatial geometry of field inductors in the domain region of thermal treatment with the use of induction heating of mobile metal straps [9–11]. The basis of these works is the obtained accurate analytical solution to the task of the field of the alternating sinusoidal current flowing along a closed circuit near to the electro-conducting half-space. The solution was obtained without limits of the geometry and orientation of the circuit, electric and physical properties of the medium (electrical conduction γ and relative permeability μ) and field frequency ω [12, 13]. This enables determining all characteristics of the electromagnetic field in any point of the dielectric and conducting regions.

For the pulse current, the obtained solution is a frequency spectrum of the electromagnetic field created by a current with a set frequency spectrum. Time dependencies can be obtained by performing the inverse Fourier transform. In this case, the solution is represented by triple improper integrals, which, despite of the analytical form of the expressions, is rather difficult.

Calculations become easier with the expansion of the expressions into bounded asymptotic series [14]. However, for pulse fields, the calculation is limited by some time interval from the beginning of the current pulse. Since, in general, the current pulse changes the most quickly and reaches the biggest values during the short time interval, the electromagnetic field is determined exactly in this the most important stage.

In the general case, the found solution enables the calculations for the arbitrary time dependence of a circuit current. At the same time, it seems reasonable that the analysis of the features of field distribution is conducted for standard form current pulses, which are the most frequently realized. Therefore, the aim of the work is to obtain the concrete computational expressions and to analyze the possibilities of applying current pulses in the circuits of arbitrary configuration located near to the conducting half-space, where eddy currents are induced. Among the standard pulses, the following ones are considered: exponentially decaying current pulse, pulse with the finite edge increasing speed represented by the difference between two exponents, decaying oscillating pulse.

2. Mathematical model

In [13] in general case in the dielectric half-space the following expressions are obtained for the complex magnitudes of magnetic $\dot{\boldsymbol{H}}_e$ and electrical $\dot{\boldsymbol{E}}_e$ fields of the system "current $\dot{I}_0$ flowing along the circuit l of arbitrary configuration in the dielectric half-space – conducting half-space in which induced eddy currents flow" (Fig.1):

$$\dot{\boldsymbol{H}}_e = \dot{\boldsymbol{H}}_0 + \dot{\boldsymbol{H}}_1 + \dot{\boldsymbol{H}}_2 = = -\frac{\dot{I}_0(i\omega)}{4\pi}\oint_l \left[\frac{\boldsymbol{t}\times\boldsymbol{r}}{r^3} - \frac{\boldsymbol{t}_1\times\boldsymbol{r}_1}{r_1^3} - \boldsymbol{t}_1\times\nabla\left(\frac{\partial \dot{G}_e(i\omega)}{\partial z}\right)\right] dl, \tag{1}$$

$$\dot{\boldsymbol{E}}_e = \dot{\boldsymbol{E}}_0 + \dot{\boldsymbol{E}}_1 + \dot{\boldsymbol{E}}_2 = = -\frac{\mu_0 i\omega \dot{I}_0(i\omega)}{4\pi}\oint_l \left(\frac{\boldsymbol{t}}{r} - \frac{\boldsymbol{t}_1}{r_1} - \boldsymbol{e}_z\times\left[\boldsymbol{t}_1\times\nabla \dot{G}_e(i\omega)\right]\right) dl. \tag{2}$$

where $\boldsymbol{e}_z$ is a unit vector in the axis z direction. In (1) and (2) two first components correspond to the field of a set current of the closed circuit l, which flows in a dielectric medium, and the field of the same circuit with the current reflecting from the flat boundary between the media. The sum of these two components fully determines the field at the ideal skin effect, for which the depth of field penetration is equal to zero ($\delta = \sqrt{2/\omega\mu\mu_0\gamma} \to 0$). If the condition of the ideal skin effect is not fulfilled, the vector field components $\dot{\boldsymbol{H}}_2 = \dot{\boldsymbol{H}}_2(i\omega)$ and $\dot{\boldsymbol{E}}_2 = \dot{\boldsymbol{E}}_2(i\omega)$ do not equal zero. Their frequency spectrum is different from the frequency spectrum of an output circuit. The values of the components of the vector fields in each point of the space depend not only on frequency and the electrophysical properties of the medium, but also on the reciprocal location of outflow points on the circuit and an observance point in the space, that is, on the coordinate difference $z+h$ along a vertical direction and ρ on the plain. A function being the component of expressions (1) and (2) looks like the following

$$\dot{G}_e(i\omega) = 2\int_0^\infty \frac{e^{-(z+h)\vartheta} J_0(\vartheta\rho)}{\vartheta + \sqrt{\vartheta^2 + i\omega\mu_0\mu\gamma}\big/\mu} d\vartheta, \tag{3}$$

where $J_0(\cdot)$ is the zero-order Bessel function of the first kind

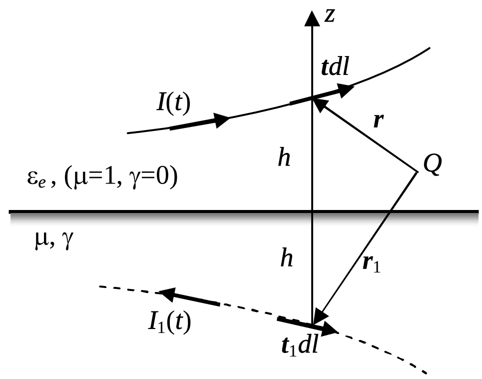

Fig. 1. Part of the circuit with current and its reflection from the surface.

Using expressions (1) and (2) for the frequency spectrum, the time dependencies of field intensity for current pulses can be found by performing the inverse Fourier transform:

$$\boldsymbol{H}_e(t) = -\frac{1}{4\pi}\oint_l \left[\left(\frac{\boldsymbol{t}\times\boldsymbol{r}}{r^3} - \frac{\boldsymbol{t}_1\times\boldsymbol{r}_1}{r_1^3}\right) I_0(t) - \boldsymbol{t}_1\times\nabla\left(\frac{\partial V_H(t)}{\partial z}\right)\right] dl, \tag{4}$$

$$\boldsymbol{E}_e(t) = -\frac{\mu_0}{4\pi}\oint_l \left[\left(\frac{\boldsymbol{t}}{r} - \frac{\boldsymbol{t}_1}{r_1}\right)\frac{\partial I_0(t)}{\partial t} - \boldsymbol{e}_z\times\left[\boldsymbol{t}_1\times\nabla V_E(t)\right]\right] dl. \tag{5}$$

Here functions $V_H(t)$ and $V_E(t)$ are

$$V_H(t) = \frac{2}{\pi}\int_0^\infty \cos(\omega t)\,\mathrm{Re}\left[\dot{I}_0(i\omega)\dot{G}_e(i\omega)\right] d\omega, \tag{6}$$

$$V_E(t) = \frac{2}{\pi}\int_0^\infty \cos(\omega t)\,\mathrm{Re}\left[i\omega\dot{I}_0(i\omega)\dot{G}_e(i\omega)\right] d\omega. \tag{7}$$

Formula (4) and (5) together with (3), (6) and (7) in general case provide the analytical expressions in the form of quadrature for the calculation of pulse electromagnetic fields. At the same time, the solution represented by the triple improper integral and, moreover, the presence of the complex magnitude of the current frequency spectrum implies an additional integral procedure of the direct Fourier transform.

The noted circumstances show that the simplification of calculations is an important task for the efficient use of the given analytical approach. Such a simplification can be realized with the use of an asymptotic expansion [14, 15], which is performed for the problem being considered in the case of a strong skin effect [16].

The expansion of the electromagnetic field into a limited asymptotic series for a certain frequency of the frequency spectrum is performed by a parameter $\varepsilon_1 = \frac{\mu\delta}{\sqrt{2}r_1} = \frac{\mu}{r_1\sqrt{\omega\mu\mu\gamma}} < 1$, which is proportional to the ratio between the field penetration depth δ and the distance r_1 between the observation point and the point of the outflow on the mirror reflected circuit. Therefore, this parameter shows the relation between the remoteness of the field sources and the penetration depth.

As the value of the parameter ε_1 depends on the frequency, the size limit of the small parameter is connected with the field frequency limit which means that the frequency must be bigger than some limit value

$$f > f_m = \frac{\mu}{2\pi\mu_0\gamma r_1^2\varepsilon_m^2}, \qquad (8)$$

where ε_m is the chosen permissible value of the small parameter.

Moreover, each member of the asymptotic series is determined with the error increasing with the increase in the number of a series member. This fact limits also the field frequency for each ***n-th*** series member

$$f > f_n = \frac{\mu}{2\pi\mu_0\gamma r_1^2\varepsilon_n^2}, \qquad (9)$$

where ε_n is determined by the permissible relative error Δ_n of taking into account the series member.

For low frequencies, the condition of infinitesimality of parameters ε_m and ε_n is not fulfilled. Taking into account the fact, that the frequency spectrum of the current pulse contains the whole frequency spectrum, we can state that the condition of infinitesimality is not fulfilled for the low frequency part of the spectrum. It means that the calculations can be performed on the limited time interval instead of the whole time period. This time interval can be estimated for the possibility of the asymptotic expansion as $t < 1/f_m$ and as $t < 1/f_n$ for each series member [13].

The expansion of the function $\dot{G}_e(i\omega)$ (3) into small asymptotic series and performing the inverse Fourier transform for each member of the series enables the considerable reduction in the amount of calculations. It allows the use of line integrals over the length of closed contour instead of triple integrals. Proper expressions for calculating the intensity of magnetic and electrical fields in the dielectric medium were obtained in [14]. Their important feature is that the dependences of each member of the series on coordinates and time are determined separately. Functions $V_H(t)$ and $V_E(t)$ in (4)–(7) are as follows:

$$V_H(t) = \sum_{n=0}^{N} V_{Hn}(t) = \sum_{n=0}^{N} \frac{n+1}{2\Gamma((n+3)/2)} g_n(\mu, \gamma, \boldsymbol{r}_1) P_n(t), \qquad (10)$$

$$V_E(t) = \sum_{n=0}^{N} V_{En}(t) = \sum_{n=0}^{N} \frac{1}{2\Gamma((n+1)/2)} g_n(\mu, \gamma, \boldsymbol{r}_1) Q_n(t), \qquad (11)$$

where $\Gamma(\cdot)$ is a gamma function.

Factors determining the dependence of the electromagnetic field on the coordinates of space points are

$$g_n = g_n(\mu, \gamma, \boldsymbol{r}_1) = (-1)^n 2a_n(\mu)\left(\frac{\mu}{\sqrt{\mu_0\mu\gamma}}\right)^{n+1} \frac{\partial^{(n)}}{\partial z^n}\left(\frac{1}{r_1}\right), \qquad (12)$$

where $a_n(\mu)$ are the coefficients of the power series expansion over the small parameter of the inverse value of a denominator of an integrand in (3). Factors determining the dependence on time are a bit different for the intensity of magnetic and electrical fields:

$$P_n(t) = \int_0^t (t-\tau)^{(n-1)/2} I_0(\tau) d\tau, \qquad (13)$$

$$Q_n(t) = \int_0^t \left[\left.\frac{dI_0(\varsigma)}{d\varsigma}\right|_{\varsigma=t-\tau} \tau^{(n-1)/2}\right] d\tau. \qquad (14)$$

The last circumstance, which should be taken into account while applying the approximate method of calculation with the use of asymptotic expansion, refers to the fact that in this method the low-frequency part of signal spectrum is not considered. If the frequency spectrum of the current pulse $\dot{I}_0(i2\pi f)$ is set, the ratio of root-mean-square values of the magnitude of the frequency spectrum S_f, that is, the energy datum of the spectrum being considered, can be used for qualitative estimation of the extent of considering the spectrum:

$$S_f = \sqrt{\int_{f_{\min}}^{\infty} \left|\dot{I}(i2\pi f)\right|^2 df} \Bigg/ \sqrt{\int_0^{\infty} \left|\dot{I}(i2\pi f)\right|^2 df}, \qquad (15)$$

where $f_{\min}$ either becomes f_m referring to the chosen permissible value of the small parameter ε_m, or takes one of the values of minimal frequency f_n for the members of asymptotic series.

3. Asymptotic series for the electromagnetic field of the standard current pulses

The obtained solutions to either general problem of calculating the electromagnetic field, or approximate solution on the basis of the method of asymptotic expansion allow the use of them for arbitrary time dependencies of the circuit currents. However, during the investigations the mathematical models for "standard" current pulses are used. These "standard" pulses are considered to be the following: an exponentially decaying current pulse; pulse represented by the difference between two decaying exponents, exponentially decaying oscillating pulse. For such current pulses of the circuit, let us analyze the major features of the use of approximate analytical approach to the calculation of the electromagnetic field.

Time dependencies of the current can be conveniently analyzed with the use of dimensionless values of frequency $f^* = f/f_b$ and time $t^* = t/t_b$. The basic value of the frequency f_b and, correspondingly, time $t_b = 1/f_b$ is to be linked with the penetration depth. It means that the chosen value of the basic frequency should be such, that the penetration depth in the conducting medium is equal to the characteristic dimension of the electromagnetic system which is the distance of the circuit element with current to the boundary of media division $f_b = \left(\pi h^2 \mu\mu_0 \gamma\right)^{-1}$.

3.1. Exponentially decaying current pulse

The edge interval of the exponentially decaying current pulse of the output circuit is equal to zero. In this case, the time dependence is determined only by the value of a decay coefficient α:

$$I(t) = I_m \cdot I^*(t), \quad I^*(t) = \exp(-\alpha t) = \exp(-\alpha^* t^*), \quad (16)$$

where $\alpha^* = \alpha / f_b$, $t^* = t f_b$.

The frequency spectrum of the current pulse (16) is known [18]:

$$\dot{I}^*(i\omega) = \frac{1}{\alpha + i\omega} = \frac{1}{f_b} \frac{1}{\alpha^* + i\omega^*}, \quad (17)$$

where $\omega^* = 2\pi f^*$.

From (15) it follows that the value of the extent of taking into account the signal spectrum, while the approximate calculation method is being used for the exponentially decaying function, is determined by an analytical expression:

$$S_f = \sqrt{1 - \frac{2}{\pi} \operatorname{arctg} \frac{2\pi f^*_{\min}}{\alpha^*}}. \quad (18)$$

As it can be seen, the indexes of using the spectrum for different $f_{\min}$ depend on the decay coefficient. In Fig. 2, the dependencies of value S_f on normalized quantity α^* are shown.

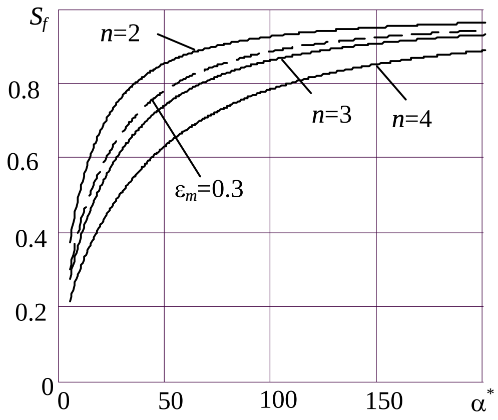

Fig. 2. Index S_f for a pulse $I^*(t^*) = \exp(-\alpha^* t^*)$ at $\Delta_n = 1$.

The figure shows that for more speedy exponential decay of the current pulse the index of taking into account its spectrum increases. A dashed curve illustrates a value S_f realized when the calculations are limited by time t_m and corresponds to the boundary value $\varepsilon_m = 0{,}3$. At this time moment the series members at $n \geq 3$ should be eliminated from the calculations, and overall accuracy is determined by the left members.

As the pulse (16) increases by momentum discontinuity to the maximum value, it is impossible to calculate the electrical field intensity for it, since the presence of a time derivative in (5) causes unlimited big values. Although, it does not concern the magnetic field intensity. In this case, in (10) time integrals (13) are tabulated ones [12], and functions $P_n(t)$ are represented as

$$P_n(t) = \int_0^t (t-\tau)^{(n-1)/2} I_0(\tau) d\tau = I_m e^{-\alpha t} \left[(-\alpha)^{-(n+1)/2} \gamma\left(\frac{n+1}{2}, -\alpha t\right) \right], \quad (19)$$

where $\gamma(\beta_1, \beta_2)$ is an incomplete gamma function.

The product in brackets can be shown as a series, and in this case, the following expression can be used instead the special functions:

$$P_n(t) = I_m e^{-\alpha_1 t} t^{(n+1)/2} \sum_{k=0}^{\infty} \frac{(\alpha t)^k}{k! \left(k + \dfrac{n+1}{2}\right)}. \quad (20)$$

3.2. Current pulse represented by the difference between two decaying exponents

The pulses being considered are characterized by the final value of the edge duration of the output current pulse of the circuit (Fig. 3, *a*). The law of time changes of such a pulse can be described by the following dependency:

$$I(t) = I_m \cdot I^*(t), \quad I^*(t) = I_1^*(t^*) - I_2^*(t^*) = \\ = I_m^*\left[\exp(-\alpha_1^* t^*) - \exp(-\alpha_2^* t^*)\right]. \quad (21)$$

Here maximum value of I^* equals one, and so the dimensionless quantity I_m^* which depends on the values of decay coefficients α_1^* and α_2^* has been left. It is also assumed that $\alpha_1^* < \alpha_2^*$.

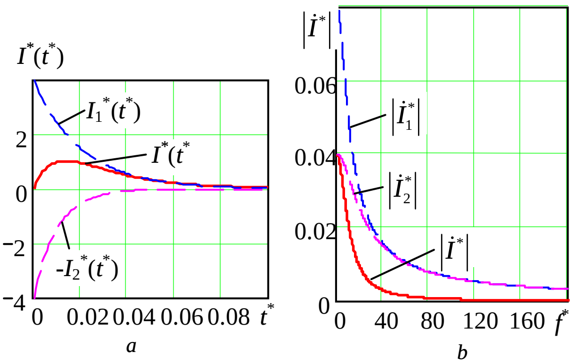

Fig. 3. Current pulse $I^*(t^*) = 4\left[exp(-50\,t^*) - exp(-100\,t^*)\right]$

a – time dependencies; b – amplitude-frequency responses.

Fig. 4 illustrates the extent of considering the frequency spectrum of the pulse (21) depending on the decay coefficient α_1^*, which in the first place determines the whole pulse duration at different values of α_2^* affecting the pulse edge duration. The calculations have not been performed for each series member, as in Fig. 2, but for limited whole pulse duration $t_m^* = 0{,}18$, which is determined by the chosen value $\varepsilon_m = 0{,}3$.

Taking into account the final duration of the pulse edge, the more the extent of the consideration of its spectrum decreases, the longer the pulse edge comparing to the duration of the whole pulse is. This feature can be explained by the fact that the frequency spectrum of the pulse contains more low frequencies than the previous pulse. For example, the current pulse $I^*(t^*) = = 4\left[exp(-50t^*) - exp(-100t^*)\right]$, shown in Fig. 3, *a*, has the frequency spectrum with the high frequency range being considerably narrower than for each component of the whole pulse (Fig. 3, *b*).

It should be mentioned that in the case of relatively small values of the decay coefficient α_2^*, the index S_f considerably differentiates from one. Therefore, the application of the method of asymptotic expansion is possible, but only by the time, when current value cannot still be considered small. Although, here, it is possible to take into account not the whole of the current pulse, but only some of its properties, for example, reaching the maximum value by its electromagnetic field.

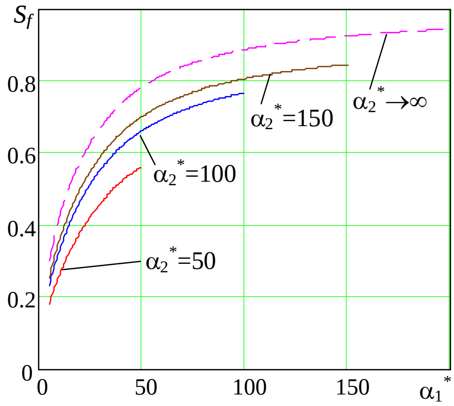

Fig. 4. Index of considering the pulse spectrum $I^*(t^*) = I_m^*\left[\exp(-\alpha_1^* t^*) - \exp(-\alpha_2^* t^*)\right]$ *at* $\varepsilon_m = 0{,}3$.

The pulse (21) is a sum of two exponents, so the expressions for the intensity of the magnetic field are also determined by the sum of two components, whose time depending functions are similar to those noted in (19) and (20).

Since the current pulse (21), as opposed to (16), does not change by a step at the initial moment, the calculation of the intensity of the electrical field for such a pulse does not cause the peculiarity at $t \to 0$.

For this case the expressions for determining the intensity of electromagnetic field are introduced. In expression (11) functions $Q_n(t)$ (14) can be shown as follows:

$$Q_n(t) = -I_m \begin{bmatrix} e^{-\alpha_2 t}(-\alpha_2)^{-(n+1)/2}\gamma\left(\dfrac{n+1}{2}, -\alpha_2 t\right) - \\ -e^{-\alpha_1 t}(-\alpha_1)^{-(n+1)/2}\gamma\left(\dfrac{n+1}{2}, -\alpha_1 t\right) \end{bmatrix}. \quad (22)$$

Instead of special functions intended for the analysis, their representation as a power series can be useful. For this (22) can be used in a following form:

$$Q_n(t) = -I_m t^{(n+1)/2} \begin{bmatrix} \alpha_2 e^{-\alpha_2 t} \displaystyle\sum_{k=0}^{\infty} \frac{(\alpha_2 t)^k}{k!\left(k + \dfrac{n+1}{2}\right)} - \\ -\alpha_1 e^{-\alpha_1 t} \displaystyle\sum_{k=0}^{\infty} \frac{(\alpha_1 t)^k}{k!\left(k + \dfrac{n+1}{2}\right)} \end{bmatrix}. \quad (23)$$

3.3. Decaying oscillating pulse

Let us consider the standard current pulse (Fig. 5) which depending on time changes according to the law:

$$I_0(t) = I_m I^*(t), \; I^*(t) = \exp(-\alpha t)\sin\beta t = \exp(-\alpha^* t^*)\sin\beta^* t^*, \quad (24)$$

де $\alpha^* = \alpha / f_b$, $\beta^* = \beta / f_b$.

The expression for the frequency spectrum for normalized values is:

$$\dot{I}^*(i\omega)=\frac{\beta}{(\alpha+i\omega)^2+\beta^2}=\frac{1}{f_b}\frac{\beta^*}{\left(\alpha^*+i\omega^*\right)^2+\left(\beta^*\right)^2}. \quad (25)$$

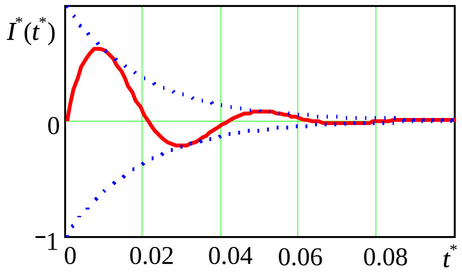

Fig. 5. Decaying oscillating current pulse.

At the initial moment, the current step is absent. Therefore, for such a pulse, the results of calculating the intensity of either magnetic, or electrical field will be correct.

For determining the values of integral index S_f taking into account the limitation of the frequency spectrum of the signal, we can chose, as previously, the variant with the biggest error at the expansion into the asymptotic series, when $r_1 = h$. The values of the small parameter are chosen $\varepsilon_m = 0.3$ and $\mu = 1$.

The dependencies of the index S_f on the decay coefficient α^* at different values of the ratio of oscillation frequency to the decay coefficient β/α are shown in Fig. 6.

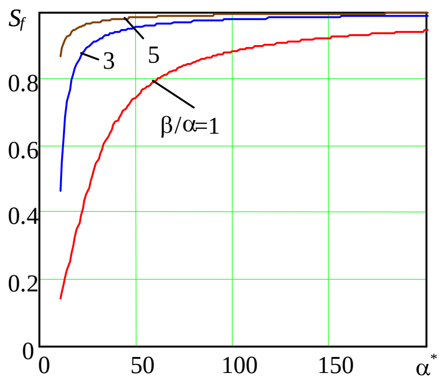

Fig. 6. Index of spectrum taken into account in the case of decaying oscillating pulse ($\varepsilon_m = 0.3$, $r_1 = h$, $\mu = 1$).

As the dependencies in Fig. 6 show, the more the rate β/α is, the more the index of the extent of using the spectrum S_f is. For values $\beta/\alpha < 1$, the pulse decays completely during the oscillation period. At $\beta/\alpha > 1$, the value of the index S_f proves to be bigger, than of that without current oscillations. It can be explained by the fact that in this case the frequency spectrum contains a wider high frequency range comparing with the exponentially decaying pulse.

For the pulse shape (24) the analytical representation of time integrals is also possible. Here, it is convenient to use a complex symbolic form for describing the pulse (24)

$$I_0(t) = I_m e^{-\alpha t}\sin\beta t \div I_m e^{-(\alpha - i\beta)t}. \quad (26)$$

In this case, time dependencies $P_n(t)$ (13) of the function $V_H(t)$ (10) determining the intensity of the magnetic field can be found as an imaginary part of appropriate complex expressions $P_n = \mathrm{Im}\left(\tilde{P}_n\right)$:

$$P_n(t) = \mathrm{Im}\left\{I_m e^{-\eta t}\left[(-\eta)^{-(n+1)/2}\gamma\left(\frac{n+1}{2}, -\eta t\right)\right]\right\}, \quad (27)$$

where $\eta = \alpha - i\beta$.

As in (20), the last expression can be represented as a series:

$$P_n(t) = I_m t^{(n+1)/2} e^{-\alpha t}\sum_{k=0}^{\infty}\frac{t^k|\eta|\sin(\beta t - k\,\mathrm{arctg}\,\beta/\alpha)}{k!\left(k+\dfrac{n+1}{2}\right)}. \quad (28)$$

At last, let us find time dependencies $Q_n(t)$ (14) for the series members of the function $V_E(t)$ (11) determining the intensity of the electrical field. Repeating the transforms performed for obtaining expression (22) for real variables and using the complex value $\eta = \alpha - i\beta$, we can find with the use of special functions

$$Q_n(t) = \mathrm{Im}\left\{-I_m \eta e^{-\eta t}\left[(-\eta)^{-(n+1)/2}\gamma\left(\frac{n+1}{2}, -\eta t\right)\right]\right\}, \quad (29)$$

or as a series

$$Q_n(t) = -I_m t^{(n+1)/2} e^{-\alpha t}\sum_{k=0}^{\infty}\frac{t^k|\eta|^{k+1}\sin(\beta t - (k+1)\mathrm{arctg}\,\beta/\alpha)}{k!\left(k+\dfrac{n+1}{2}\right)}. \quad (30)$$

Finally, let us note that the calculations of time integrals in (13) and (14) are not difficult, regardless of possible peculiarities of subintegral functions. Although, the advantage of analytical approaches allows the thorough analysis of the influence of different factors on the results obtained after the distribution of the electromagnetic field, as well as promotes the well-grounded development of mathematical models for investigating the complex pulse electromagnetic systems.

4. Conclusions

1. Analytical solving of the general task of the calculations of three-dimensional quasi-stationary electromagnetic field of the pulse current flowing near to the conducting object with flat surface needs a great amount of calculations, since it uses the inverse Fourier transform, and obtained expressions need determination of triple improper integrals. Therefore, expedient approach under the condition of strong skin effect is the application of the developed method of asymptotic expansion for pulse processes. In spite of the fact that the approximate asymptotic solution is true only in the limited time interval from the beginning of the pulse, as a rule just during this time interval the field changes the most quickly and reaches the maximum values.

2. The expressions for the intensities of the electromagnetic field in the asymptotic approximation enable taking into account the influence of geometric factors and pulse time dependencies for each series member separately, which considerably simplifies the calculations. For the analysis of the pulse electromagnetic processes, it is advisable to obtain results for standard current pulses, which are realized in many concrete cases. Appropriate dependencies with the use of special functions and their representation as series are obtained for the exponentially decaying pulse; for the pulse represented by the difference between two decaying exponents, exponentially decaying oscillating pulse.

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АНАЛІТИЧНИЙ МЕТОД ВИЗНАЧЕННЯ ЕЛЕКТРОМАГНІТНОГО ПОЛЯ СТАНДАРТНИХ ІМПУЛЬСІВ СТРУМУ, ЯКИЙ ПРОХОДИТЬ ПОБЛИЗУ ЕЛЕКТРОПРОВІДНОГО ТІЛА

Юрій Васецький

Представлено математичну модель, яка основана на розвинутій теорії аналітичного рішення тривимірних квазістаціонарних задач імпульсного струму, який проходить поблизу електропровідного тіла з плоскою поверхнею. Застосовувана математична модель передбачає наближене розв’язання з використанням асимптотичного розкладання для розрахунку напруженостей магнітного і електричного полів у випадку швидкоплинних електромагнітних процесів. Відзначено, що розрахунок наближеним методом обмежений певним проміжком часу від початку імпульсу, однак, зазвичай саме протягом цього проміжку часу поле змінюється найшвидше і досягає максимальних значень. Розглянуто електромагнітне поле при проходженні стандартних імпульсів струму: експоненціально загасаючий імпульс; імпульс, представлений різницею двох згасаючих експонент; коливальний експоненціально згасаючий імпульс. Для них проаналізовано основні особливості застосування наближеного аналітичного методу розрахунку поля. Знайдено інтегральні показники для врахування обмежень по частоті і часу залежно від параметрів імпульсів. Залежності від часу отримано з використанням спеціальних функцій та їхнього представлення у вигляді рядів.

Yuriy Vasetsky. *Education and title*: Moscow Power Engineering Institute (Technical University) (1973), Institute of Electrodynamics, Ph. D. Degree (1981); D. Sc. Degree (1995) in Theoretical Electrical Engineering; the title of Professor (2007).

Professional Affiliation: Leading research scientist of Institute of Electrodynamics, National Academy of Sciences of Ukraine; Prof. National Aviation University, Department of General and Applied Physics.

Author of more than 160 scientific papers in the fields of electrodynamics and electrophysics of electromagnetic systems. Author of three books and a manual on electrodynamics for theory, analytical and asymptotic methods in electrodynamics problems. The results and methods are used for the development of technological equipment, high current magnetic systems, high voltage equipment.