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DYNAMIC PROPERTIES OF TRANSDUCERS TESTING USING WHITE NOISE EXCITATION. PART 1: TIME DOMAIN

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This article deals with experimental non-parametric methods of parameters qualifying dynamic properties of transducers of voltage/voltage type using external stochastic input function (white noise) and cross-correlation function.

Introduction. Experimental non-parametric methods of testing dynamic properties of transducers using external input function taking into consideration the type of applied testing signals can be divided into:

a) deterministic - in which the input function are determined testing signals;

b) probabilistic – in which stationary random signals are the input function (usually white noise).

In each of previously mentioned methods of testing one can operate, reckoning and analysing transducer's response in the time or frequency domain. Appling this criterium a more detailed classification of methods can be produced, in which the basis of reckoning the parameters determining dynamic properties of transducers is constituted by [1-6, 8-13]:

a1) the response to step function (the most common), impulse function or velocity rush;

a2) frequency characteristics;

b1) cross-correlation function of intput and output signals;

b2) power spectral density function of output signal or cross-spectral density function of intput and output signals.

In this work it is assumed that the input signal is white noise type $N(0,\sigma_x)$, its autospectrum is uniform and wide enough in comparison to examined frequency band system and the system itself is stationary and linear. To compute the cross-correlation function Brüel & Kjær 3550 [9] digital signal analyser has been applied. In figure 1 there is a block diagram of a measurement system.

Determination of transducers dynamics parameters on the basis of the course of cross-correlation function. Cross-correlation function $R_{xy}(\tau)$ is connected with the impulse response of system k(t) by dependence [1–4]:

$$R_{xy}(\tau) = \int_0^\infty k(t) R_x(t-\tau) dt, \qquad (1)$$

where: $R_x(\tau)$ - autocorrelation function of intput signal. For the white noise characterised by constant power spectral density function $G_x(\omega) = a = \text{const}$, $R_x(\tau) = a\delta(\tau)$ and the dependence (1) can be written:

$$R_{xy}(\tau) = ak(\tau). \qquad (2)$$

When occur internal noise of examined object (noncorrelated with input signal), can be shown that equation (2) takes the form [4]:

$$R_{xy}(\tau) = ak(\tau) + c, \qquad (3)$$

where c – constant.

Constant c can be easly separate from course $R_{xy}(\tau)$.

2.1. Inertial transducer of the 1st order

Impulse response of inertial transducer of the 1st order for $x(t) = A\delta(t)$ is given by:

$$k(t) = \frac{kA}{T} \cdot e^{-\frac{t}{T}}, \qquad (4)$$

where: A = const – intput impulse surface, T – time-constant of transducer.

The typical course of cross-correlation function $R_{xy}(\tau)$ determined for the 1[st] order transducer model is shown in figure 2. After the substitution (4) in dependence (2) the system of equations (5) can be written for two points of characteristic $R_{xy}(\tau)$ of coordinates $[\tau_1, R_{xy}(\tau_1)]$, $[\tau_2, R_{xy}(\tau_2)]$:

$$\begin{cases} R_{xy}(\tau_1) = a\dfrac{kA}{T}\cdot e^{-\frac{\tau_1}{T}} \\ R_{xy}(\tau_2) = a\dfrac{kA}{T}\cdot e^{-\frac{\tau_2}{T}} \end{cases} . \tag{5}$$

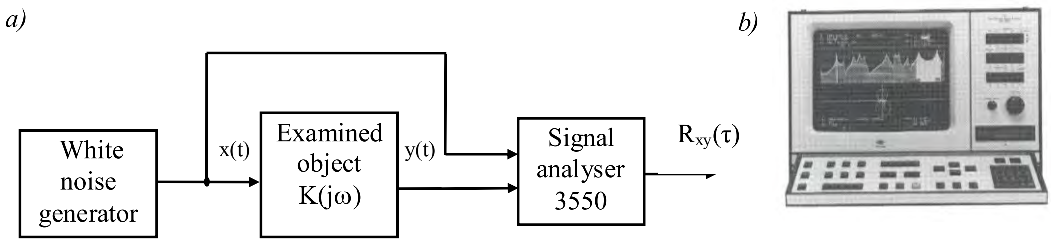

Fig. 1. A block diagram of a measurement system (a), digital signal analyser Brüel & Kjær 3550 (b)

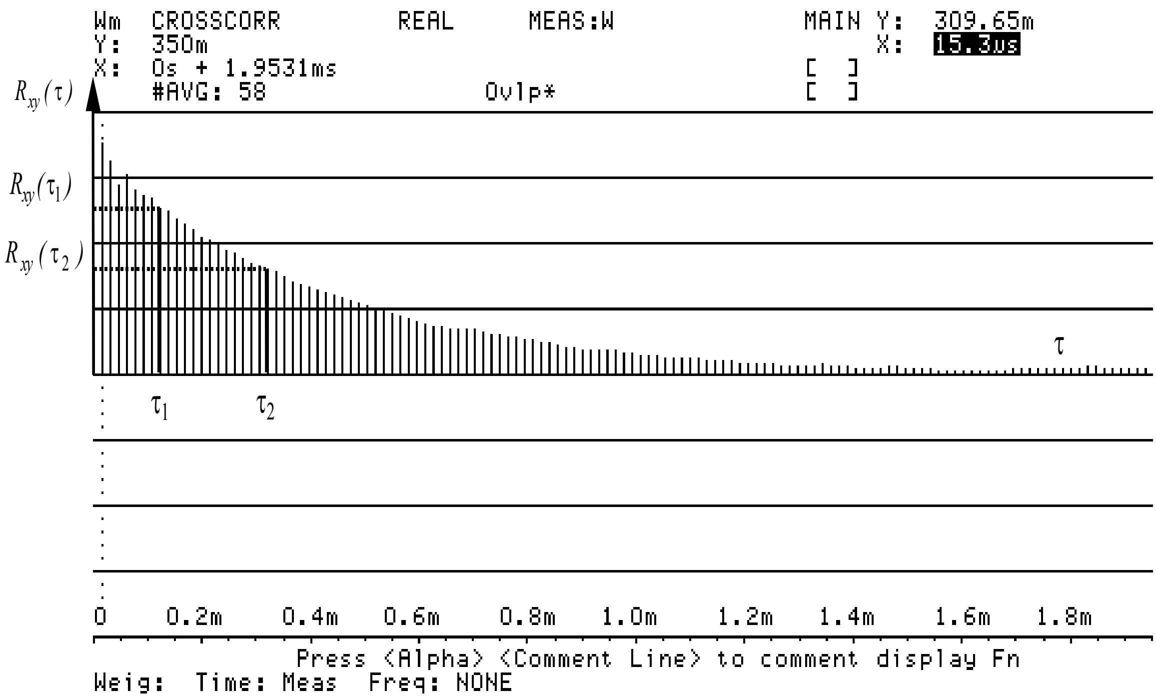

Fig. 2. Cross-correlation function $R_{xy}(\tau)$ of input and output signal for the 1[st] order transducer model

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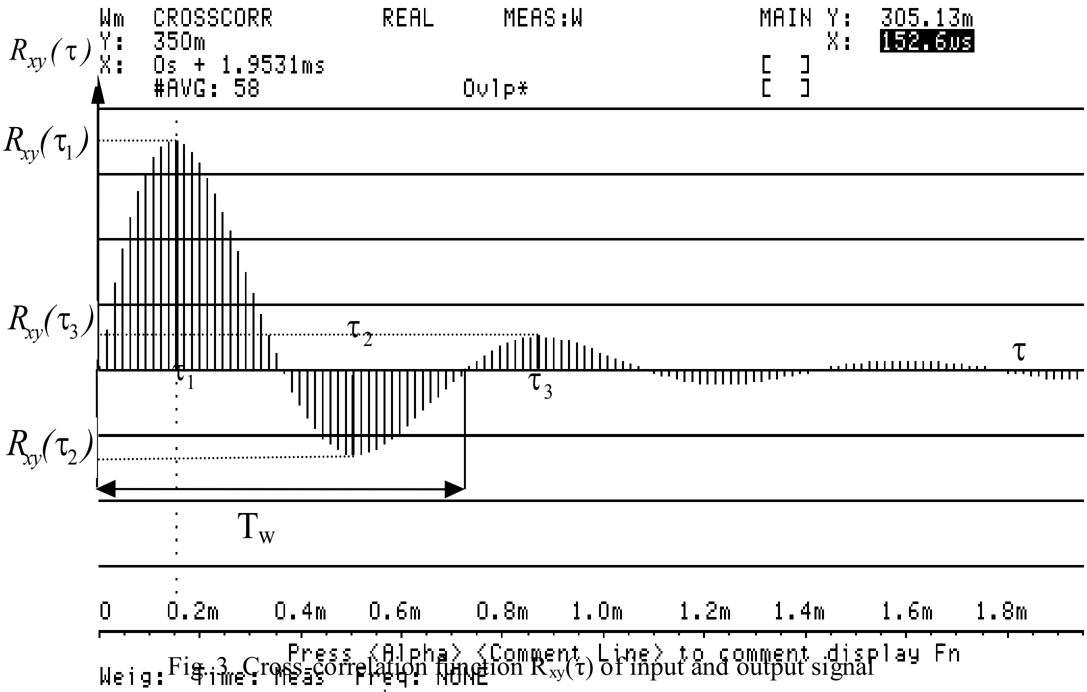

Fig. 3. Cross-correlation function $R_{xy}(\tau)$ of input and output signal for the 2nd order oscillatory transducer model

After dividing of both sides and transformating the equations (5) there appears the dependence which allows to determine time-constant of transducer:

$$T = \frac{\tau_2 - \tau_1}{\ln\left(\dfrac{R_{xy}(\tau_1)}{R_{xy}(\tau_2)}\right)}. \qquad .(6)$$

In table 1 there is the comparison of time-constant achieved as an outcome of examining of two transducer models of the 1st order appling step input function and white noise excitation (mean values of 10 independent runs). In the case of step function the time-constant is obtained by the course of step response [7].

Table 1

Time-constant comparison of two transducers models of the 1st order

Parameter	Transducer No	Step excitation	Noise excitation
T [μs]	1	480	451
	2	102	103

2.2. Oscillatory transducer of the 2nd order

There are two parameters defining the dynamics of the 2nd order oscillatory transducer and they are: the damping ratio ξ and the undamped natural pulsation ω_0. The first of these values is calculated on the basis of the analysis of the course of the impulse response of the system from dependence [4]:

$$\xi = \frac{\ln\left|\dfrac{A_n}{A_{n+1}}\right|}{\sqrt{\pi^2 + \ln^2\left|\dfrac{A_n}{A_{n+1}}\right|}}, \qquad (7)$$

where: A_n, A_{n+1} – amplitudes of following oscilations in response course k(t).

The typical course of cross-correlation function $R_{xy}(\tau)$, obtained for oscillatory transducer model is shown in figure 3. Taking into account dependence (2), the equalization (7) can be shown in the following way:

$$\xi = \frac{\ln\left|\dfrac{R_{xy}(\tau_1)}{R_{xy}(\tau_2)}\right|}{\sqrt{\pi^2 + \ln^2\left|\dfrac{R_{xy}(\tau_1)}{R_{xy}(\tau_2)}\right|}}. \qquad (8)$$

where: $R_{xy}(\tau_1)$, $R_{xy}(\tau_2)$ – amplitudes of the 1st and 2nd oscillation in the course of cross-correlation function (figure 3).

The natural pulsation ω_0 can be obtained on the basis of measurement of period T_w of the course $R_{xy}(\tau)$ from equation:

$$\omega_0 = \frac{2\pi}{T_w\sqrt{1-\xi^2}}. \qquad (9)$$

In table 2 there is the comparison of damping ratio and natural pulsation achieved as an outcome of examination of two 2nd order transducer models applying step and noise excitation. In the case of step input function ξ and ω_0 are calculated on the basis of measurements of parameters of step responses.

Table 2

Comparison of parameters ξ and ω_0 for two transducers models of the 2nd order

Parameter	Transducer No	Step excitation	Noise excitation
ξ [-]	1	0,539	0,548
	2	0,294	0,291
ω_0 [rad/s]	1	8778	8714
	2	9005	8968

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