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DISKRETE-CONTINUUM MODELING OF LEGS EXTERNAL OSTEOSYNTHESIS MECHANICS

У роботі розглядаються ряди числових схем для обчислення складних конструкцій фіксаторів кісток. Комплекс числових схем забезпечує дискретно безперервний тип обчислень, який надає можливість в адаптивному режимі обчислювати навантаження не лише в елементах конструкції фіксатора, але і в місцях з'єднання фіксатора з кісткою. Також розглядаються числові схеми для дослідження елементів з'єднувачів, які також будуються на основі кінематичних гіпотез. Це дослідження розвиває сучасну методологію прогнозування і контролю, засновану на теорії комплексного континіуму.

Ключові слова – фіксатор кісток, моделювання, оптимізація

The numerical schemes (NS) row is considered for complex fixer-bone constructions. Complex NS are provided of discretely-continual type that enables in the adaptive mode to calculate tension not only in the fixer construction elements, but in places of most their concentration –in fixer-bone joints. It is considered also numerical schemes for research of joint elements that also are got on the basis of kinematics hypotheses. Present research develops a modern prediction and control methodology, based on complex continuum theory.

Keywords – fixer-bone constructions, modeling, optimization

Introduction

The development of models for flexible multibody systems aims at an accurate description of deformations and high accuracy as well as efficiency of the used discretization method. The discrete models are totally inadequate to calculate the natural frequencies of vibration of the complicated machines constructions with accuracy and therefore, for a sufficiently accurate determination of its dimensional characteristics so as to determine such frequencies. It is therefore necessary in practice to dimension these constructions through more complex modeling [1-5]. In particular, concentrated mass and rigidity calculation methods may be adopted based on an even more accurate theoretical determination. The numerical schemes (NS) row is considered for such complex vibroexitated constructions. Methods of decomposition and the NS synthesis are considered on the basis of new methods of modal synthesis [1-5]. Complex NS are provided of discretely-continual type that enables in the adaptive mode to calculate tension not only in the construction elements, but in places of most their concentration –in joints. It is considered also numerical schemes for research of joint elements that also are got on the basis of kinematics hypotheses. On the basis of simple and more complex NS research of local tensions on verge of stratified structure at the different kinds of its fixing is conducted. Traditional design methodology, based on discontinuous models of structures and machines is not effective for high frequency vibration. Present research develops a modern prediction and control methodology, based on complex continuum theory and application of special frequency characteristics of structures. Complex continuum theory allows to take into consideration system anisotropy, supporting structure strain effect on equipment motions and to determine some new effects that are not described by ordinary mechanics of the continuum theory.

Discrete-continuum modeling Let us consider the following discrete-continuum scheme for legs external osteosynthesis (Fig.1)

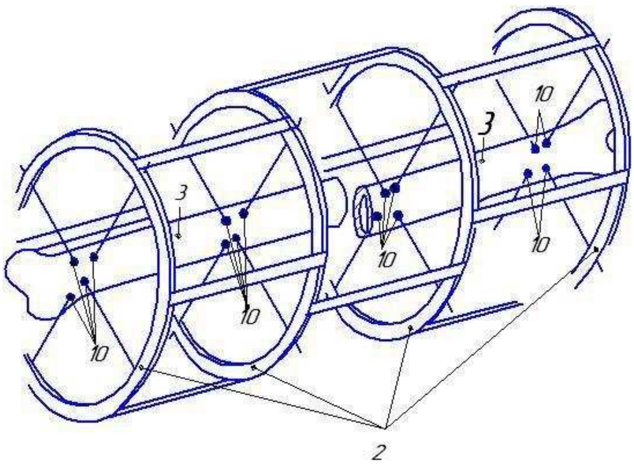

Fig.1. Scheme for legs external osteosynthesis

Problem of f complicated designs deformation and strain is considered for the purports of complex analysis. The problem is solved on the basis of four modified methods of modal synthesis. The basis of these methods is in deriving solving set of equations in a normal form at minimum application of matrix operations. The essence of the first method consists in reviewing knots of junctions (bone-needles) as compact elastic elements for which inertial properties are taken into account without reviewing their strain, and massive connected parts - as deformable elements, their inertion being taken into account on the basis of modal expansion. The second one consists in choice of a base extended element and alignment of the elements joined to it with the help of a special choice of their coordinate functions. The third - in the combined choice of a system of coordinate functions for a knot of junction which after matrix transformations without application of global operations allows to receive solving set of equations of a dynamic equilibrium in a normal form. The fourth - in reviewing junctions as elastic extended elements, which are modeled by the specified equations in view of shift, compression and modifications of the form during a strain. Some aspects of this common numerical scheme are discussed near,

Bone material properties

The type of material symmetry possessed by bone tissue is often ambiguous. Within the accuracy we have to measure the elastic constants of bone, a given specimen may be represented as, say, orthotropic. However, the orthotropy may not be of high degree in all directions and the specimen may be represented, for all practical purposes, as transversely isotropic or, even, isotropic. The question addressed, and rigorously answered, in this work is the following: Given a set of orthotropic elastic constants for a specific specimen, what bounds do these elastic constants impose on the effective transversely isotropic and the effective isotropic elastic constants of that specimen? In the cases considered thus far for bone, the bounds have been so tight that the effective values of the elastic constants of the higher symmetries have been determined from those of the lower symmetry, as will be illustrated below. This work is undertaken to build a greater database for the transversely isotropic elastic constants of bone. The longer-term project is to assemble the numerical values of parameters for a transversely isotropic model of bone poroelasticity. A set of numerical values of parameters for an isotropic model of bone poroelasticity is given in [6]. In part I various displacement models have been developed by considering combinations of displacement fields for in-plane and transverse displacements inside a mathematical sub-layer to investigate the phenomenon of vibrations in laminated composite plates. Numerical evaluations obtained for vibrations in isotropic, orthotropic and composite laminated plates have been used to determine the efficient displacement field for economic analysis of wave propagation and vibrations in laminated composite plate. A number of nondestructive evaluation techniques have been proposed for the determination of material properties of laminated composite parts [1-5]. A number of researchers used experimental eigenfrequencies to identify elastic constants of laminated composites. In [7,8] a two-step method to identify the elastic constants of orthotropic materials is developed.

Experimental design

Fig. 2 shows the experimental set up for the vibration measurement in the bone material beam system.

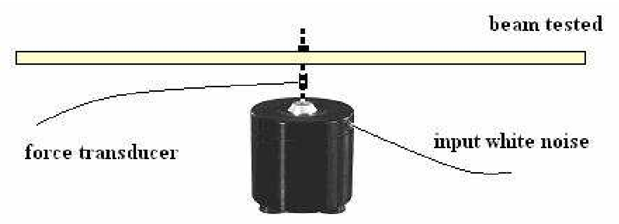

Fig 2. Experimental set up for vibration measurements of sandwich beam

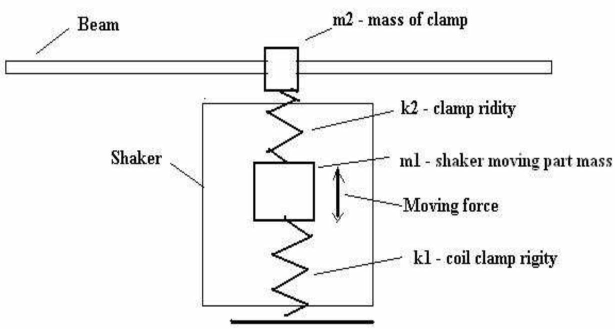

Fig. 3. Mechanical scheme for vibration measurements of sandwich beam

The Microflown probe and laser probe are both used for non-contacting testing. If placed close to the beam, the Microflown devise should measure beam velocity, although the magnitude may not be quite correct and will partly depend on the distance of the probe from the beam. Fig.3 shows the discrete-continuum scheme of beam – shaker system. In the experiment the beam was excited with white noise by a shaker mounted at the middle. The experimental Bruel&Kjaer PULSE system analyzes the signals using dual FFT.

Theoretical-experimental comparison

The range of numerical experiments must be done to be sure that our theoretical approach is correct. The FRF for this beams are presented in Fig.4 obtained by various range of approximations for various frequency domains (various beam length). The discrete-continual numerical scheme based on Fig. 3 is applied.

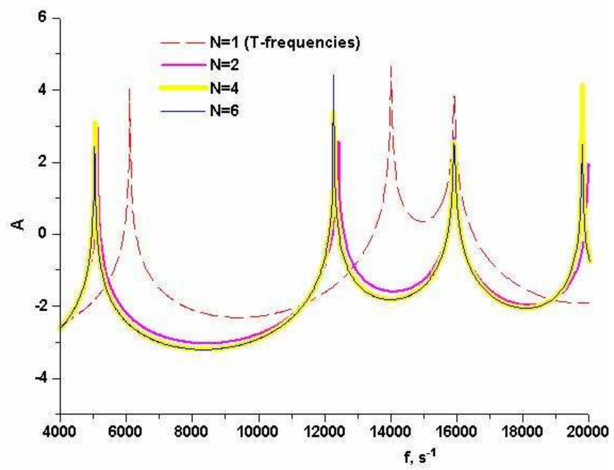

Fig.4. Model depended FRF (length of beam L=0.06m)

Elastic modules identification

A detailed sensitivity study has shown that the solution of the one level optimization problem is very sensitive to the variations of one group of modules match less sensitive to another modules. Since the existence of noise in measurement data is inevitable, the above formulation of the material constants identification problem then becomes inappropriate if the determination of accurate elastic constants is desired. This may produce erroneous results even when the variations of the measured frequencies are relatively small. The two or more level identification method is proposed. It contains two or more and various minimization technique for every step. The first level optimization problem is based on genetic algorithms and includes maximum set of parameters: elastic modules, unknown explicitly geometrical and mechanical parameter of sample joint. Again the above second level minimization problem is solved using the nonlinear optimization technique submerged to a set of parameters that less influence the frequencies.

A possible solution is the application of an optimization strategy to determine the specific parametric values that best fit the. In general, a parameter estimation method is an iterative process that takes a known input u(t) of a system, evaluates it into a mathematical model with starting sets of parameter values, compares the result with the optimization function (thus obtaining a value of error between them), and then it modifies the values of the model parameters, expecting the new set to get closer to the optimum solution. The complexity and high dimensionality of some models lead to the use of a heuristic search method. In this matter, Genetic Algorithms has proven to be a suitable optimization tool for a wide selection of problems. The general form of optimization function is

$$F_C = \sum_i^N C_i\left(f_i^{\exp}, f_i^c\right) + \sum_j^M D_j\left(a_j^{\exp}, a_j^c\right) + \sum_{i,j} M_j\left(E_{ij}^{\exp}, E_{ij}^c\right) + \sum_k^K G_k\left(g_k^{\exp}, g_k^c\right) \quad (1)$$

Here: $\sum_i^N C_i\left(f_i^{\exp}, f_i^c\right)$ - the experimental – calculated frequencies difference; $\sum_j^M D_j\left(a_j^{\exp}, a_j^c\right)$ – the experimental – calculated vibration forms difference; $\sum_{i,j} M_j\left(E_{ij}^{\exp}, E_{ij}^c\right)$– the experimental – apriority known elastic modules difference; $\sum_k^K G_k\left(g_k^{\exp}, g_k^c\right)$ – the apriority known – theoretically refined geometrical and joint mechanical properties difference. Functions C,D,G may be different. Such most commonly used form is given in [7] for FC

$$F_C = \sum_i^N \frac{\left|f_i^{\exp} - f_i^c\right|}{f_i^{\exp}} \quad (2)$$

More complicated form is used in [8]

$$F_C = \sum_i^N \alpha_i \frac{\left|f_i^{\exp} - f_i^c\right|^2}{f_i^{\exp}}$$

The first level optimization problem is based on genetic algorithm and includes parameters:

Elastic modules (uniform foam material beam) $C_{xx}, C_{xz}, C_{zz}, G$ (see (2));

Geometrical parameter L – beam length, mass of beam clamp m2 and rigidity of clamp k2 .

On the first step – genetic optimisation the form of optimization function is

$$F_C = \sum_i^N \frac{\left|f_i^{\exp} - f_i^c\right|}{f_i^{\exp}} + \frac{\left|L_a - L_c\right|}{L_a} + \frac{\left|m2_a - m2_c\right|}{m2_a} + \frac{\left|k2_a - k2_c\right|}{k2_a}$$

The details of clamp condition influence, discrete elements – beam interaction and sandwich vibration modeling may be found in [1-5]. Results of identification for bone material beam are presented in Fig.5

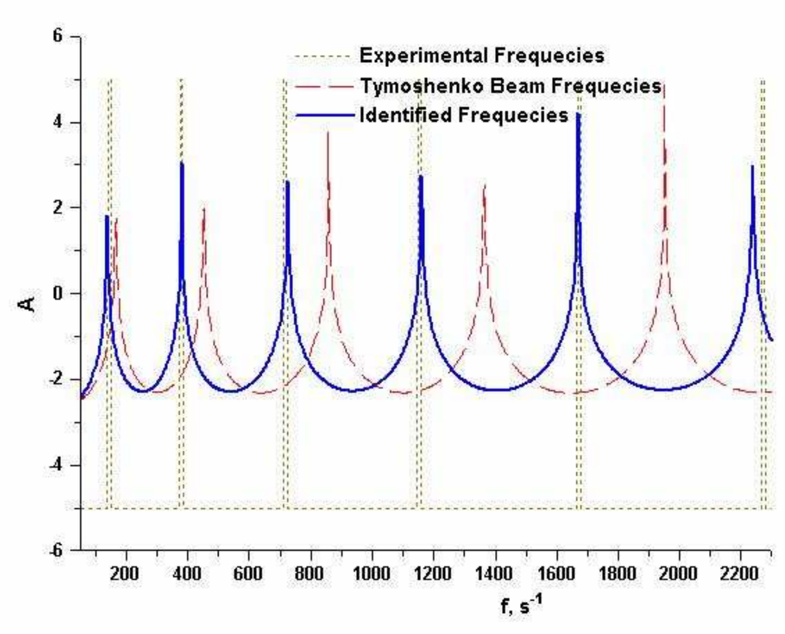

Fig.5. Resulting FRF for beam elastic module identification

Refined clamp condition

Consider bending of the needles in elastic joint with the bone [1]. The material of the needles assumed to be more rigid than that of the interlayer (Fig.6).

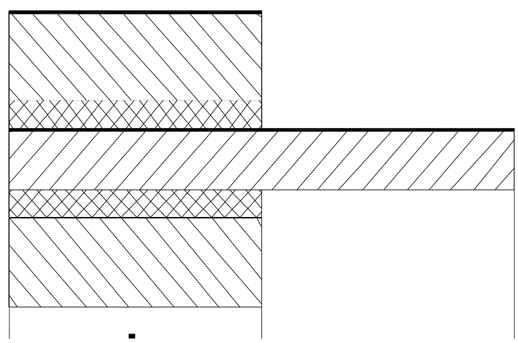

Fig.6. Clamped scheme

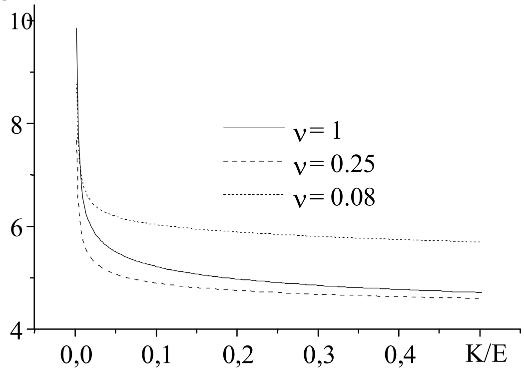

Fig.7 Angles of beam deflection near to clamp (K/E – clamp rigidity to Young modulus ratio, $\nu = C_{xx}/C_{zz}$ *)*

The material of the bone is assumed to be anisotropic, with elasticity modulus Cxx, Cxz, Czz, and Gxz. The material of the interlayer is assumed to be isotropic and incompressible (rubber-like material). The equilibrium equations are derived on the basis of the kinematics hypothesis:

$$u = u_{ij} \cdot x^i \cdot z^{j-1}, \text{ and } w = w_{ij} \cdot x^i \cdot z^{j-1}, \qquad (3)$$

by substitution of Eq.(1) into the

$$\int_{V_p} (\sigma_{xx}\delta\varepsilon_{xx} + \sigma_{zz}\delta\varepsilon_{zz} + \tau_{xz}\delta\varepsilon_{xz})dV + \int_0^L (K^+(x)w^+ + K^-(x)w^-\delta w^-)dx +$$

$$+ \int_0^L (K_u^+(x)u^+\delta u^+ + K_u^-(x)u^- \cdot \delta u^-)dx - \int_{-H_p}^{H_p} (N(z)\delta u + T(z)\delta w) \cdot dz = 0$$

we obtain the set of the linear algebraic equations on u_{ij} and w_{ij}.Here K+-\, K+-u \ are respectively the normal and tangential elastic coefficients of the elastic interlayer such as used in the Winkler foundation. These may be found by the same method [1-5].

In Fig. 7 (which is for an isotropic needle and varying relationships, Cz/Cx) the angles of beam deflection near to clamp are shown. In Fig. 6 may be seen that by absolutely rigid clamp the angle of beam deflection is not equal to zero. For Euler beam clamp we may write $K_\Sigma = \dfrac{K_e K_c}{K_e + K_c}$, $K_e = \dfrac{L_s^3}{3EI}$, where K_Σ – integral rigidity, K_e – rigidity of rigidly clamped Euler beam (L_s -beam length, EI - beam bending rigidity), K_c – clamp rigidity.

Conclusions

In this paper various displacement models have been developed by considering combinations of displacement fields for bone-needles interaction in external osteosynthesis. The numerical method developed follows a semi-analytical approach with the analytical field assumed in the longitudinal direction and a layer-wise displacement field assumed in the transverse direction. The present work aims at developing a simple numerical technique, which can produce very accurate results in comparison with the available analytical solutions and also to decide upon the level of refinement in higher order theory that is needed for accurate and efficient analysis of bone-needles clamp conditions.

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