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PENDULUM TYPE DYNAMIC VIBRATION ABSORBER APPLICATIONS

Розглянуто методи декомпозиції і синтезу числових схем на базі дискретно-неперервних модальних методів та виконано аналіз шляхів дослідження поведінки складних механізмів, зважаючи на їхню взаємодію з системою поглиначів динамічної вібрації.

Ключові слова – Вібрація, поглинач, маятник

The main aim of this paper is pendulum type dynamic vibration absorbers attached to the elongated element design with taking into account complex machines dynamic Methods of decomposition and the numerical schemes synthesis are considered on the basis discrete-continuum modal methods. Investigation of complicated machines in view of their interaction with system of dynamic vibration absorbers is under discussion.

Keywords – Vibration, absorber, pendulum

Introduction

Machines will typically introduce both acoustic and vibration energy into any fluids or structures surrounding the machinery. This is dangerous for both for its construction strength and human health. From two general classes of tools used to assess and optimize machines acoustic performance: test based methods and Computer Aided Engineering based methods, the second should be discussed in this paper. Large elongated elements, particularly such elements as big masts of fire machines or derricks elongated elements of agricultural machines, are dynamically unbalanced during operation due to their exposure to various factors. It is often impossible to balance these elements to reduce the vibration to an acceptable level.

The paper contemplates the provision of dynamic vibration absorbers (DVA) or any number of such absorbers [1,2]. Such originally designed absorbers reduce vibration selectively in maximum vibration mode without introducing vibration in other modes. In order to determine the optimal parameters of an absorber the need for complete modelling of machine dynamics is obvious. Present research has developed a modern prediction and control methodology, based on a complex continuum theory and the application of special frequency characteristics of structures.

Former investigations

The two most popular computational methods used in structural dynamics are: the finite element method (FEM) and the boundary element method (BEM). While investigating higher frequency ranges for acoustic applications and using finite elements, structures are decomposed into smaller and smaller elements. The mesh size is chosen so that its largest dimension does not exceed the wavelength of the vibration. Going in this direction, when dealing with complex and large structures, the number of elements often becomes prohibitive. The calculation of eigenvalues in the range of medium frequency becomes cumbersome and time consuming.

Since the dynamic characteristics of some structural systems may be predicted by using a beam carrying single or multiple concentrated elements, the literature concerned is plenty. In [3] the vibration analysis of a uniform cantilever beam with point masses by an analytical-and-numerical-combined method is performed The frequency equations of a Bernoulli–Euler beam to which several spring–mass systems are attached in span were investigated in [4]. In [5] a spring–mass system and gave many numerical examples with different boundary conditions. The approach presented in. [5] was based on the method which divided the beam into segments from the point attached to the spring–mass system. For the vibration analysis of beams with various attachments, various classical analytical methods are presented to solve the similar problems [6-10]. The hybrid methods and lumped-mass (model) transfer matrix method are one of the known approaches in early years [11–14]. From reviews of the existing literature [3–15], one finds that the information regarding the vibration analysis of a non-uniform beam with various boundary conditions and carrying multiple sets of pendulum type concentrated elements is rare, thus, the purpose of this paper is to extend the theories of [15-20] to the presented structures.

Aspects of modeling

The numerical schemes (NS) row for the complex vibroexcitated construction and methods of decomposition and the NS synthesis are considered in our paper on the basis of new methods of modal synthesis [15-23]. Complex NS of discretely-continua type are used. In the adaptive mode they can be used to calculate tension not only in the stratified elements, but also in places of its highest concentration in joints. The problem of DVA design may be divided into such steps (Fig.1)

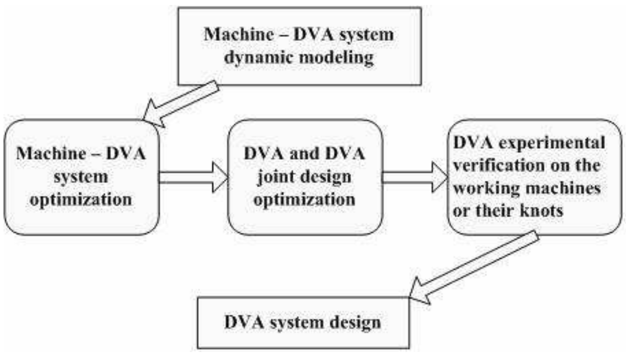


Fig.1. DVA – machine system scheme design

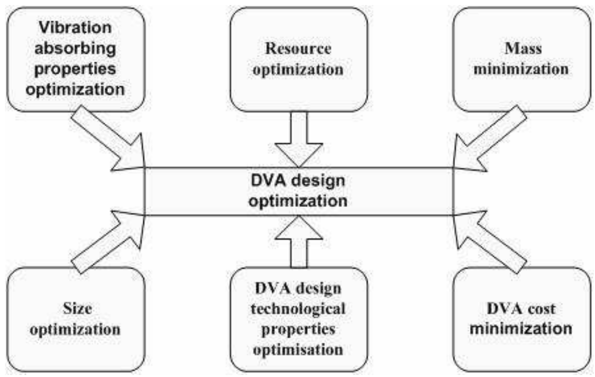


Fig.2. DVA design scheme optimization

In Fig.2. steps of DVA design are presented. It is not a comprehensive list of criterions. This list may be completed, for example, by such criterions: damage control, aesthetic design etc. Not at the lust place must be DVA design simplicity, especially for the theoretical purposes. Most important from them is criterion “vibration absorbing properties optimization”. This important criterion should be discussed later. It is not enough attention paid to such an important criterion as DVA resource optimization. In order optimal parameters of dynamic vibration absorber (DVA) to be determinate the complete modeling of dynamics of machine is obvious. The two degrees of freedom model is totally inadequate to calculate the vibration frequencies of the construction with accuracy and therefore, for a sufficiently accurate determination of its dimensional characteristics so as to determine such frequencies. It is therefore necessary in practice to dimension the construction through more complex modeling. In particular, concentrated mass and rigidity calculation methods may be adopted based on an even more accurate theoretical determination.

Dynamic equations

Problem of vibration fields modeling of complicated designs deformation and strain is considered for the purposes of dynamic absorption. The problem is solved on the basis of modified method of modal synthesis. The basis of these methods is in deriving solving set of equations in a normal form at minimum application of matrix operations. The essence of the first method consists in reviewing knots of junctions as compact discrete elements A_i^n for which inertial properties are taken into account without reviewing their strain, and massive connected parts - as deformable elements A_i^c, their inertion being taken into account on the basis of modal expansion.

For every point X=(x,y,z) of A_i^c we have

$$U_i(t,X)=\begin{bmatrix} q_{1i}(t)\varphi_{1i}(X) \\ \dots \\ q_{ni}(t)\varphi_{ni}(X) \end{bmatrix}, \tag{1}$$

Here $\varphi_{1i}(X),\dots,\varphi_{ni}(X)$ are coordinate functions, $q_{1i}(t),\dots,q_{ni}(t)$ – corresponding independent time functions. By variation of strain U_i^c and kinetic K_i^c energies for A_i^c we have

$$\delta U_i^c = (K_i^{uc}\cdot q_i)^T\cdot\delta q_i\ ,\ \ \delta K_i^c = (M_i^{uc}\cdot q_i)^T\cdot\delta q_i\ . \tag{2}$$

Here

$$q_i = [q_{1i}, q_{2i},\dots,q_{ni}]^T\ .$$

By variation of strain U_i^n and kinetic K_i^n energies for connecting and attached discrete element A_i^n we have

$$\delta U_i^n = k_{ij}(q_{ij}^n(t)-q_j(t)\varphi_j(X_{ij}))\cdot(\delta q_{ij}^n(t)-\delta q_j(t)\varphi_j(X_{ij})) \tag{3}$$

Here X_{ij} are point of contact of discrete element A_i^n and continual element A_j^c and k_{ij} – corresponding rigidity of connection. For the mass-less joints of continual elements we must add to the strain energy such terms

$$\begin{aligned}\delta U_i^n = k_{ij}(q_i(t)\varphi_i(X_{ij})-q_j(t)\varphi_j(X_{ij}))\cdot \\ \cdot(\delta q_i(t)\varphi_i(X_{ij})-\delta q_j(t)\varphi_j(X_{ij}))\end{aligned} \tag{4}$$

Kinetic energy variation of discrete one-mass element A_i^n is

$$\delta K_i^n = m_i\,\dot{q}_i^n\cdot\delta\,\dot{q}_i^n\ . \tag{5}$$

By Hamilton-Ostrogradsky variation equation

$$\int_{t_0}^{t_1}(\delta U-\delta K)dt = 0\quad ,$$

equating terms by independent variation parameters in (2-5) we obtain [9-12]

$$(M\ \ddot{q}\ +\overline{K}\ \cdot q\ )\cdot\delta q\ =0\quad , \tag{6}$$

a set of ordinary differential equations.

Beam modeling

For the beam modeling let us consider nonuniform Timoshenko beam. The kinematical hypothesis are (for pure bending) are

$$U(X,Y,Z,t)=\gamma(x,t)\cdot Z,\quad W(X,Y,Z,t)=w(x,t). \tag{7}$$

By substitution of (7) into the variation Hamilton-Ostrogradsky equation

$$\int_0^L\left(EI\frac{\partial\gamma}{\partial x}\delta\frac{\partial\gamma}{\partial x}+GF\left(\gamma+\frac{\partial W}{\partial x}\right)\delta\gamma+\rho I\frac{\partial^2\gamma}{\partial t^2}\delta\gamma++GF\left(\gamma+\frac{\partial W}{\partial x}\right)\delta\frac{\partial W}{\partial x}+\rho F\frac{\partial^2 W}{\partial t^2}\delta W\right)dx=F, \tag{8}$$

and taking the power series expansion for the functions

$$\gamma(x,t)=\sum_{1}^{N} q_i(t)\gamma_i(x)\ ,\ w(x,t)=\sum_{1}^{N} p_i(t)\gamma_i(x)\ , \tag{9}$$

we obtain a set of ordinary differential equations for unknown time dependent functions (written in matrix form)

$$[M]\frac{d^2\vec{r}}{dt^2}+[C]\vec{r}=\vec{f} \tag{10}$$

(Here [M] and [C] are well known mass and rigidity matrix, $\vec{r}=\left(\vec{q},\vec{p}\right)$ – vector of unknown functions, $\vec{f}$ vector of outer forces.)

Pendulum modeling

Let us consider two pendulums. The first – ordinary massive pendulum (Fig.3), and second – two-mass pendulum with an additional spring (Fig.4). For the pendulum type DVA let us consider such a model (Fig.3)

The additional variations of the kinetic and potential energies caused by elastically suspended pendulum are

$$\delta K_m = M\left(\frac{\partial X_m}{\partial x}\delta\left(\frac{\partial X_m}{\partial x}\right)+\frac{\partial Y_m}{\partial x}\delta\left(\frac{\partial Y_m}{\partial x}\right)\right)\ ,\ \delta U_m = KL\delta L + Mg\ , \tag{11}$$

$$L=\sqrt{\left(X_m+W\cos(\alpha)\right)^2+\left(L_0-Y_m+W\sin(\alpha)\right)^2}\ ,$$

$$\delta U_m = K\frac{dL}{L}\begin{bmatrix}(W+L_0+\cos(\alpha)X_m-\sin(\alpha)Ym)\delta W\\ +(W\cos(\alpha)+X_m)\delta X_m+(-L_0-W\sin(\alpha)+Y_m)\delta Y_m\end{bmatrix}. \tag{12}$$

For the completely rigid pendulums their effective length (length of equivalent mathematical pendulum) are:

$L_e=\dfrac{\left(2/3\, m_c+2M\right)}{m_c+2M}$ for the pendulum in Fig.3; $L_e=\dfrac{m_1L_1^2+m_2L_2^2\, a_1^2/a_2^2}{m_1L_1^2+m_2\, L_1^2/L_2+ka_1^2tg^2\left(b/a_2\right)}$ in Fig.4.

Here for the first pendulum M is a concentrated mass and m_c is a mass of the beam; for the second pendulum m_1 , m_2 are the masses of first and second pendulum, L_1, L_2 – their effective lengths, a_1,a_2 are lengths of the elements 8 and 7, b is a distance between pendulums and k is a rigidity of the spring 13 (Fig.4).

Combined now the set of equation for beam (10) and (11,12) we obtain the complete system of dynamic equations

$$[M_R]\frac{d^2\vec{R}}{dt^2}+[C_R]\vec{R}=\vec{f} \tag{13}$$

Here $[M_R]$ and $[C_R]$ are complete mass and rigidity matrix, $\vec{r}=\left(\vec{q},\vec{p},X_m,Y_m\right)$ – complete vector of unknown functions, $\vec{f}$ the same vector of outer forces.

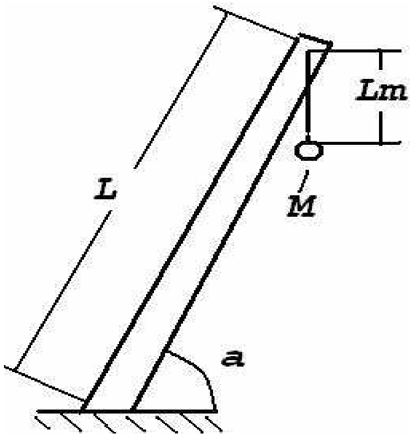


Fig.3. Mast – single pendulum system

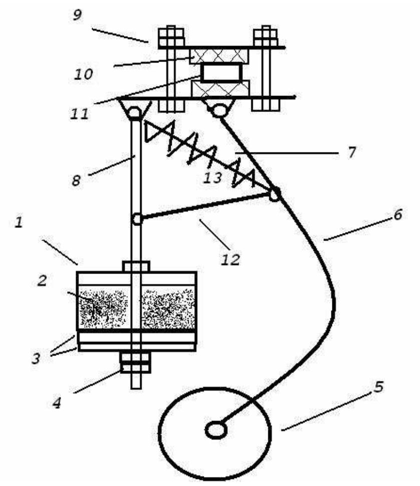


Fig.4. Two mass pendulum

Numerical results

Let us at first consider the case of elastically connected DVA. If the rigidity of elastic element is *k* and mass is *m* and the additional kinetic and potential energies variations are:

$$\delta K = M\left(\frac{\partial X_a}{\partial x}\delta\left(\frac{\partial X_a}{\partial x}\right)\right), \qquad \delta U = k(X_a - W)(\delta X_a - \delta W)$$

The set of dynamic equations has now the same form as in (13). For the proportional viscous damping this equations may be written in such form

$$[M_R]\frac{d^2\vec{R}}{dt^2} + [D]\frac{d\vec{R}}{dt} + [C_R]\vec{R} = \vec{f} \tag{14}$$

In Fig.5. results are presented for impact loading of beam with the elastically connected DVA for various masses of DVA. DVA are appropriately optimized (Fig.7). In Fig.7. the results of ptimization are presented. The evaluation function was $F_e = \max_{T>5c}(W(T))$.

The mass of the tapered beam was 150kg and length 15m

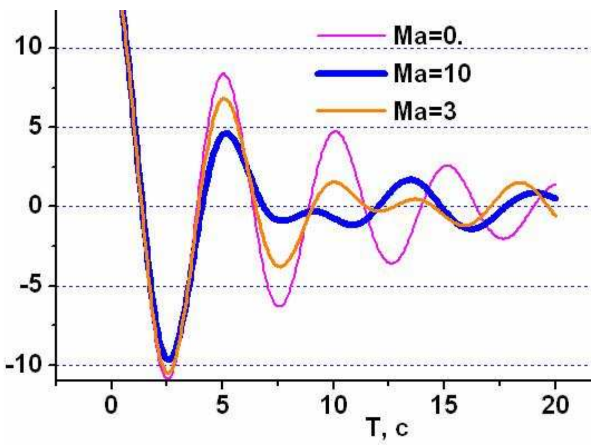


Fig.5. Dynamic response by impact (elastically connected DVA)

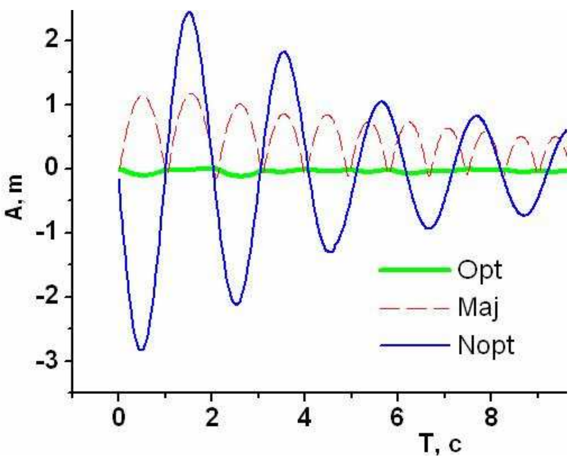
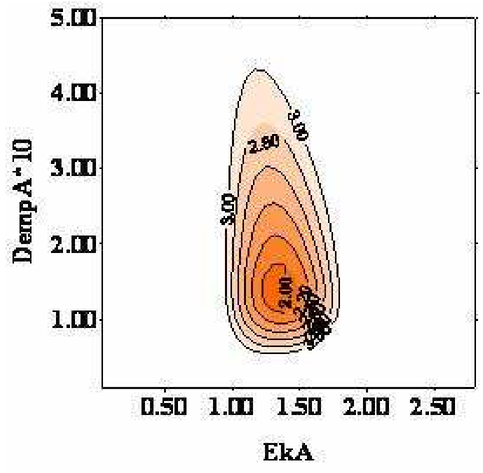
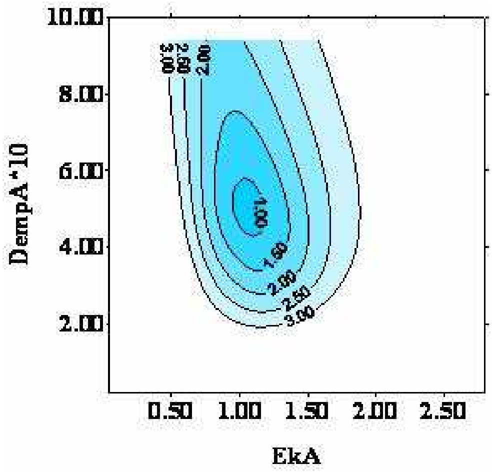


Fig.6. Dynamic response by impact (pendulum DVA)



(a)



(b)

Fig.7. The evaluation functions for elastically clamped DVA: (a) M=3kg, (b) M=10 kg

Conclusion

As the model of many actual systems in the literature, Timoshenko tapered beams with console supporting conditions and DVA of various type additions are used. However, in these applications the DVA's are frequently assumed to be elastically clamped. In the present study, a pendulum type DVA with one or two masses is modeled with the transverse vibrating rod, attached to the tip of a cantilevered beam together with an additional mass, thus composing the system under study. The dynamic equation of this combined system is derived. Comparison of the numerical results with the elastically clamped DVA and pendulum type DVA case reveals the fact that this second is more preferable for some parameter combinations.

1. Timoshenko S.P. Oscillations in engineering. - M., The Science, 1967– 444pp. 2. Korenev, B. G. and Reznikov, L. M. 1993 . "Dynamic Vibration Absorbers: Theory and Technical Applications," Wiley , UK . J.S. 3. 3. Wu, T.L. Lin, Free vibration analysis of a uniform cantilever beam with point masses by an analytical-and-numerical-combined method, Journal of Sound and Vibration 136 (1990) 201–213. 4. M. Gurgoze, On the alternative formulations of the frequency equations of a Bernoulli–Euler beam to which several spring–mass systems are attached in span, Journal of Sound and Vibration 217 (1998) 585–595. 5.D.V. Bambill, C.A. Rossit, Forced vibrations of a beam elastically restrained against rotation and carrying a spring–mass system, Ocean Engineering 29 (2002) 605–626. 6. M.J. Maurizi, R.E. Rossi, J.A. Reyes, Vibration frequencies for a uniform beam with one end spring-hinged and subjected to a translational restraint at the other end, Journal of Sound and Vibration 48 (4) (1976) 565–568. 7. A. Rutenberg, Vibration frequencies for a uniform cantilever with a rotational constraint at a point, ASME Journal of Applied Mechanics 45 (1978) 422–423. 8. N.G. Stephen, Vibration of a cantilevered beam carrying a tip heavy body by Dunkerley's method, Journal of Sound and Vibration 70 (3) (1980) 463–465. 9. J.H. Lau, Vibration frequencies and mode shapes for a constrained cantilever, American Society of Mechanical Engineers Journal of Applied Mechanics 51 (1984) 182–187. 10. W.H. Liu, C.C. Huang, Vibrations of a constrained beam carrying a heavy tip body, Journal of Sound and Vibration 123 (1) (1988) 15–29.] 11. R. Firoozian, H. Zhu, A hybrid method for the vibration analysis of rotor-bearing systems, Proceedings of the Institute of Mechanical Engineers 205 (Part C) (1991) 131–137. 12. A.C. Lee, Y. Kang, S.L. Liu, A modified transfer matrix for linear rotor-bearing systems, Journal of Applied Mechanics, Transactions of the ASME (1991) 776–783. 12. O.S. Sener, H.N. Ozguven, Dynamic analysis of geared shaft systems by using a continuous system model, Journal of Sound and Vibration 166 (3) (1993) 539–556. 14. M. Aleyaasin, M. Ebrahimi, R. Whalley, Multivariable hybrid models for rotor-

bearing systems, Journal of Sound and Vibration 233 (2000) 835–856. 15. B.M.Diveyev. Rational modeling of dynamic processes in complete constructions. Lviv National Polytechnic University. Production processes automatization in machin- and device design. – "Lvivska Politechnika". № 41. 2007. – pp.103-108. (In Ukrainian). 16. Diveiev Bogdan. Rotating machine dynamics with application of variation-analytical methods for rotors calculation. Proceedings of the XI Polish – Ukrainian Conference on "CAD in Machinery Design – Implementation and Education Problems.", Warsaw, June 2003, pp.7-17. 17. Diveyev B. Vibroprocesses optimization by means of semiautomatic vibration absorber. Production processes automatization in machin- and device design. "Lvivska Politechnika", 2005.–p.71-76. (In Ukrainian). 18. Stocko Z.A., Diveyev B.M., Sokil B.I., Topilnyckyj V.H. Mathematical model of vibroactivity regulation of technological machines. Machine knowledge. Lviv. №2, 2005.–pp .37-42. (In Ukrainian).19. Diveiev Bogdan. Rotating machine dynamics with application of variation-analytical methods for rotors calculation. Proceedings of the XI Polish – Ukrainian Conference on "CAD in Machinery Design – Implementation and Education Problems.", Warsaw, June 2003, pp.7-17. 20. Kernytskyy I., Diveyev B., Pankevych B., Kernytskyy N. 2006. Application of variation-analytical methods for rotating machine dynamics with absorber. Electronic Journal of Polish Agricultural Universities, Civil Engineering, Volume 9, Issue 4. Available Online http://www.ejpau.media.pl/

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INVESTIGATION METHODS OF SENSITIVITY IN THE POLIOPTIMAL CONSTRUCTION TO THE SMALL PARAMETER CHANGES

Аналіз з приводу впливу геометричних відхилень і особливостей матеріалу видовбаної конічної дискової пружини на результати її параметричної оптимізації було представлено в роботі. Маленька варіація функції – значення критерію з відхиленнями запевняє коректність нелінійної оптимізації, що передбачається, моделюють з багатьма decision змінними, що розглядають виробничу можливість об'єкта проекту. Методи аналізу структур, що управляють, і повного відмітного аналізу помилки були застосовані для того, щоб оцінити зміни нелінійної функції - критерій на весняній характеристиці.

Ключові слова – дискова пружина, поліоптимальна конструкція, оптимізація.

The analysis concerning the influence of the geometrical deviations and construction material features of the slotted conical disc spring upon the results of its parametrical optimization has been presented in the work. Small variation of the function – criterion value with deviations assures the correctness of the assumed nonlinear optimization model with many decision variables considering the production possibility of the design object. The methods of interval analysis and total differential analysis of the error have been applied in order to evaluate the changes of the nonlinear function – criterion upon the spring characteristic.

Keywords – spring disk, polyoptimal construction, optimization.

Introduction

Analysis concerning the influence of geometrical deviations and material construction features of the slotted conical disc spring upon the results of its parametrical optimization is the main object of this work. In the considered case of searching the optimal construction features <Ci> with maximal constant force