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Predicate Data Model in the Form of a Linear Space

V. Shliakhov, G. Chetverykov, I. Bozhko, N. Shliakhova

Department of Higher Mathematics, Department of Software Engineering
Kharkiv National University of Radioelectronics, Ukraine
14, Nauky ave., Kharkiv, Ukraine
e-mail: chetvergg@gmail.com, ivan.bozhko@nure.ua, natalia.shliakhova@nure.ua

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Abstract. The restriction of the input set in the form of a positive cone of the space <L, R> is not always correct. For instance, while studying the organ of vision, people are limited not only to positive, but also to radiation with not very high energies, because excessively intense can disturb the visual organ. In this particular case, a convex body of a linear space is a fairly acceptable model of the set of input signals. Therefore, we consider linear predicates with this domain of definition.

Key words: predicate model, linear space, predicate algebra, algebraic structure

INTRODUCTION

The input signals set of the object represents some algebraic structure in many practical occasions. It is explained by the fact that usually there are certain connections between the elements of this set, which can be interpreted as algebraic operations. The correct recognition of the corresponding structure largely determines the adequacy of the mathematical model as a whole. In scope of comparator identification, this recognition must be done in the language of experimentally verifiable properties of relations or predicates. Without dwelling on the experimental part, which goes beyond the scope of this paper, we give a theoretical solution of this problem for an algebraic structure such as a linear space over a certain field. This structure is widespread in practice. The domain of definition of operators, which will be studied in the future, is a linear space in this paper. The basics of the given topic are described in [1] and [2–6].

PREDICATE DATA MODEL

We will assume that the input signal processing system realizes by its behavior four predicates defined on the corresponding Cartesian power of the set M (input signals): one one-placed predicate $P(x)$, one two-placed predicate $E(x,y)$ and two three-placed predicates $S(x,y,z), T(x,y,z)$. Characters x,y,z indicate the input signals of the system. The output signals of the system are elements 0 and 1, which are the values of the listed predicates.

The predicate $P(x)$ forms classes of prototypes of coefficients that can be adopted as the coefficients themselves. The predicate $E(x,y)$ is an equivalence predicate given on $M \times M$. It forms classes of prototypes of vectors that can be taken as vectors themselves. The predicate $S(x,y,z)$ is given on P^3, it determines the coefficients addition operation. The predicate $T(x,y,z)$ is given on $P \times M \times M$, it determines the operation of multiplication of the coefficients by vector. Consider the set M on which the relations $E(x,y), S(x,y,z), P(x), T(x,y,z)$, satisfying the following conditions are given:

1) $E(x,x) = 1$;
2) $E(x,y) = 1 \Rightarrow E(y,x) = 1$;
3) $E(x,y) = 1, E(y,z) = 1 \Rightarrow E(x,z) = 1$;
4) $\forall x,y \exists\, z : S(x,y,z) = 1$;
5) $S(x,y,z) = 1, S(x,y,z') = 1 \Rightarrow E(z,z') = 1$;
6) $S(x,y,z) = 1, S(x',y,z) = 1 \Rightarrow E(x,x') = 1$;
7) $S(x,y,z) = 1, S(x,y',z) = 1 \Rightarrow E(y,y') = 1$;
8) $S(x,y,z) = 1 \Rightarrow S(y,x,z) = 1$;
9) $S(x,y,z) = 1, E(z,z') = 1 \Rightarrow S(x,y,z') = 1$;
10) $S(x,y,z) = 1, E(y,y') = 1 \Rightarrow S(x,y',z) = 1$;
11) $S(x,y,z) = 1, E(x,x') = 1 \Rightarrow S(x',y,z) = 1$;
12) $S(x,y,z) = 1, S(z,t,r) = 1, S(y,t,p) = 1 \Rightarrow$
$\Rightarrow S(x,p,r) = 1$;
13) $\exists 0 : S(x,y,x) = 1 \Rightarrow E(y,o) = 1$;
14) $\forall x \exists (-x) : S(x,-x,y) = 1 \Rightarrow E(y,0) = 1$;
15) $P(0) = 1$;
16) $P(x) = 1, P(y) = 1, S(x,y,z) = 1 \Rightarrow P(z) = 1$;
17) $P(x) = 1, E(x,y) = 1 \Rightarrow P(y) = 1$;

18) $\forall x, y \exists z : P(x) = 1 \Rightarrow T(x,y,z) = 1$;
19) $P(x) = 0, P(y) = 0 \Rightarrow T(x,y,z) = 0$;
20) $P(x) = 1, P(y) = 1, T(x,y,z) = 1 \Rightarrow P(z) = 1$;
21) $T(x,y,z) = 1 \Rightarrow T(x,y,z) = 1$;
22) $T(x,y,z) = 1, T(x,y,z') = 1 \Rightarrow E(z,z') = 1$;
23) $T(x,y,z) = 1, T(x,y',z) = 1 \Rightarrow E(y,y') = 1$;
24) $T(x,y,z) = 1, T(x',y,z) = 1 \Rightarrow E(x,x') = 1$;
25) $T(x,y,z) = 1, T(z,z') = 1 \Rightarrow T(x,y,z') = 1$;
26) $T(x,y,z) = 1, T(y,y') = 1 \Rightarrow T(x,y',z) = 1$;
27) $T(x,y,z) = 1, T(x,x') = 1 \Rightarrow T(x',y,z) = 1$;
28) $T(x,y,z) = 1, P(x) = 1, E(z,0) = 1 \Rightarrow E(x,0) = = 1$;
29) $E(z,0) = 1, T(x,y,z) = 1 \Rightarrow E(z,0) = 1$;
30) $T(x,y,z) = 1, P(x) = 1, P(y) = 1, T(z,p,r) = = 1, T(y,p,t) = 1 \Rightarrow T(x,t,r) = 1$;
31) $T(x,y,z) = 1, T(x',y,z') = 1, S(x,x',t) = = 1, P(x) = P(x') = 1, S(z,z',p) = 1 \Rightarrow T(t,y,p) = 1$;
32) $P(x) = 1, T(x,y,z) = 1, T(x,y',z') = = 1, S(z,z',t) = 1,$
$S(y,y',p) = 1 \Rightarrow T(x,p,t) = 1$; $\exists 1 : P(1) = 1, T(y,x,x) = 1 \Rightarrow E(1,y) = 1$;
33) $P(x) = 1\ \exists x^{-1} : T(x, x^{-1}, y) = 1 \Rightarrow E(1,y) = 1$;
34) $\exists \{t_i\}_{i-1}^{n} : \forall x\ \exists \{y_i(x)\}_{i=1}^{n}$:
a. $P(y_i(x)) = 1$;
b. $T(y_i(x), t_i, z_i) = 1, S(z_1, z_2, r_1) = = 1, S(z_1, z_3, r_2) = 1, \dots, S(z_{n-2}, z_n, r_{n-1}) = 1 \Rightarrow E(x, r_{n-1}) = 1$;
c. $\forall \{h_i,(x)\}_{i=1}^{n}$, satisfying a and b $\Rightarrow E(\square_i, y_i(x)) = 1$.
35) $P(x) = 1, P(z) = 1, S(x,y,z) = 1 \Rightarrow P(y) = 1$.

In this case the set M is divided into equivalence classes by the relation $E(x,y)$. The equivalence classes will be denoted by A, B, C, R, T, …, and all the set of classes will be denoted by N. Then, as it is shown in the work, $E(x,y)$ can be represented as

$E(x,y) = D(Fx, Fy)$, where D is an equality predicate given on $N \times N$, and $F : M \to N$ (and $Fx = = Fy \Leftrightarrow E(x,y) = 1$).

Our aim is to show that the given relations induce the structure of n -dimensional linear space on equivalence classes.

Statement 1. If we introduce an operation (addition) on the equivalence classes by the rule $A + + B = C$ if and only if $\forall x, y, z$:

$x \in A,\ y \in B,\ z \in C,\ S(x,y,z) = 1,$ then the definition is correct and with respect to the operation N forms an abelian group.

Proof. First we show the correctness of the definition. We arbitrarily select two equivalence classes A,B $\in N$ and two representatives of each class $x \in A, y \in B$. Then the property 4 implicates that there is $z \in C$, for which $S(x,y,z) = 1$. It means that $A + B = C$. Thus, the operation is defined on any pair A,B $\in N$, moreover, uniquely. Let $C' \neq C$, and $A + B = C,\ A + B = C'$.

Then for an arbitrary $z' \in C'$ we have $S(x,y,z') = = 1$. Considering that $S(x,y,z) = 1$, from property 5 we get $E(z,z') = 1$ or $z' \in C$. Hence, $C \cap C' \neq \emptyset$ and since different classes have an empty intersection, then $C \in C'$. There is a contradiction. Now we show that the class Z does not depend on the choice of $x \in A$ and $y \in B$. Let $x, x' \in A$ and $y, y' \in B$. Then since $S(x,y,z) = 1$ and $E(x',x) = 1$, then on the basis of property 11 we get $S(x',y,z) = 1$. Further, taking into account property 10 and the equation $E(y',y) = 1$, we get that $S(x',y',z) = = 1$, but this also means that the addition operation does not depend on the choice of the elements in the classes A and B. Hence, the operation we introduced is correct.

We show that with respect to this operation N forms an Abelian group.

Let $A + B = C$. Then for any $x \in A, y \in B, z \in C$, $S(x,y,z) = 1$. In this case, property 8 implies $S(x,y,z) = 1$ or $A + B = C$. Thus, $A + B = B + A$, the operation is commutative.

It is also associative. Let $(A + B) + C = R$, $A + +B = T$, $B + C = G$. Then for the representatives of classes the equalities $S(x,y,t) = 1, S(y,z,g) = = 1, S(t,z,r) = 1$. Taking into account property 12, we obtain $S(x,g,r) = 1$. It means $A + G = R$ or $A + (B + +C) = R$, i.e.

$(A + B) + C = A + (B + C)$, then the operation is associative. Consider property 13. It states that there is $O \in M$ such that for any x $S(x,O,x) = 1$. Hence, $A + O = A$ (O is an equivalence class, which O belongs to). Moreover, O is unique, because if there is $O' \neq O$, then for $y \in O'$ we get $S(x,y,z) = 1$ and the second part of the property 13 implies $E(y,O) = 1$, i.e. $O' = O$. Thus, among N there is only one element O, which performs the role of zero relative to this operation.

Finally, let us dwell on the existence of the inverse element. We choose an arbitrary class A and its representative $x \in A$. Then by property 14 we have: there is $-x$, for which $S(x,-x,y) = 1$ implies $E(y,0) = 1$. Let $-x \in -A$, then $A + (-A) = B$, where $y \in B$, but with $E(y,0) = 1$ we get $y \in 0$ or $B = 0$.

Thus, $A + (-A) = 0$, and $-A$ is unique. Since if the equation is correct for some other class C, then $S(x,z,0) = 1, S(x,-x,y) = 1$ and $E(y,0) = 1$. Then from property 9 we get $S(x,-x,0) = 1$, and from property 6 we get $-E(-x,z) = 1$, i.e. $-x \in C$ or $-A = C$.

The statement has been proven.

Statement 2. The relation $P(x)$, given on M, determines its subset M', which is the union of equivalence classes, and the set of classes included in M' form a subgroup of the group of all classes with respect to the addition operation.

Proof. To prove the first part of the assertion of the lemma, it is necessary to show that for any equivalence class S the following is correct: $A \cap M'$ is either an empty set or A. Let $x \in A$, then if $P(x) = 1$ and $E(x,y) = 1$, then property 17 implies $P(y) = 1$, i.e. $A \subset M'$. If $P(x) = 0$, then for any $y \in A : P(y) = 0$, since otherwise if $P(y) = 1, E(x,y) = 1$, then from the property 17 we obtain $P(x) = 1$. It is a contradiction. Hence, if $P(x) = 0$, then $A \cap M' = \emptyset$. Thus, $M` = \{x : P(x) = 1\}$ is a union of equivalence classes. We denote the set of these classes by N'. Let us prove that $N' \subset N$ is a subgroup with respect to the addition of classes. Let $A, B \in N'$ and $A + B =$ C, $P(x) = 1, P(y) = 1$ and $S(x,y,z) = 1$.

Property 16 asserts that $P(z) = 1$, consequently, $z \in N'$. Hence the operation of addition does not lead beyond N'. Property 15, which states that $P(0) = 1$, means that $0 \in N'$. Now, let us show that the inverse element belongs to N'. For this we consider $A \in N'$ and $-A$. Let $-A$ does not belong to N`. Then $P(-x) = 0, P(x) = 1, P(0) = = 1$ and $S(x, -x, 0) = 1$. But the last set of equalities contradicts property 36. The assertion is proved.

Statement 3. If on equivalence classes belonging to N', we introduce the operation (multiplication) by the rule: AB $= C$, if and only if $T(x, y, z) = 1$, for $\forall x \in A$, $y \in B, z \in C$, then this definition will be correct, and with respect to these operations of addition and multiplication, the equivalence classes N` form a field.

Proof. The correctness of the definition of the introduced operation is clarified as follows. From properties 18 and 20 it follows that for any $A, B \in N'$ and their arbitrary elements $x \in A$ and $y \in B$ there is z, for which $T(x, y, z) = 1$ и $P(z) = 1$. Therefore, by virtue of this definition of the operation of multiplication, there is a class $C \subset N'$, for which AB $= C$ and $y \in B$. Let for some z' the following equation is correct $T(x, y, z') = 1$, but then the property 22 and the equation $T(x, y, z) = 1$ imply that $E(z, z') = 1$, i.e. $z' \in C$. On the other hand, if initially we chose $x \neq x' \in A$ and $y \neq y' \in B$, then the equalities $E(x, x') = 1, E(y, y') = 1$, $T(x, y, z) = 1$ and the properties 26, 27 imply $T(x', y', z) = 1$. Thus, the class C does not depend on the original choice of the elements of the classes A and B. Consequently, the definition of the operation of multiplication is correct. We now show that with respect to the operations of addition and multiplication, the set of classes N' forms a field. It follows from Statement 2 that N' is an abelian addition group. Let us prove that by multiplication N' is also an abelian group. Consider two arbitrary classes $A, B \in N'$ and let AB $\in C$. The last equation means that $T(x, y, z) = = 1$ for $x \in A, y \in B, z \in C$, but the property 21 in this case implies $T(x, y, z) = 1$, i.e. BA $= C$. Hence, the operation is commutative. Property 30 implies its associativity. Indeed, let us consider (AB)C and let AB $= R$, RC $= T$ and BC $= P$. Then representatives of these classes will satisfy $T(x, y, z) = 1, T(r, z, t) = = 1, T(y, z, p) = 1$, but according to the property 30 we obtain $T(x, p, t) = 1$, i.e. (AB)$C = A$(BC).

Consider the property 33. It states that for $\forall x \in A \subset N'$ there is $x^{-1} \in A^{-1} \subset N'$ such that for any $z \in C$, $T(x, x^{-1}, y) = 1$ and $T(y, z, z) = 1$ are correct. If $z \in M'$, the last two equations mean that (AA^{-1})$C = C$, and AA$^{-1} = B \in N'$ according to the property 20. Let us show that A^{-1} does not depend on the class A. Indeed, let$x_1 \neq x$ and $x, x_1 \in A$, then

$E(x_1, x) = 1, T(x, x^{-1}, y) = 1$, $T(x_1, x_1^{-1}, y_1) = 1, T(y, z, z) = 1, T(y_1, z, z) = 1$.

From these equations, on the basis of the properties 25, 27, 24 we get:

$E(y, y_1) = 1, T(x_1, x^{-1}, y) = 1$, $T(x_1, x^{-1}, y) = = 1, T(x_1, x^{-1}, y_1) = 1$ and $E(x_1^{-1}, x^{-1}) = 1$, i.e. $x_1^{-1} \in A^{-1}$. Let $A_1 = A_2$ and $A_1A_1^{-1} = B_1$ and $A_2A_2^{-1} = B_2$. Then $T(x_1, x_1^{-1}, y_1) = 1$, $T(x_2, x_2^{-1}, y_2) = 1$, and taking into account the property 32, we get $\forall z : T(y_1, z, z) = T(y_2, z, z) = 1$. Now we use property 24, then $E(y_1, y_2) = 1$, consequently $B_1 = B_2$. Thus, for any $A \subset N' : A A^{-1} = B$ does not depend on A and since for $\forall z \in N'$ we have (AA^{-1})$C = C$, then AA^{-1} performs the role of one with respect to multiplication. Therefore, in future we denote AA$^{-1} = E$.

Finally, we can conclude that with respect to the operation of multiplication and addition of a set of classes N' form groups. These operations are also interconnected so that N' is a field. We will show this. In fact, we checked all the axioms of the field, except distributivity. This axiom follows from property 31, since for arbitrary classes $A, A', B, C, C', T, P \subset N'$ from the equalities AB $= C$, $A'B = C$, $A + A' = T$, $C = C = P$it follows for their representatives that $T(x, y, y) = T(x', y, z') = = S(x, x', t) = S(z, z', p) = 1$ and from 31 we have $T(t, y, p) = 1$, i.e. TB $= P$, consequently AB $+ A'B = = (A + A')B$. Hence, distributivity is satisfied, which completes the proof of the assertion.

RESULTS AND DISCUSSIONS

We summarize the results of our assertions. Specified relationships:

1) partition the original set into equivalence classes;

2) these equivalence classes form a set N, on which the operation of addition is induced and with respect to it the set N is a group;

3) in the set N, it is possible to allocate a subset $N' \subset N$, on which the initial relations induce the operation of multiplication, with respect to the operations of multiplication and addition, the set N' is a field.

Now we can formulate and prove the theorem, which is the goal of this article.

Theorem 1. The set of equivalence classes N is a finite-dimensional linear space over a field N' with the operation of addition of vectors defined in Statement 1 and with the operation of multiplying a vector by an element of the field defined in Statement 3.

Proof. To begin with, we note that the multiplication operation induced by the relation T and introduced for the elements of the field N', similarly to the way it is done in statement 3, can be correctly defined for elements N, i.e. multiplying vectors by the elements of the field N`. The proof of this fact repeats the corresponding arguments in the proof of statement 3. Let us proceed to the proof of the theorem.

We have already shown that with respect to the operation of addition, the set of elements (hereinafter referred to as their vectors, but denoted by capital letters, since they are equivalence classes) N form a group. There is also a field N and the operation of multiplying the elements of the field by a vector. We show that this operation has the following properties:

1) if $A \in N', B, C \in N$, then $A(B + C) =$ AB + AC;
2) if $A, B \in N', C \in N$, then (AB)$C = A$(BC);
3) if $A, B \in N', C \in N$, then $(A + B)C =$ AC + BC;

4) for $O, E \in N'$ takes place $OA = O$ and $EA = A$ for any $A \in N$.

The first of the above properties follows from the property of relations 32, similarly to the way it was done in Statement 3. The third property and the second as a matter of fact are proved by us in the Statement 3 when it was a question of associativity and distributivity of the operations of addition and multiplication. Let us consider property 4. The fact that $EA = A$ is implied from the property 33 of relations. To justify the second equation, we can use property 29, from which it follows: if we consider $OA = C$, then for the elements it is $y' \in O$ and $S(z, -z, y) = 1$, consequently $E(y', y) = 1$. Then $T(y', x, t) = 1$, i.e. $OA = T$ and $E(y, t) = 1$ or $T = O$. Hence, $OA = O$.

CONCLUSIONS

Thus, we have shown that the set N is a linear space over the field N'. Let us prove its finite dimensionality. It is written in property 35. It follows that there are such elements $t_1 \in T_1, \ldots, t_n \in T_n$ that for any $x \in A$ there are unique $y_1(x) \in B_1(A), \ldots, y_n(A) \in$ $\in B_n(A)$, for which (on the right we will write what is done for classes)

$A)$ $P(y_i) = 1$, i.e. $B_i(A) \in N$ are the elements of the field;

$B)$ $T(y_i(x), t_i, z_i) = 1$, i.e. $B_i(A)T_i =$ $= Z_i$; $S(z_1, z_2, r_1) = 1$, i.e. $C_1 + C_2 = R_1$ etc.;

$S(r_{n-2}, z_n, r_{n-1}) = 1$, i.e. $R_{n-2} + C_n = R_{n-1}$ or $C_1 + C_2 + \ldots + C_n = R_{n-1}$.

Then by property 35 it follows that $E(x, r_{n-1}) = 1$, consequently, $R_{n-1} = A$.

Finally, we obtain the expansion by the basis $T_1, \ldots, T_n$

$$A = B_1(A)T_1 + \ldots + B_n(A)T_n.$$

The uniqueness of the classes $B_i(A)$ (unlike the elements $y_i(x)$, which are mentioned in property 35) follows from c) of property 35. The theorem is proved.

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