COMPUTATIONAL PROBLEMS OF ELECTRICAL ENGINEERING

Vol. 9, No. 1, 2019

ANALYTICAL DESIGN OF DYNAMIC SYSTEM REGULATORS TAKING INTO ACCOUNT THE EFFECT OFDISTURBING FACTORS

Orest Lozynskyi, Volodymyr Moroz, Roman Biletskyi, Yurii Biletskyi
Lviv Polytechnic National University, Lviv, Ukraine
orest.y.lozynskyi@lpnu.ua, volodymyr.i.moroz@lpnu.ua, yurii.o.biletskyi@lpnu.ua

Abstract: This article deals with the problem of synthesis of optimal by the minimal value of integral-quadratic criterion dynamic systems, which are described by a model of equations in state variables.

Based on finding the Lyapunov matrix and optimization equations, we propose a method for synthesizing a set of feedback loop coefficients with respect to state variables that provide the minimal value of integral quadratic criteria when exposed to external coordinate disturbances. Finding the feedback loop coefficients with respect to state variables of the dynamic system using the proposed method extends the optimization methods of such systems by integral-quadratic criteria in a vector-matrix description taking into account the action of external influences. The synthesis of the coefficients carried out on the example of a second-order dynamic system is also given.

This method makes it possible to find the dependences of these coefficients on the initial coordinates of the dynamic system, as well as to synthesize a functional converter whose influence in the feedback loops optimizes the given system

Key words: regulator design, dynamic systems, disturbance, Bellman dynamic programming, initial conditions, the Lyapunov matrix.

1. Introduction

Analytical design of regulators is the method of synthesis of the optimal regulator for a certain object with stated limitations and a given optimality criterion. For the most part, the problem of building an optimal system is mathematically formulated as a variational problem.

Optimization of dynamic systems based on variation calculus methods can be performed by searching the system parameters that provide the minimum of a given functional of quality, i.e. it is an analytical design which consists in finding extremals as a solution to the Euler or Euler-Poisson equations and parameters of these extremals [1].

Another problem of optimization based on classical variation methods is to form the optimal control law, i.e. to synthesize a dynamic system regulator that implements the required control influences. From the perspective of classical variations calculus, the problem of choosing the law of control $\vec{u} = u\left(x_1; x_2; \ldots x_n\right)$ that minimizes the functional $J = \int_0^\infty F\left(x_1, x_2, \ldots x_n, u\right) dt$ of the dynamic system quality $dx_i/dt = f_i\left(x_1, x_2, \ldots x_n, u\right)$ is a Lagrange variational problem [1].

2. Synthesis of dynamic system regulators taking into account initial conditions

In addition to classical methods of variation calculus, non-classical methods of variation calculus, in particular the Bellman dynamic programming method, should be recognized as a powerful tool for optimizing dynamic systems. It is applicable to both stationary and non-stationary systems, and at the same time it can be successfully applied to the problem of formulating the coordinate trajectories under determined or undetermined final state [2].

Analytical design of regulators based on this method makes it possible to find control influences of the system described by the following vector-matrix equation:

$$\begin{cases} \dot{\vec{x}} = \mathbf{A}\vec{x} + \mathbf{B}\vec{u} \\ \vec{y} = \mathbf{C}\vec{x} \end{cases}, \qquad (1)$$

where $\vec{x}$ is the n-dimensional vector of state variables; $\vec{u}$ denotes the m-dimensional vector of control influence; $\vec{y}$ stands for the l-dimensional vector of output variables; **A** is the $n \times n$-dimensional matrix of the system; **B** represents the $n \times m$-dimensional matrix of control influences; **C** is the $l \times n$ matrix, where $l < n$, establishes a relation between the full state vector $\vec{x}$ and the l-dimensional vector of output variables $\vec{y}$.

These control influences provide the minimal value of a system functioning criterion as an integral from quadratic form

$$J = \int_0^\infty \left[\vec{y}^T \mathbf{R}_1 \vec{y} + \vec{u}^T \mathbf{R}_2 \vec{u} \right] dt \to \min , \qquad (2)$$

and are found by solving the algebraic Riccati matrix equation.

In the above quadratic criterion, the positively determined matrices $\mathbf{R}_1$ and $\mathbf{R}_2$ have dimensions ($l \times l$) and ($m \times m$), respectively, and in some cases they are identity matrices. It is clear that for non-stationary systems, matrices **A**, **B** and **C** are time dependent, so integral quadratic criterion (2) has integration boundaries ($t_1 \div t_2$) and the solution is sought from the Riccati differential matrix equation.

Based on this system description and the quadratic quality functional, we have the following task – to find a control that transits the system from the initial state $\vec{x}(t=0)=\vec{x}_0$ to the final state $\vec{x}(t=\infty)=0$ and provides the minimum of quadratic quality functional. Note that a quadratic functional is a functional of errors. So, if the balance state of the system is denoted by the vector $\vec{x}_\mathrm{d}=0$, then any deviation of the vector $\vec{x}$ from the vector $\vec{x}_\mathrm{d}$ is an error. In this formulation of the problem, we are not interested in why and how the system has gone into the state $\vec{x}_0$. We should form a transition from the state $\vec{x}_0$ to the state $\vec{x}(\infty)=0$, ensuring the minimal value of the accepted functional of quality. This is implemented by a linear quadratic regulator $\vec{u}=\mathbf{K}\vec{x}$, according to which the scheme in Fig. 1 should be closed by a feedback loop with a matrix **K** (dashed line) that will provide the optimal transition trajectory (Fig. 2) from the state $\vec{x}_0$ to the state $\vec{x}(\infty)=0$.

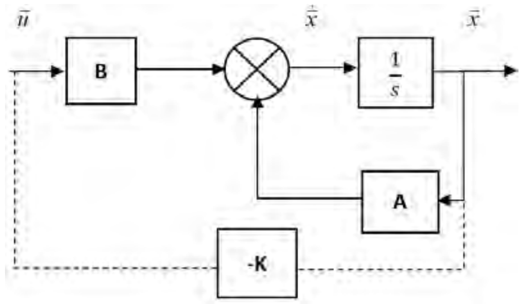


Fig. 1. Block diagram of a closed-loop system.

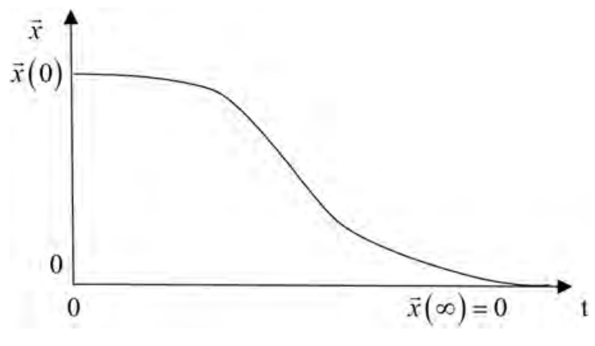


Fig. 2. Transition function from initial to final state.

At the same time a linear stationary continuous controlled dynamic system is generally described by an ordinary first-order vector linear differential equation [3]

$$\begin{cases} \dot{\vec{x}}=\mathbf{A}\vec{x}+\mathbf{B}\vec{u}+\mathbf{D}_1\vec{w} \\ \vec{y}=\mathbf{C}\vec{x}+\mathbf{D}_2\vec{w} \end{cases}, \qquad (3)$$

where $\vec{w}$ is the vector of disturbing or reference influences; $\mathbf{D}_1$ is the port-matrix for disturbing or reference influences; $\mathbf{D}_2$ is the matrix of output signal measurement inaccuracies.

To begin with, we shall assume that $\mathbf{D}_2 = 0$.

The direct use of the regulator synthesis methods based on the Bellman dynamic programming is impossible in this case because system (3) differs from the vector-matrix equations of system (1). Therefore, this system (3) is proposed to be considered from the following point of view. Let us assume that $\mathbf{D}_1\vec{w}$ is nothing but specific increments of the state variables at the initial moment $t = 0$, so at time $t = 0$ the state variables have non-zero increments and under the action of a synthesized regulator the system must transit to the state $\vec{x}(\infty)=0$, providing a minimal value of the accepted quadratic criterion. Hence, system (3), being limited in the first stage by state variables optimization, will be optimized taking into account the initial conditions of systems that are described by state variable models.

Also, in our case, we assume that the control $\vec{u}$ is a linear combination of state variables $\vec{u}=\mathbf{K}\vec{x}$. Substituting in vector-matrix equation (1) the expression for vector $\vec{u}$, we shall get:

$$\dot{\vec{x}}=\mathbf{A}\vec{x}+\mathbf{B}\vec{u}=\mathbf{A}\vec{x}+\mathbf{BK}\vec{x}=\mathbf{H}\vec{x},$$

where $\mathbf{H}=\left|\mathbf{A}+\mathbf{BK}\right|$ represents the matrix obtained by summing the matrices **A** and **BK**.

Let us accept the quality evaluation as a functional of the state vector

$$J=\int_0^\infty \vec{x}^T\vec{x}dt\to\min.$$

Now we shall assume that there is some function $V=x^T\mathbf{P}x$, the derivative of which is

$$\dot{V}=-\vec{x}^T\vec{x},$$

that is

$$\frac{d\left(\vec{x}^T\mathbf{P}\vec{x}\right)}{dt}=-\vec{x}^T\vec{x},$$

where the matrix **P** must be determined.

Choosing, without loss of generality, the matrix **P** symmetric and executing differentiation we shall obtain:

$$\frac{d\left(\vec{x}^T\mathbf{P}\vec{x}\right)}{dt}=\dot{\vec{x}}^T\mathbf{P}\vec{x}+\vec{x}^T\mathbf{P}\dot{\vec{x}}.$$

Substituting the value of $\dot{\vec{x}}$ in the expression obtained will result in:

$$\begin{aligned}\frac{d\left(\vec{x}^T\mathbf{P}\vec{x}\right)}{dt}&=\left(\mathbf{H}\vec{x}\right)^T\mathbf{P}\vec{x}+\vec{x}^T\mathbf{PH}\vec{x}= \\ &=\vec{x}^T\mathbf{H}^T\mathbf{P}\vec{x}+\vec{x}^T\mathbf{PH}\vec{x}=\vec{x}^T\left(\mathbf{H}^T\mathbf{P}+\mathbf{PH}\right)\vec{x}\end{aligned}.$$

Now, if we assume that ($\mathbf{H}^T\mathbf{P}+\mathbf{PH}$) = $-\mathbf{I}$, where **I** is the identity matrix, then the derivative of the function *V* takes the form above. Now by replacing in the quality functional expression the quadratic form $\vec{x}^T\vec{x}$ by its value through the derivative of *V* and following integration, we shall obtain:

$$J = \int_0^\infty \left[-\frac{d\left(\vec{x}^T\mathbf{P}\vec{x}\right)}{dt} \right] dt = -\vec{x}^T\mathbf{P}\vec{x}\Big|_0^\infty = \vec{x}^T\left(0\right)\mathbf{P}\vec{x}\left(0\right).$$

Here the need for system stability is taken into account, that is why $\mathbf{P}(\infty) = 0$ and $\vec{x}\ (\infty) = 0$.

Thus, to synthesize a system that provides the minimum of quadratic functional and takes into account the vector of initial conditions, it is necessary to consider the following two equations [4]:

$$\begin{aligned} &J = \int_0^\infty \left[\vec{x}^T\mathbf{P}\vec{x} \right] dt = \vec{x}^T\left(0\right)\mathbf{P}\vec{x}\left(0\right) \\ &\mathbf{H}^T\mathbf{P} + \mathbf{H}^T\mathbf{P} = -\mathbf{I} \end{aligned}. \tag{4}$$

Therefore, the synthesis procedure is reduced to the following stages:

– assuming the matrix **H,** which includes the matrix of feedback loop coefficients, known, we determine the matrix **P** which satisfies the second equation of system (4);

– employing the known methods we minimize *J*, i.e. find the values of the feedback loop coefficients that provide the functional minimum.

It is clear that if the initial conditions change, then each time it will be necessary to recalculate the coefficients with respect to the state variables that optimize the system. It is also necessary to think of a mechanism for correcting the feedback loop coefficients. Regarding the on-line recalculation of the feedback loop coefficients, we do not see any fundamental difficulties here, since the components of matrix **P** are found invariantly from the initial conditins as functions of the feedback loop coefficients in general, analytic form. Further, when we begin to formulate an expression for the quality functional *J,* the initial conditions values take effect, that is, these initial conditions of state variables at a particular starting moment of time will only be the coefficients for the component of the matrix **P**, expressed through the feedback loop coefficients. For the numerical values of those coefficients to be found, it is required to substitute the component values of the matrix **P** and state variables increments in the expression of the functional and conduct its minimization using any of the known methods. As a result, we shall obtain

$$k_i = f_i\left(x_1\left(0\right), x_2\left(0\right), \dots x_m\left(0\right)\right),$$

where *m* is the amount of state variables.

3. Design of second-order system regulators

The procedure of finding numerical values of feedback loop coefficients will be illustrated using a simple example. Let us have a system [4, 5]:

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= u \end{aligned}.$$

Considering that the system control influence is found as a set of closed loops by state variables

$$u = -k_1x_1 - k_2x_2.$$

For such a system its matrix **A**, as well as the control influences vector **B** will be as follows

$$\mathbf{A} = \begin{vmatrix} 0 & 1 \\ 0 & 0 \end{vmatrix}; \qquad \mathbf{B} = \begin{vmatrix} 0 \\ 1 \end{vmatrix},$$

so
$$\begin{vmatrix} \dot{\vec{x}}_1 \\ \dot{\vec{x}}_2 \end{vmatrix} = \begin{vmatrix} 0 & 1 \\ 0 & 0 \end{vmatrix} \cdot \begin{vmatrix} x_1 \\ x_2 \end{vmatrix} + \begin{vmatrix} 0 \\ 1 \end{vmatrix} \vec{u},$$

then the matrix

$$\mathbf{H} = \mathbf{A} + \mathbf{BK} = \begin{vmatrix} 0 & 1 \\ 0 & 0 \end{vmatrix} + \begin{vmatrix} 0 & 0 \\ -k_1 & -k_2 \end{vmatrix} = \begin{vmatrix} 0 & 1 \\ -k_1 & -k_2 \end{vmatrix},$$

here

$$\mathbf{BK} = \begin{vmatrix} 0 \\ 1 \end{vmatrix} \cdot \begin{vmatrix} -k_1 & -k_2 \end{vmatrix} = \begin{vmatrix} 0 & 0 \\ -k_1 & -k_2 \end{vmatrix}.$$

For the sake of simplicity, choose $k_1 = 1$ and find k_2 so that it might ensure the condition of quality evaluation minimization.

So

$$\mathbf{H}^T\mathbf{P} + \mathbf{HP} = -\mathbf{I},$$

and with $k_1 = 1$

$$\mathbf{H}^T = \begin{vmatrix} 0 & -k_1 \\ 1 & -k_2 \end{vmatrix} = \begin{vmatrix} 0 & -1 \\ 1 & -k_2 \end{vmatrix},$$

we can write the last equation as

$$\begin{vmatrix} 0 & -1 \\ 1 & -k_2 \end{vmatrix} \cdot \begin{vmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{vmatrix} + \begin{vmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{vmatrix} \cdot \begin{vmatrix} 0 & 1 \\ -1 & -k_2 \end{vmatrix} = \begin{vmatrix} -1 & 0 \\ 0 & -1 \end{vmatrix}.$$

According to the aforementioned expression it is easy to find

$$p_{12} = p_{21} = \frac{1}{2};\ p_{22} = \frac{1}{k_2};\ p_{11} = \frac{k_2^2 + 2}{2k_2}.$$

Now let us write an expression for integral quality evaluation as

$$J = \begin{vmatrix} x_1\left(0\right) & x_2\left(0\right) \end{vmatrix} \cdot \begin{vmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{vmatrix} \cdot \begin{vmatrix} x_1\left(0\right) \\ x_2\left(0\right) \end{vmatrix} = x_1^2\left(0\right)p_{11} +$$
$$+ x_1\left(0\right)x_2\left(0\right)p_{12} + x_1\left(0\right)x_2\left(0\right)p_{21} + x_2^2\left(0\right)p_{22}.$$

Substituting the found values of the matrix **P** components in the expression for *J*, we shall get:

$$J = \frac{x_1^2(0)\left(k_2^2+2\right)+2k_2x_1(0)x_2(0)+2x_2^2(0)}{2k_2}.$$

Minimizing the quality evaluation by the value of k_2, we shall obtain

$$k_2 = \sqrt{2+2\frac{x_2^2(0)}{x_1^2(0)}}.$$

Thus, way we have obtained a closed loop coefficient by a state variable as a function of initial conditions x_1 and x_2 in general form.

Now, simply assuming $x_1(0) = x_2(0) = 1$, we can find the minimal value of a quality functional as $J = 3$.

It is clear that this example only illustrates the synthesis method and in complicated systems the process of finding feedback loop coefficients in general case, i.e. through the initial values of coordinates, requires the application of computer hardware to find the components of the matrix **P,** and to minimize the functional *J* in the case of setting the larger amount of closed loop coefficients. It is quite easy to show that when the matrices **H** and $\mathbf{H}^{\mathrm{T}}$ take the form

$$\mathbf{H} = \begin{vmatrix} 0 & 1 \\ -k_1 & -k_2 \end{vmatrix};\ \mathbf{H}^{\mathrm{T}} = \begin{vmatrix} 0 & -k_1 \\ 1 & -k_2 \end{vmatrix},$$

the second equation of system (4) will be written as:

$$\begin{vmatrix} 0 & -k_1 \\ 1 & -k_2 \end{vmatrix} \cdot \begin{vmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{vmatrix} + \begin{vmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{vmatrix} \cdot \begin{vmatrix} 0 & 1 \\ -k_1 & -k_2 \end{vmatrix} = \begin{vmatrix} -1 & 0 \\ 0 & -1 \end{vmatrix}$$

and correspondingly

$$\begin{vmatrix} -k_1p_{21} & -k_1p_{22} \\ p_{11}-k_2p_{21} & p_{12}-k_2p_{22} \end{vmatrix} + \begin{vmatrix} -k_1p_{12} & p_{11}-k_2p_{12} \\ -k_1p_{22} & p_{21}-k_2p_{22} \end{vmatrix} = \begin{vmatrix} -1 & 0 \\ 0 & -1 \end{vmatrix}.$$

The equation for calculation of the matrix **P** components through the coefficients k_1 and k_2 will be obtained as:

$$\begin{aligned} &-k_1p_{21}-k_1p_{12}=-1, \\ &p_{11}-k_2p_{21}-k_1p_{22}=0, \\ &p_{12}-k_2p_{22}+p_{21}-k_2p_{22}=-1. \end{aligned}$$

The solutions to these equations are as follows:

$$p_{12}=p_{21}=\frac{1}{2k_1};\ p_{22}=\frac{k_1+1}{2k_1k_2};\ p_{11}=\frac{k_1^2+k_2^2+k_1}{2k_1k_2}.$$

Then, the quality functional is

$$J = \begin{vmatrix} x_1(0) & x_2(0) \end{vmatrix} \cdot \begin{vmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{vmatrix} \cdot \begin{vmatrix} x_1(0) \\ x_2(0) \end{vmatrix} = \frac{x_1^2(0)k_2^2}{2k_1k_2} + $$
$$+\frac{x_1^2(0)k_1^2x_1^2(0)k_1+2k_2x_1(0)x_2(0)+x_2^2(0)k_1+x_2^2(0)}{2k_1k_2}.$$

and the problem of finding, in general form, the following functions $k_1 = f_1\left[x_1(0);x_2(0)\right]$ and $k_2 = f_2\left[x_1(0);x_2(0)\right]$ requires the solution of the two minimization equations below:

$$\begin{aligned} \frac{dJ}{dk_1} &= \frac{x_1^2(0)k_2^2+2k_2x_1(0)x_2(0)-x_1^2(0)k_1+x_2^2(0)}{2k_1^2k_2}, \\ \frac{dJ}{dk_2} &= \frac{x_1^2(0)k_1^2+\left[x_1^2(0)+x_2^2(0)\right]k_1-x_1^2(0)k_2^2+x_2^2(0)}{2k_1k_2^2} \end{aligned} \quad (5)$$

Found from those equations previously mentioned functions $k_1 = f_1\left[x_1(0);x_2(0)\right]$ and $k_2 = f_2\left[x_1(0);x_2(0)\right]$ for the given matrices of the system **A** and **B** will depend on the initial conditions of state variables.

From the first minimization equation of system (5) we have found the value of coefficient k_2 as the absolute value of

$$k_2 = \frac{x_2(0)}{x_1(0)}+k_1.$$

As for the solution of the second minimization equation of the same system (5), to obtain it in the general form as a simple function of the initial conditions is almost impossible because it is a complex nonlinear equation of connection between k_1 and k_2:

$$\begin{aligned} k_1 = &\frac{-\left[x_1^2(0)+x_2^2(0)\right]\pm}{2x_1^2(0)} \\ &\frac{\pm\sqrt{\left[x_1^2(0)+x_2^2(0)\right]^2+4x_1^2(0)\left[x_2(0)+k_1x_1(0)\right]^2}}{2x_1^2(0)} \end{aligned} \quad (6)$$

It can be solved only for specific numerical values of $x_1(0)$ and $x_2(0)$, that is, by numerical methods using specialized programs. Obviously, equation (6) is obtained by substituting the expression for k_2 found from the first minimization equation.

By the way, assuming that $x_1(0) = x_2(0) = 1$, as in the first example, then the absolute value of the coefficient k_1 is found as $k_1 = 1$. The difference is that in the first example, we set the same value, and here we got it by synthesis.

4. Analysis of the results obtained in the synthesis of regulators for the second-order system

In order to check the results obtained, let us plot a 3D surface of the quality functional for the selected object depending on the feedback loop coefficients under the initial conditions $x_1(0) = x_2(0) = 1$ (Fig. 3). For the given object, it is seen that under the same initial conditions, J acquires a minimal value with the same values of feedback loop coefficients. In this way we can build the dependences of the quality functional and its derivative on the feedback loop coefficients, which we assume to be equal to k (Fig. 4). For this system, the quality functional is asymptotically approximated to its minimal value 1, but the feedback loop coefficients should be calculated not only from the perspective of providing the functional minimum, but also taking into account the technical limitations of the system and its regulations.

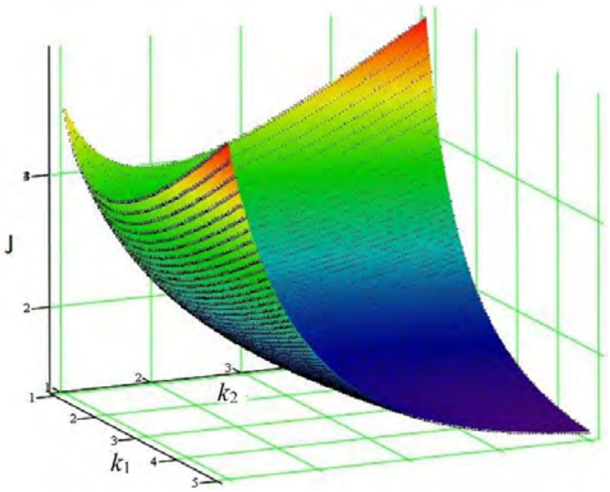


Fig. 3. The description of a macromodel object.

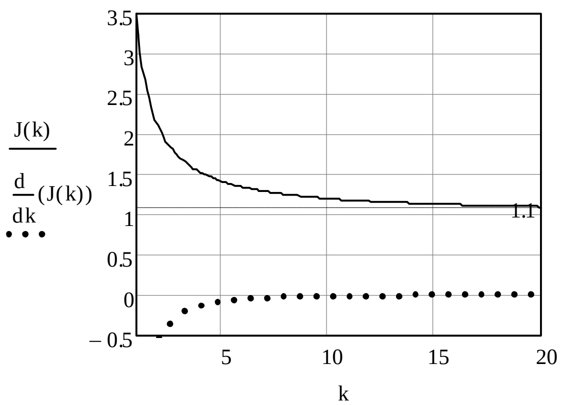


Fig. 4. Changes in quality variable and its differential during transition process.

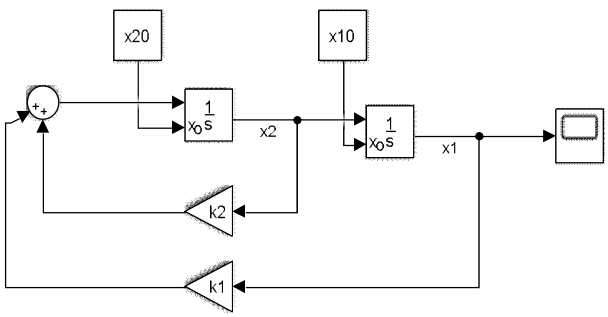


Fig. 5. Computer model in MATLAB/Simulink of the sample object.

For a more detailed analysis a computer model of the selected object was constructed (Fig. 5) and studies of transients were conducted at various values of the feedback loop coefficients k_1 and k_2 (Fig. 6). As we can see, the above calculations and mathematical analysis make it possible to achieve the desired and practically realizable result.

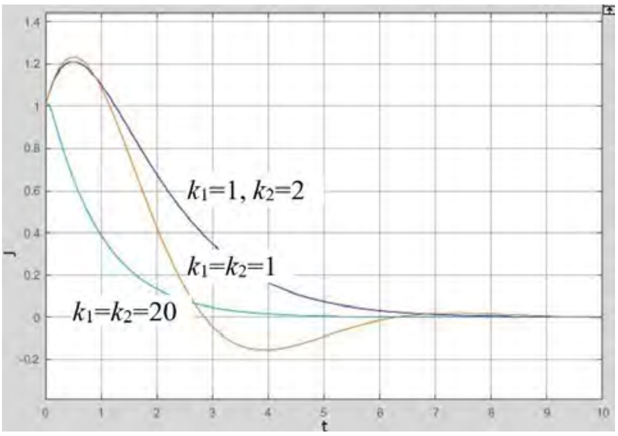


Fig. 6. Changes in quality variable during transition process in three variations of closed loop coefficients.

Therefore, the next step is to develop a digital program that will enable the implementation of these procedures. The input elements of these programs should be the matrices of systems **A** and **B**, the initial conditions of state variables and the matrix of **K** – feedback loop coefficients by the state variables in the general form.

From the examples considered above, it is clear that the actual minimum of the quality evaluation depends on the initial conditions and the type of quality criterion accepted as the basis of synthesis procedure. Using the illustrated methods, it is possible to perform a synthesis involving the selection of several feedback loop values, and to solve a synthesis problem for higher-order systems. However, as already stated, to determine the matrix **P** and to solve the minimization equations $dJ/dk_i = 0$, it is necessary to develop special computer programs, as well as methods of forming the feedback loop coefficients if, as a result of the analysis of the minimization equations, the realizable solution at least one of them does not exist, or the solution returns infinity.

6. Conclusion

The check of optimal systems synthesis has been performed on the basis of initial conditions under which the matrix of feedback loop coefficients is determined from the system optimization by the initial coordinate values, whereby the initial coordinate condition is the value resulted from this or that disturbance.

This approach makes it possible to obtain dependencies of feedback loop coefficients of the dynamic optimized system on the initial values of system coordinates due to disturbances and, if necessary, to make 'on-line' adjustments of these coefficients.

References

[1] V. Vanko, O. Ermolina and G. Kuvyrkin, *Variational calculus and optimal control: Textbook for high schools,* 2nd ed. Moscow, Russia: The Bauman University Publishing House, 2001. (Russian).

[2] N. Egupov, *Methods of the classical and modern theory of automatic control: Synthesis of regulators and theory of optimization of automatic control systems. Textbook.* Moscow, Russia: The Bauman University Publishing House, 2000. (Russian).

[3] B. Poljak and P. Shherbakov, *Robust stability and control.* Moscow, Russia: Nauka, 2002. (Russian)

[4] R. Dorf and R. Bishop, *Modern control systems.* Translated by V. Kopylov, Moscow, Russia: Binom. Laboratory of knowledge, 2002. (Russian).

[5] O. Lozynskyj, A. Lozynskyj, Ya. Marushhak, Ya.Paranchuk and V. Tcyapa, *Synthesis of linear optimal dynamic systems: Tutorial*, Lviv, Ukraine: Publishing House of Lviv Polytechnic National University, 2016. (Ukrainian).

АНАЛІТИЧНЕ КОНСТРУЮВАННЯ РЕГУЛЯТОРІВ ДИНАМІЧНИХ СИСТЕМ З ВРАХУВАННЯМ ДІЇ ЗБУРЮВАЛЬНИХ ФАКТОРІВ

Орест Лозинський, Володимир Мороз,
Роман Білецький, Юрій Білецький

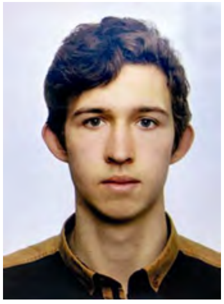
У цій статті розглянуто проблему синтезу оптимальних за мінімальним значенням інтегрально-квадратичного критерію динамічних систем, які описуються моделлю рівнянь у змінних стану.

На основі знаходження матриці Ляпунова і рівнянь оптимізації, запропоновано метод синтезу набору коефіцієнтів зворотних зв’язків за змінними стану, які забезпечують мінімальне значення інтегрального квадратичного критерію при дії на систему зовнішніх координатних збурень. Знаходження коефіцієнтів зворотних зв’язків за змінними стану динамічної системи запропонованим способом розширює методи оптимізації таких систем за інтегрально-квадратичними критеріями при векторно-матричному описі з урахуванням дії зовнішніх впливів. Наведено синтез цих коефіцієнтів на прикладі динамічної системи другого порядку.

Цей метод дає змогу знаходити залежності цих коефіцієнтів від початкових значень координат динамічної системи, а також синтезувати функціональний перетворювач, вплив якого в колах зворотних зв’язків оптимізує дану систему.

Orest Lozynskyi – Honoured Scientist of Ukraine, Professor, Dr. Sc. in Engineering, professor of the Department of Electromechatronics and Computerized Electromechanical Systems at Lviv Polytechnic National University. Graduated from Lviv Polytechnic National University, Department of Electric drive and Automation of Industrial Installations. His research activities are directed towards solving scientific problems and applied tasks in the field of automation, synthesis, analysis and optimal control of the modes of dynamic systems.

Volodymyr Moroz – Professor, Dr. Sc. in Engineering, professor of the Department of Electromechatronics and Computerized Electromechanical Systems at Lviv Polytechnic National University. Graduated from Lviv Polytechnic National University, Department of Electric Drive and Automation of Industrial Installations. His research and teaching interests are in the areas of control theory, computer simulation of the electromechanical systems, digital control systems.



Roman Biletskyi – Student of master degree of Electromechanical Automation Systems and Electric Drive Engineering at Lviv Polytechnic National University. His research is in the areas of control theory, digital control systems and practical implementations of optimization theory.

Yurii Biletskyi – Ph. D. in Engineering Sciences, senior lecturer of the Department of Electromechatronics and Computerized Electromechanical Systems at Lviv Polytechnic National University. Graduated from Lviv Polytechnic National University, Department of Electric Drive and Automation of Industrial Installations. His research and teaching interests are in the areas of control theory, energy-based approaches of control system synthesis, renewable energy sources.