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POSITIVITY AND STABILITY OF DESCRIPTOR LINEAR SYSTEMS WITH INTERVAL STATE MATRICES

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Abstract: The positivity and asymptotic stability of descriptor linear continuous-time and discrete-time systems with interval state matrices and interval polynomials are investigated. Necessary and sufficient conditions for the positivity of descriptor continuous-time and discrete-time linear systems are established. It is shown that the convex linear combination of polynomials of positive linear systems is also the Hurwitz polynomial. The Kharitonov theorem is extended to the positive descriptor linear systems with interval state matrices. Necessary and sufficient conditions for the asy mptotic stability of descriptor positive linear systems have been also established. The considerations have been illustrated by numerical examples.

Key words: interval, positive, descriptor, linear, system, stability, extension, Kharitonov theorem.

1. Introduction

A dynamical system is called positive if its state variables take nonnegative values for all nonnegative inputs and nonnegative initial conditions. The positive linear systems have been investigated in [1, 5, 11], and the positive nonlinear systems in [6, 7, 9, 17, 18].

Examples of positive systems are industrial processes involving chemical reactors, heat exchangers and distillation columns, storage systems, compartmental systems, water and atmospheric pollution models. A variety of models having positive linear behavior can be found in engineering, management science, economics, social sciences, biology, medicine, etc.

Positive linear systems with different fractional orders have been addressed in [3, 12, 14]. Descriptor (singular) linear systems have been analyzed in [9, 13, 15, 16, 20, 23] and the stability of a class of nonlinear fractional-order systems in [6, 18, 26]. Fractional positive continuous-time linear systems and their reachability have been addressed in [10]. The application of the Drazin inverse to analysis of descriptor fractional discrete-time linear systems has been presented in [8], and the stability of discrete-time switched systems with unstable subsystems in [24]. The robust stabilization of discrete-time positive switched systems with uncertainties has been addressed in [25]. A comparison of three methods of the analysis of descriptor fractional systems has been presented in [22]. The stability of linear fractional order systems with delays has been analyzed in [2], and the simple conditions for practical stability of positive fractional systems have been proposed in [4]. The stability of interval positive continuous-time linear systems has been addressed in [19].

In this paper, the positivity and the asymptotic stability of descriptor continuous-time and discrete-time linear systems with interval state matrices will be investigated.

The paper is organized as follows. In section 2, the positivity of descriptor continuous-time linear systems is addressed. A convex linear combination of Hurwitz polynomials and extension of the Kharitonov theorem for linear continuous-time systems is presented in section 3. The stability of descriptor positive linear systems with interval state matrices is analyzed in section 4. The positive descriptor discrete-time linear systems are addressed in section 5, and their stability in section 6. The stability of descriptor positive discrete-time linear systems with interval state matrices is investigated in section 7. Concluding remarks are given in section 8.

The following notations will be used: $\Re$ – the set of real numbers, $\Re^{n\times m}$ – the set of $n\times m$ real matrices, $\Re_+^{n\times m}$ – the set of $n\times m$ real matrices with nonnegative entries, and $\Re_+^n = \Re_+^{n\times 1}$, M_n – the set of $n\times n$ Metzler matrices (real matrices with nonnegative off-diagonal entries), I_n – the $n\times n$ identity matrix.

2. Positivity of descriptor continuous-time linear systems

Consider the autonomous descriptor continuous-time linear system

$$E\dot{x} = Ax, \tag{2.1}$$

where $x = x(t) \in \Re^n$ is the state vector and $E, A \in \Re^{n\times n}$.

It is assumed that

$$\det[Es - A] \neq 0 \text{ for some } s \in \mathbf{C}, \tag{2.2}$$

where **C** is the field of complex numbers.

In this case system (2.1) has a unique solution for admissible initial conditions, $x_0 = x(0) \in \Re^n$.

It is well-known [20] that if (2.2) holds, then there exists a pair of nonsingular matrices $P, Q \in \Re^{n\times n}$ such that

$$P[Es - A]Q = \begin{bmatrix} I_{n_1}s - A_1 & 0 \\ 0 & Ns - I_{n_2} \end{bmatrix},$$

$$A_1 \in \Re^{n_1\times n_1},\ N \in \Re^{n_2\times n_2}, \tag{2.3}$$

where $n_1 = \deg\{\det[Es - A]\}$, and N is the nilpotent matrix, i.e. $N^m = 0$, $N^{m-1} \neq 0$ (μ is the nilpotency index).

To simplify the considerations, it is assumed that the matrix N has only one block. The nonsingular matrices P and Q can be found, for example, by the use of elementary row and column operations [20]:

1. Multiplication of any i-th row (column) by the number $c \neq 0$. This operation will be denoted $L[i \times c]$ ($R[i \times c]$).
2. Addition to any i-th row (column) of the j-th row (column) multiplied by any number $c \neq 0$. This operation will be denoted $L[i + j \times c]$ ($R[i + j \times c]$).
3. Interchange of any two rows (columns). This operation will be denoted $L[i, j]$ ($R[i, j]$).

Definition 2.1. Descriptor system (2.1) is called (internally) positive if $x(t) \in \Re_+^n$, $t \geq 0$ for all admissible nonnegative initial conditions $x(0) \in \Re_+^n$.

Definition 2.2. A real matrix $A = [a_{ij}] \in \Re^{n\times n}$ is called a Metzler matrix if its off-diagonal entries are nonnegative, i.e. $a_{ij} \geq 0$ for $i \neq j$. The set of $n \times n$ Metzler matrices will be denoted M_n.

Theorem 2.1. Descriptor system (2.1) is positive if and only if the matrix E has solely linearly independent columns and the matrix $A_1 \in M_{n_1}$.

Proof. Knowing $n_1 = \deg\{\det[Es - A]\}$ and $\operatorname{rank} E$, we may find the nilpotency index $m = \operatorname{rank} E - n_1 + 1$ of the matrix N. Using a column permutation of E, we choose its n_1 linearly independent columns as its first columns. Next using elementary row operations, we transform the matrix E to the form $\begin{bmatrix} I_{n_1} & 0 \\ 0 & N \end{bmatrix}$ and the matrix A to the form $\begin{bmatrix} A_1 & 0 \\ 0 & I_{n_2} \end{bmatrix}$.

From (2.3) it follows that system (2.1) has been decomposed into two independent subsystems

$$\dot{x}_1 = A_1 x_1,\ x_1 \in \Re^{n_1} \tag{2.4}$$

and

$$N\dot{x}_2 = x_2,\ x_2 \in \Re^{n_2}, \tag{2.5a}$$

where

$$Q^{-1}x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \tag{2.5b}$$

and Q and Q^{-1} are permutation matrices.

It is well-known [5, 11] that the solution $x_1 = e^{A_1 t} x_1(0)$ of (2.4) is not negative if and only if $A_1 \in M_{n_1}$ and the solution x_2 of (2.5) is zero for $t > 0$.

Definition 2.3. [5, 11] Positive system (2.4) is called asymptotically stable if

$$\lim_{t\to\infty} x_1(t) = 0 \text{ for all admissible } x_1(0) \in \Re_+^{n_1}. \tag{2.6}$$

Theorem 2.2. [5, 11, 13] positive system (2.4) is asymptotically stable if and only if one of the equivalent conditions is satisfied:

1) All coefficients of the polynomial

$$\det[I_{n_1}s - A_1] = s^{n_1} + a_{n_1-1}s^{n_1-1} + \ldots + a_1 s + a_0 \tag{2.7}$$

are positive, i.e. $a_k > 0$ for $k = 0, 1, \ldots, n_1 - 1$.

2) All principal minors $\bar{M}_i$, $i = 1, \ldots, n_1$ of the matrix $-A_1$ are positive, i.e.

$$\bar{M}_1 = |-a_{11}| > 0,\ \bar{M}_2 = \begin{vmatrix} -a_{11} & -a_{12} \\ -a_{21} & -a_{22} \end{vmatrix} > 0,\ \ldots, \\ \bar{M}_{n_1} = \det[-A_1] > 0. \tag{2.8}$$

3) There exists a strictly positive vector $l = [l_1 \ \ \mathbf{L} \ \ l_{n_1}]^T$, $l_k > 0$, $k = 1, \ldots, n_1$ such that

$$A_1 l < 0 \text{ or } A_1^T l < 0. \tag{2.9}$$

If $\det A \neq 0$, then we may choose $l = -A_1^{-1}c$, where $c \in \Re^{n_1}$ is any strictly positive vector.

Example 2.1. Consider descriptor system (2.1) with the matrices

$$E = \begin{bmatrix} 0 & 0 & 0 & 2 \\ 0 & 1 & 0 & -2 \\ 1 & -2 & 0 & 0 \\ 0 & 0 & 0 & -2 \end{bmatrix}, \quad A = \begin{bmatrix} 0 & 1 & 0 & -4 \\ 1 & -4 & 0 & 4 \\ 0 & 6 & 1 & 0 \\ 1 & -1 & 0 & 4 \end{bmatrix}. \tag{2.10}$$

Condition (2.2) for (2.10) is satisfied since

$$\det[Es - A] = \begin{vmatrix} 0 & -1 & 0 & 2s+4 \\ -1 & s+4 & 0 & -2s-4 \\ s & -2s-6 & -1 & 0 \\ -1 & 1 & 0 & -2s-4 \end{vmatrix} = \tag{2.11}$$

$$= -2s^2 - 10s - 12$$

and $n_1 = 2$. In this case $\operatorname{rank} E = 3$ and $m = \operatorname{rank} E - n_1 + 1 = 2$.

To transform the matrix $Es - A$ with (2.10) to the desired form

$$\begin{bmatrix} I_2 s - A_1 & 0 \\ 0 & Ns - I_2 \end{bmatrix} \tag{2.12}$$

with $A_1 = \begin{bmatrix} -2 & 1 \\ 0 & -3 \end{bmatrix}$, $N = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$, the following elementary column operations $R[4 \times \frac{1}{2}]$, $R[4,1]$ and elementary row operations $L[2 + 4 \times (-1)]$, $L[4 + 1 \times 1]$, $L[3 + 2 \times 2]$ have been performed.

In this case, the matrices Q and P have the form

$$Q = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \frac{1}{2} & 0 & 0 & 0 \end{bmatrix}, \quad P = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 2 & 1 & -2 \\ 1 & 0 & 0 & 1 \end{bmatrix}. \tag{2.13}$$

Note that the matrix A_1 defined by (2.12) is the stable Metzler matrix and the descriptor system with (2.10) is positive and asymptotically stable.

3. Convex linear combination of Hurwitz polynomials and extension of the Kharitonov theorem

Consider the set (family) of the n-degree polynomials

$$p_n(s) := a_n s^n + a_{n-1} s^{n-1} + \dots + a_1 s + a_0 \tag{3.1a}$$

with the interval coefficients

$$\underline{a_i} \le a_i \le \overline{a_i}, \; i = 0,1,\dots,n. \tag{3.1b}$$

Using (3.1), we define the following four polynomials

$$\begin{aligned} p_{1n}(s) &:= \underline{a_0} + \underline{a_1} s + \overline{a_2} s^2 + \overline{a_3} s^3 + \underline{a_4} s^4 + \underline{a_5} s^5 + \dots, \\ p_{2n}(s) &:= \underline{a_0} + \overline{a_1} s + \overline{a_2} s^2 + \underline{a_3} s^3 + \underline{a_4} s^4 + \overline{a_5} s^5 + \dots, \\ p_{3n}(s) &:= \overline{a_0} + \underline{a_1} s + \underline{a_2} s^2 + \overline{a_3} s^3 + \overline{a_4} s^4 + \underline{a_5} s^5 + \dots, \\ p_{4n}(s) &:= \overline{a_0} + \overline{a_1} s + \underline{a_2} s^2 + \underline{a_3} s^3 + \overline{a_4} s^4 + \overline{a_5} s^5 + \dots. \end{aligned} \tag{3.2}$$

Theorem 3.1. (Kharitonov Theorem) Set of polynomials (3.1) is asymptotically stable if and only if the four polynomials (3.2) are asymptotically stable.

Proof. The proof is given in [21, 20].

The polynomial

$$p(s) := s^n + \overline{a_{n-1}} s^{n-1} + \dots + \overline{a_1} s + \overline{a_0} \tag{3.3}$$

is called Hurwitz if its roots s_i, $i = 1,\dots,n$ satisfy the condition $\operatorname{Re} s < 0$ for $i = 1,\dots,n$.

Definition 3.1. The polynomial

$$p(s) := (1-k) p_1(s) + k p_2(s) \text{ for } k \in [0,1] \tag{3.4}$$

is called a convex linear combination of the polynomials

$$\begin{aligned} p_1(s) &= s^n + a_{n-1} s^{n-1} + \dots + a_1 s + a_0, \\ p_2(s) &= s^n + b_{n-1} s^{n-1} + \dots + b_1 s + b_0. \end{aligned} \tag{3.5}$$

Theorem 3.2. Convex linear combination (3.4) of Hurwitz polynomials (3.5) of the positive linear system is also a Hurwitz polynomial.

Proof. By Theorem 2.2, polynomials (3.5) are Hurwitz if and only if

$$a_i > 0 \text{ and } b_i > 0 \text{ for } i = 0,1,\dots,n-1. \tag{3.6}$$

Convex linear combination (3.4) of Hurwitz polynomials (3.5) is a Hurwitz polynomial if and only if

$$(1-k) a_i + k b_i > 0 \tag{3.7}$$

for $k \in [0,1]$ and $i = 0,1,\dots,n-1$.

Note that conditions (3.7) are always satisfied if (3.6) holds.

Therefore, convex linear combination (3.4) of Hurwitz polynomials (3.5) of the positive linear system is always a Hurwitz polynomial.

Example 3.1. Consider convex linear combination (3.4) of the Hurwitz polynomials

$$\begin{aligned} p_1(s) &= s^2 + 5s + 2, \\ p_2(s) &= s^2 + 3s + 4. \end{aligned} \tag{3.8}$$

Convex linear combination (3.4) of polynomials (3.8) is a Hurwitz polynomial since

$$\begin{aligned} &(1-k)5 + 3k = 5 - 2k > 0 \text{ and} \\ &(1-k)2 + k4 = 2 + 2k > 0 \text{ for } k \in [0,1]. \end{aligned} \tag{3.9}$$

The above considerations for two polynomials (3.5) of the same order *n* can be extended to two polynomials of different orders [19].

Consider the set of positive interval linear continuous-time systems with the characteristic polynomials

$$p(s) = p_n s^n + p_{n-1}s^{n-1} + ... + p_1 s + p_0\,, \qquad (3.10a)$$

where

$$0 < \underline{p_i} \le p_i \le \overline{p_i},\ \ i = 0,1,...,n. \qquad (3.10b)$$

Theorem 3.3. The positive interval linear system with characteristic polynomial (3.10a) is asymptotically stable if and only if conditions (3.10b) are satisfied.

Proof. By the Kharitonov Theorem, the set of polynomials (3.10) is asymptotically stable if and only if polynomials (3.2) are asymptotically stable. Note that the coefficients of polynomials (3.2) are positive if conditions (3.10b) are satisfied. Therefore, by Theorem 2.2, the positive interval linear system with the characteristic polynomials (3.10a) is asymptotically stable if and only if conditions (3.10b) are satisfied.

Example 3.2. Consider the positive linear system with the characteristic polynomial

$$p(s) = a_3 s^3 + a_2 s^2 a_1 s + a_0 \qquad (3.11a)$$

with the interval coefficients

$$\begin{aligned} &0.5 \le a_3 \le 2,\ \ 1 \le a_2 \le 3,\\ &0.4 \le a_1 \le 1.5,\ \ 0.3 \le a_0 \le 4. \end{aligned} \qquad (3.11b)$$

By Theorem 3.3, the interval positive linear system with (3.11) is asymptotically stable since the coefficients a_k, $k = 0, 1, 2, 3$ of polynomial (3.11a) are positive, i.e. the lower bounds are positive.

4. Stability of descriptor positive linear systems with interval state matrices

Consider the autonomous descriptor positive linear system

$$E\dot{x} = Ax\,, \qquad (4.1)$$

where $x = x(t) \in \Re^n$ is the state vector, $E \in \Re^{n\times n}$ is constant (exactly known) and $A \in \Re^{n\times n}$ is an interval matrix defined by

$$\underline{A} \le A \le \overline{A} \text{ or equivalently } A \in [\underline{A}, \overline{A}]. \qquad (4.2)$$

It is assumed that

$$\det[Es - \underline{A}] \ne 0 \text{ and } \det[Es - \overline{A}] \ne 0 \qquad (4.3)$$

and the matrix *E* has only linearly independent columns.

If these assumptions are satisfied, then there exist two pairs of nonsingular matrices (P_1, Q_1), (P_2, Q_2) such that

$$P_1[Es - \underline{A}]Q_1 = \begin{bmatrix} I_{\underline{n}_1}s - \underline{A}_1 & 0 \\ 0 & \underline{N}s - I_{\underline{n}_2} \end{bmatrix},$$

$$\underline{A}_1 \in \Re^{\underline{n}_1\times \underline{n}_1},\ \underline{N} \in \Re^{\underline{n}_2\times\underline{n}_2},\ \underline{n}_1 + \underline{n}_2 = n\,, \qquad (4.4a)$$

and

$$P_2[Es - \overline{A}]Q_2 = \begin{bmatrix} I_{\overline{n}_1}s - \overline{A}_1 & 0 \\ 0 & \overline{N}s - I_{\overline{n}_2} \end{bmatrix},$$

$$\overline{A}_1 \in \Re^{\overline{n}_1\times \overline{n}_1},\ \overline{N} \in \Re^{\overline{n}_2\times\overline{n}_2},\ \overline{n}_1 + \overline{n}_2 = n\,, \qquad (4.4b)$$

where $\underline{n}_1 = \deg\{\det[Es - \underline{A}]\}$ and $\overline{n}_1 = \deg\{\det[Es - \overline{A}]\}$.

Theorem 4.1. If the assumptions are satisfied, then interval descriptor system (4.1) is positive if and only if

$$\underline{A}_1 \in M_{\underline{n}_1} \text{ and } \overline{A}_1 \in M_{\overline{n}_1}\,. \qquad (4.5)$$

Proof. The proof is similar to the proof of Theorem 2.1.

Definition 4.1. Descriptor interval positive system (4.1) is called asymptotically stable (Hurwitz) if the system is asymptotically stable for all matrices *E*, *A*, $A \in [\underline{A}, \overline{A}]$.

Theorem 4.2. If the matrices $\underline{A}$ and $\overline{A}$ of positive system (4.1) are asymptotically stable, then their convex linear combination

$$A = (1-k)\underline{A} + k\overline{A} \text{ for } 0 \le k \le 1 \qquad (4.6)$$

is also asymptotically stable.

Proof. By condition (2.9) of Theorem 2.2, if the positive systems are asymptotically stable, then there exists a strictly positive vector $l \in \Re_+^n$ such that

$$\underline{A}l < 0 \text{ and } \overline{A}l < 0\,. \qquad (4.7)$$

Using (4.6) and (4.7), we obtain

$$Al = [(1-k)\underline{A} + k\overline{A}]l = (1-k)\underline{A}l + k\overline{A}l < 0 \qquad (4.8)$$

for $0 \le k \le 1$. Therefore, if the matrices $\underline{A}$ and $\overline{A}$ are asymptotically stable, and (4.7) hold, then the convex linear combination is also asymptotically stable.

Theorem 4.3. Interval descriptor positive system (4.1) with (4.2) and the matrix E with only linearly independent columns is asymptotically stable if and only if there exists a strictly positive vector $l \in \Re_+^n$ such that

$$P_n \underline{A} l < 0 \text{ and } P_n \overline{A} l < 0, \tag{4.9}$$

where P_n is the submatrix of P consisting of its first n rows.

Proof. If, by assumption, the matrix E has only linearly independent columns, then $l = Q l_q \in \Re_+^n$ with all positive components for any $l_q \in \Re_+^n$ with all positive components. By condition (2.9) of Theorem 2.2 and Theorem 4.2, interval descriptor positive system (4.1) with (4.2) is asymptotically stable if and only if conditions (4.9) are satisfied.

Example 4.1. (Continuation of Example 2.1) Consider descriptor positive system (4.1) with the matrix E of the form (2.10) and the interval matrix A with

$$\underline{A} = \begin{bmatrix} 0 & -1 & 0 & -2 \\ 1 & -3 & 0 & 2 \\ 0 & 4 & 1 & 0 \\ 1 & -1 & 0 & 2 \end{bmatrix}, \quad \overline{A} = \begin{bmatrix} 0 & -1 & 0 & -6 \\ 1 & -5 & 0 & 6 \\ 0 & 8 & 1 & 0 \\ 1 & -1 & 0 & 6 \end{bmatrix}. \tag{4.10}$$

The matrices $(E, \underline{A})$ and $(E, \overline{A})$ satisfy assumptions (4.3) and the matrix E given by (2.10) has only linearly independent columns.

In this case

$$n = \deg\{\det[Es - \underline{A}]\} = \deg \begin{vmatrix} 0 & -1 & 0 & 2s+2 \\ -1 & s+3 & 0 & -2s-2 \\ s & -2s-4 & -1 & 0 \\ -1 & 1 & 0 & -2s-2 \end{vmatrix}, \tag{4.11a}$$
$$= \deg(-2s^2 - 6s - 4) = 2$$

$$n = \deg\{\det[Es - \overline{A}]\} = \deg \begin{vmatrix} 0 & -1 & 0 & 2s+6 \\ -1 & s+5 & 0 & -2s-6 \\ s & -2s-8 & -1 & 0 \\ -1 & 1 & 0 & -2s-6 \end{vmatrix} \tag{4.11b}$$
$$= \deg(-2s^2 - 14s - 24) = 2$$

and from (2.12), we have

$$P_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \end{bmatrix}. \tag{4.12}$$

Using (4.9) and (4.12) for $l = [1 \;\; 1 \;\; 1 \;\; 1]^T$, we obtain

$$P_2 \underline{A} l = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} 0 & -1 & 0 & -2 \\ 1 & -3 & 0 & 2 \\ 0 & 4 & 1 & 0 \\ 1 & -1 & 0 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} -3 \\ -2 \end{bmatrix} < \begin{bmatrix} 0 \\ 0 \end{bmatrix} \tag{4.13a}$$

and

$$P_2 \overline{A} l = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} 0 & -1 & 0 & -6 \\ 1 & -5 & 0 & 6 \\ 0 & 8 & 1 & 0 \\ 1 & -1 & 0 & 6 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} -7 \\ -4 \end{bmatrix} < \begin{bmatrix} 0 \\ 0 \end{bmatrix}. \tag{4.13b}$$

Therefore, by Theorem 4.3, the interval positive descriptor system is asymptotically stable.

5. Positive descriptor discrete-time linear systems

Consider the autonomous descriptor discrete-time linear system

$$E x_{i+1} = A x_i, \; i \in Z_+ = \{0, 1, \ldots\}, \tag{5.1}$$

where $x_i \in \Re^n$ is the state vector and $E, A \in \Re^{n \times n}$.

It is assumed that

$$\det[Ez - A] \neq 0 \text{ for some } z \in \mathbf{C}. \tag{5.2}$$

In this case, system (5.1) has a unique solution for the admissible initial conditions $x_0 \in \Re_+^n$.

It is well-known [20] that if (5.2) holds, then there exists a pair of nonsingular matrices $P, Q \in \Re^{n \times n}$ such that

$$P[Ez - A]Q = \begin{bmatrix} I_{n_1} z - A_1 & 0 \\ 0 & Nz - I_{n_2} \end{bmatrix},$$
$$A_1 \in \Re^{n_1 \times n_1}, \; N \in \Re^{n_2 \times n_2}, \tag{5.3}$$

where $n_1 = \deg\{\det[Ez - A]\}$ and N is the nilpotent matrix, i.e. $N^m = 0$, $N^{m-1} \neq 0$ (μ is the nilpotency index).

To simplify the considerations, it is assumed that the matrix N has only one block.

The nonsingular matrices P and Q can be found, for example, by the use of the elementary row and column operations.

Definition 5.1. [5, 11] The autonomous discrete-time linear system

$$x_{i+1} = Ax_i\,,\ A \in \Re^{n\times n} \tag{5.4}$$

is called (internally) positive if $x_i \in \Re_+^n$, $i \in Z_+$ for all $x_0 \in \Re_+^n$.

Theorem 5.1. [5, 11] System (5.4) is positive if and only if

$$A \in \Re_+^{n\times n}\,. \tag{5.5}$$

Definition 5.2. [5, 11] Positive system (5.4) is called asymptotically stable (Schur) if

$$\lim_{i\to\infty} x_i = 0 \text{ for all } x_0 \in \Re_+^{n_1}\,. \tag{5.6}$$

Theorem 5.2. [11] Positive system (5.4) is asymptotically stable if and only if one of the equivalent conditions is satisfied:

1) All coefficients of the characteristic polynomial

$$\det[I_n(z+1) - A] = z^n + a_{n-1}z^{n-1} + \ldots + a_1 z + a_0 \tag{5.7}$$

are positive, i.e. $a_k > 0$ for $k = 0,1,\ldots,n-1$.

2) There exists a strictly positive vector $l = [l_1 \ \ \mathbf{L} \ \ l_n]^T$, $l_k > 0$, $k = 1,\ldots,n$ such that

$$Al < l\,\cdot \tag{5.8}$$

Definition 5.3. Descriptor system (5.1) is called (internally) positive if $x_i \in \Re_+^n$, $i \in Z_+$ for all admissible nonnegative initial conditions $x_0 \in \Re_+^n$.

Theorem 5.3. Descriptor system (5.1) is positive if and only if the matrix E has only linearly independent columns, and the matrix $A_1 \in \Re_+^{n_1\times n_1}$.

Proof. Using the column permutation (the matrix Q), we choose n_1 linearly independent columns of the matrix E as its first columns. Next, using elementary row operations (the matrix P), we transform the matrix E to the form $\begin{bmatrix} I_{n_1} & 0 \\ 0 & N \end{bmatrix}$, and the matrix A to the form $\begin{bmatrix} A_1 & 0 \\ 0 & I_{n_2} \end{bmatrix}$. From (5.3) it follows that system (5.1) has been decomposed into the following two independent subsystems

$$x_{1,i+1} = A_1 x_{1,i}\,,\ x_{1,i} \in \Re^{n_1}\,,\ i \in Z_+ \tag{5.9}$$

and

$$Nx_{2,i} = x_{2,i}\,,\ x_{2,i} \in \Re^{n_2}\,,\ i \in Z_+ \tag{5.10}$$

where

$$Q^{-1}x_i = \begin{bmatrix} x_{1,i} \\ x_{2,i} \end{bmatrix},\ i \in Z_+ \tag{5.11}$$

and Q and Q^{-1} are permutation matrices.

Note that the solution $x_{1,i} = A_1^i x_{10}$, $i \in Z_+$ of (5.9) is nonnegative if and only if $A_1 \in \Re_+^{n_1\times n_1}$, and the solution $x_{2,i}$ of (5.10) is zero for $i = 1,2,\ldots$.

Example 5.1. Consider descriptor system (5.1) with the matrices

$$E = \begin{bmatrix} 0 & 0 & 0 & 2 \\ 0 & 1 & 0 & -2 \\ 1 & -2 & 0 & 0 \\ 0 & 0 & 0 & -2 \end{bmatrix},\ A = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & -\frac{2}{3} & 0 & -1 \\ 0 & \frac{2}{3} & 1 & 0 \\ 1 & -1 & 0 & -1 \end{bmatrix}. \tag{5.12}$$

Condition (5.2) is satisfied since

$$\det[Ez - A] = \begin{vmatrix} 0 & -1 & 0 & 2z-1 \\ -1 & z+\frac{2}{3} & 0 & -2z+1 \\ z & -2z-\frac{2}{3} & -1 & 0 \\ -1 & 1 & 0 & -2z+1 \end{vmatrix} \tag{5.13}$$
$$= -2z^2 + \frac{5}{3}z - \frac{1}{3}$$

and $n_1 = 2$. In this case $\operatorname{rank} E = 3$, and $m = \operatorname{rank} E - n_1 + 1 = 2$. Performing on the matrix

$$Ez - A = \begin{bmatrix} 0 & -1 & 0 & 2z-1 \\ -1 & z+\frac{2}{3} & 0 & -2z+1 \\ z & -2z-\frac{2}{3} & -1 & 0 \\ -1 & 1 & 0 & -2z+1 \end{bmatrix}, \tag{5.14}$$

the following column elementary operations $R[4\times\frac{1}{2}]$, $R[4,1]$ and the row operations $L[2+4\times(-1)]$, $L[4+1\times1]$, $L[3+2\times2]$, we obtain

$$A_1 = \begin{bmatrix} \frac{1}{2} & 1 \\ 0 & \frac{1}{3} \end{bmatrix},\ N = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}. \tag{5.15}$$

In this case, the matrices Q and P have the forms

$$Q=\begin{bmatrix}0&0&0&1\\0&1&0&0\\0&0&1&0\\\frac{1}{2}&0&0&0\end{bmatrix},\ P=\begin{bmatrix}1&0&0&0\\0&1&0&-1\\0&2&1&-2\\1&0&0&1\end{bmatrix}. \qquad (5.16)$$

By Theorem 5.3, descriptor system (5.1) with (5.12) is positive since $A_1 \in \Re_+^{2\times 2}$ and the matrix Q is monomial.

6. Stability of positive descriptor discrete-time linear systems

Consider descriptor system (5.1) satisfying condition (5.2).

Lemma 6.1. The characteristic polynomials of the system (5.1) and of the matrix $A_1 \in \Re^{n_1\times n_1}$ are related by

$$\det[I_{n_1}z - A_1] = c\det[Ez - A], \qquad (6.1)$$

where $c = (-1)^{n_2}\det P\det Q$.

Proof. From (5.3), we have

$$\det[I_{n_1}z - A_1] = (-1)^{n_2}\det\begin{bmatrix}I_{n_1}z - A_1 & 0\\ 0 & Nz - I_{n_2}\end{bmatrix} = \qquad (6.2)$$
$$= (-1)^{n_2}\det P\det[Ez - A]\det Q = c\det[Ez - A].$$

Theorem 6.1. Positive descriptor system (5.1) is asymptotically stable if and only if one of the following equivalent conditions is satisfied:

1) All coefficients of the characteristic polynomial

$$\begin{aligned}&\det[I_{n_1}(z+1) - A_1] =\\ &= z^{n_1} + a_{n_1-1}z^{n_1-1} + \ldots + a_1 z + a_0\end{aligned} \qquad (6.3)$$

are positive, i.e. $a_k > 0$ for $k = 0,1,\ldots,n_1-1$.

2) All coefficients of the characteristic equation of the matrix $Ez - A$

$$\begin{aligned}&\det[E(z+1) - A]\\ &= \bar{a}_{n_1}z^{n_1} + \bar{a}_{n_1-1}z^{n_1-1} + \ldots + \bar{a}_1 z + \bar{a}_0 = 0\end{aligned} \qquad (6.4)$$

are positive.

3) There exists a strictly positive vector $l = [l_1 \ \ldots \ l_{n_1}]^T$, $l_k > 0$, $k = 1,\ldots,n_1$ such that

$$A_1 l < l. \qquad (6.5)$$

4) There exists a strictly positive vector $\bar{l} = [\bar{l}_1 \ \ldots \ \bar{l}_{n_1}]^T$, $\bar{l}_k > 0$, $k = 1,\ldots,n_1$ such that

$$\bar{P}\hat{A}\bar{l} < \bar{l}, \qquad (6.6a)$$

where

$$\bar{P} = \bar{Q}_{n_1}P_{n_1}, \qquad (6.6b)$$

$\bar{Q}_{n_1} \in \Re_+^{n_1\times n_1}$ consists of n_1 nonzero rows of $Q_{n_1} \in \Re_+^{n\times n_1}$ which is built of first n_1 columns of the matrix Q defined by (5.3), $P_{n_1} \in \Re^{n_1\times n}$ consists of n_1 rows of the matrix P defined by (5.3), $\hat{A} \in \Re^{n\times n_1}$ consists of n_1 columns of $A \in \Re^{n\times n}$ corresponding to the nonzero rows of Q_{n_1}.

Proof. Proof of condition 1) follows immediately from condition 1) of Theorem 5.2. By Lemma 6.1 $\det[I_{n_1}(z+1) - A_1] = 0$ if and only if $\det[E(z+1) - A] = 0$. Therefore, the positive descriptor system (5.1) is asymptotically stable if and only if all coefficients of (6.4) are positive.

From (5.11) we have

$$A_1 = P_{n_1}AQ_{n_1} \qquad (6.7)$$

and using (5.8) we obtain

$$A_1 l = P_{n_1}AQ_{n_1}l < l \qquad (6.8)$$

for some strictly positive vector $l \in \Re_+^{n_1}$. Premultiplying (6.8) by $\bar{Q}_{n_1}$ and taking into account $\bar{Q}_{n_1}l = \bar{l}$ and eliminating from A all columns corresponding to zero rows of Q_{n_1} we obtain (6.6).

Example 6.1. (Continuation of Example 5.1) Using Theorem 6.1 check the asymptotic stability of the positive descriptor system (5.1) with the matrices (5.12).

The matrix A_1 of the system is given by (5.15) and its characteristic polynomial

$$\det[I_2(z+1) - A_1] = \begin{bmatrix}z+\frac{1}{2} & -1\\ 0 & z+\frac{2}{3}\end{bmatrix} \qquad (6.9)$$
$$= z^2 + \frac{7}{6}z + \frac{1}{3}$$

has positive coefficients. Therefore, by condition 1) of Theorem 6.1, the matrix A_1 is asymptotically stable.

Characteristic equation (6.4) of matrices (5.12)

$$\det[E(z+1)-A]=$$
$$=\begin{vmatrix} 0 & -1 & 0 & 2z+1 \\ -1 & z+\frac{5}{3} & 0 & -2z-1 \\ z+1 & -2z-\frac{8}{3} & -1 & 0 \\ -1 & 1 & 0 & -2z-1 \end{vmatrix}= \tag{6.10}$$
$$=2z^2+\frac{7}{3}z+\frac{2}{3}=0$$

has positive coefficients and by condition 2) of Theorem 6.1, the positive system is asymptotically stable.

In this case, we have

$$\bar{P}=\bar{Q}_{n_1}P_{n_1}=\begin{bmatrix} 0 & 1 \\ \frac{1}{2} & 0 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \end{bmatrix}=\begin{bmatrix} 0 & 1 & 0 & -1 \\ \frac{1}{2} & 0 & 0 & 0 \end{bmatrix},$$
$$\hat{A}=\begin{bmatrix} 1 & -1 \\ -\frac{2}{3} & 1 \\ \frac{2}{3} & 0 \\ -1 & 1 \end{bmatrix} \tag{6.11}$$

and

$$\bar{P}\hat{A}=\begin{bmatrix} 0 & 1 & 0 & -1 \\ \frac{1}{2} & 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ -\frac{2}{3} & 1 \\ \frac{2}{3} & 0 \\ -1 & 1 \end{bmatrix}=\begin{bmatrix} \frac{1}{3} & 0 \\ \frac{1}{2} & -\frac{1}{2} \end{bmatrix}. \tag{6.12}$$

Therefore, using (6.6a), (6.11) and (6.12), we obtain

$$\bar{P}\hat{A}\bar{l}=\begin{bmatrix} \frac{1}{3} & 0 \\ \frac{1}{2} & -\frac{1}{2} \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix}<\begin{bmatrix} 1 \\ 1 \end{bmatrix}, \tag{6.13}$$

and by condition (6.6), the positive system is asymptotically stable.

7. Stability of positive descriptor discrete-time linear systems with interval state matrices

Consider the autonomous descriptor positive linear system

$$Ex_{i+1}=Ax_i,\ i\in Z_+ \tag{7.1}$$

where $x_i\in\Re^n$ is the state vector, $E\in\Re^{n\times n}$ is constant (exactly known), and $A\in\Re^{n\times n}$ is an interval matrix defined by

$$\underline{A}\le A\le\overline{A} \text{ or equivalently } A\in[\underline{A},\overline{A}]. \tag{7.2}$$

It is assumed that

$$\det[Ez-\underline{A}]\ne 0\text{ , and }\det[Ez-\overline{A}]\ne 0, \tag{7.3}$$

and the matrix E has only linearly independent columns.

If these assumptions are satisfied, then there exist two pairs of nonsingular matrices (P_1,Q_1), (P_2,Q_2) such that

$$P_1[Es-\underline{A}]Q_1=\begin{bmatrix} I_{\underline{n}_1}z-\underline{A}_1 & 0 \\ 0 & \underline{N}z-I_{\underline{n}_2} \end{bmatrix},$$
$$\underline{A}_1\in\Re^{\underline{n}_1\times\underline{n}_1},\ \underline{N}\in\Re^{\underline{n}_2\times\underline{n}_2},\ \underline{n}_1+\underline{n}_2=n, \tag{7.4a}$$

and

$$P_2[Ez-\overline{A}]Q_2=\begin{bmatrix} I_{\overline{n}_1}z-\overline{A}_1 & 0 \\ 0 & \overline{N}z-I_{\overline{n}_2} \end{bmatrix},$$
$$\overline{A}_1\in\Re^{\overline{n}_1\times\overline{n}_1},\ \overline{N}\in\Re^{\overline{n}_2\times\overline{n}_2},\ \overline{n}_1+\overline{n}_2=n, \tag{7.4b}$$

where $\underline{n}_1=\deg\{\det[Ez-\underline{A}]\}$ and

$$\overline{n}_1=\deg\{\det[Ez-\overline{A}]\}.$$

Theorem 7.1. If the assumptions are satisfied, then interval descriptor system (7.1) with (7.2) is positive if and only if

$$\underline{A}_1\in\Re_+^{\underline{n}_1\times\underline{n}_1}\text{ and }\overline{A}_1\in\Re_+^{\overline{n}_1\times\overline{n}_1}. \tag{7.5}$$

Proof. The proof is similar to the proof of Theorem 5.3.

Definition 7.1. Positive descriptor interval system (7.1) is called asymptotically stable (Schur) if the system is asymptotically stable for all matrices $E,A\in[\underline{A},\overline{A}]$.

Theorem 7.2. If the matrices $\underline{A}$ and $\overline{A}$ of positive system (7.1) are asymptotically stable, then their convex linear combination

$$A=(1-k)\underline{A}+k\overline{A}\text{ for }0\le k\le 1 \tag{7.6}$$

is also asymptotically stable.

Proof. By condition 2) of Theorem 5.2, if the positive systems are asymptotically stable, then there exists a strictly positive vector $l\in\Re_+^n$ such that

$$\underline{A}l<l\text{ and }\overline{A}l<l. \tag{7.7}$$

Using (7.6) and (7.7), we obtain

$$\begin{aligned} Al&=[(1-k)\underline{A}+k\overline{A}]l \\ &=(1-k)\underline{A}l+k\overline{A}l<(1-k)l+kl=l \end{aligned} \tag{7.8}$$

for $0\le k\le 1$. Therefore, if the matrices $\underline{A}$ and $\overline{A}$ are asymptotically stable and (7.7) hold, then the convex linear combination is also asymptotically stable.

Theorem 7.3. Positive descriptor system (7.1) with the matrix E with only linearly independent columns and interval matrix A is asymptotically stable if and only if there exists a strictly positive vector $l \in \Re_+^n$ such that

$$\bar{P}\underline{A}l < l \ \text{ and } \ \bar{P}\bar{A}l < l\,, \tag{7.9}$$

where $\bar{P}$ is defined by (6.6b).

Proof. By assumption, the matrix E has only linearly independent columns and $l = Ql_q \in \Re_+^n$ is strictly positive for any $l_q \in \Re_+^n$ with all positive components. By condition 2) of Theorem 5.2 and Theorem 7.2, the positive descriptor system with interval (7.2) is asymptotically stable if and only if conditions (7.9) are satisfied.

Example 7.1. (Continuation of Example 6.1) Consider positive descriptor system (7.1) with E given by (5.12) and the interval matrix A with

$$\underline{A} = \begin{bmatrix} 0 & 1 & 0 & 0.4 \\ 0 & -0.7 & 0 & -0.4 \\ 1 & -0.6 & 1 & 0 \\ 0 & -1 & 0 & -0.4 \end{bmatrix}, \quad \bar{A} = \begin{bmatrix} 0 & 1 & 0 & 0.8 \\ 0 & -0.4 & 0 & -0.8 \\ 1 & -1.2 & 1 & 0 \\ 0 & -1 & 0 & -0.8 \end{bmatrix}. \tag{7.10}$$

We shall check the stability of the system by the use of Theorem 7.3. The matrices Q and P have the same forms (5.16) as in Example 5.1 and 6.1. Therefore, the matrix $\bar{P}$ in (7.9) in this case is the same as in Example 6.1, and it is given by (6.11). Taking into account that in this case

$$\underline{\tilde{A}} = \begin{bmatrix} 1 & 0.4 \\ -0.7 & -0.4 \\ -0.6 & 0 \\ -1 & -0.4 \end{bmatrix} \text{ and } \bar{\tilde{A}} = \begin{bmatrix} 1 & 0.8 \\ -0.4 & -0.8 \\ -1.2 & 0 \\ -1 & -0.8 \end{bmatrix} \tag{7.11}$$

and using (7.9), we obtain

$$\bar{P}\underline{\tilde{A}}\bar{l} = \begin{bmatrix} 0 & 1 & 0 & -1 \\ 0.5 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0.4 \\ -0.7 & -0.4 \\ -0.6 & 0 \\ -1 & -0.4 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0.3 & 0 \\ 0.5 & 0.2 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} < \begin{bmatrix} 1 \\ 1 \end{bmatrix} \tag{7.12a}$$

and

$$\bar{P}\bar{\tilde{A}}\bar{l} = \begin{bmatrix} 0 & 1 & 0 & -1 \\ 0.5 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0.8 \\ -0.4 & -0.8 \\ -1.2 & 0 \\ -1 & -0.8 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0.6 & 0 \\ 0.5 & 0.4 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} < \begin{bmatrix} 1 \\ 1 \end{bmatrix}. \tag{7.12b}$$

Therefore, by Theorem 7.3, the positive descriptor system is asymptotically stable.

8. Concluding remarks

The positivity and asymptotic stability of descriptor linear continuous-time and discrete-time systems with interval state matrices and interval polynomials have been investigated. Necessary and sufficient conditions for the positivity of descriptor continuous- time and discrete-time linear systems have been established (Theorems 2.1, 2.2, 5.1). It has been shown that the descriptor system is positive if and only if the matrix E has only linearly independent columns, and the matrix A_1 is a Metzler matrix, and the convex linear combination of the Hurwitz polynomials of positive linear systems is also the Hurwitz polynomial (Theorem 3.2). The Kharitonov theorem has been extended to positive descriptor linear systems with interval state matrices (Theorem 5.3). Necessary and sufficient conditions for the asymptotic stability of descriptor positive discrete-time linear systems has been also established (Theorem 7.3). The considerations have been illustrated by numerical examples.

The above considerations can be extended to positive fractional linear systems. An open problem is an extension of these considerations to standard (non-positive) descriptor linear systems.

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References

[1] A. Berman and R. J. Plemmons, *Nonnegative Matrices in the Mathematical Sciences*, SIAM, 1994.

[2] M. Busłowicz, "Stability of linear continuous-time fractional order systems with delays of the retarded type", *Bull. Pol. Acad. Sci. Tech.*, vol. 56, no. 4, pp. 319–324, 2008.

[3] M. Busłowicz, "Stability analysis of continuous-time linear systems consisting of n subsystems with different

fractional orders", *Bull. Pol. Acad. Sci. Tech.*, vol. 60, no. 2, pp. 279–284, 2012.

[4] M. Busłowicz and T. Kaczorek, "Simple conditions for practical stability of positive fractional discrete-time linear systems", *Int. J. Appl. Math. Comput. Sci.*, vol. 19, no. 2, pp. 263–169, 2009.

[5] L. Farina and S. Rinaldi, *Positive Linear Systems; Theory and Applications*, J. Wiley, New York, 2000.

[6] T. Kaczorek, "Analysis of positivity and stability of fractional discrete-time nonlinear systems", *Bull. Pol. Acad. Sci. Tech.*, vol. 64, no. 3, pp. 491–494, 2016.

[7] T. Kaczorek, "Analysis of positivity and stability of discrete-time and continuous-time nonlinear systems", *Computational Problems of Electrical Engineering*, vol. 5, no. 1, pp. 11–16, 2015.

[8] T. Kaczorek, "Application of Drazin inverse to analysis of descriptor fractional discrete-time linear systems with regular pencils", *Int. J. Appl. Math. Comput. Sci.*, vol. 23, no. 1, 29–34, 2013.

[9] T. Kaczorek, "Descriptor positive discrete-time and continuous-time nonlinear systems", *Proc. of SPIE*, vol. 9290, 2014.

[10] T. Kaczorek, "Fractional positive continuous-time linear systems and their reachability", *Int. J. Appl. Math. Comput. Sci.*, vol. 18, no. 2, pp. 223–228, 2008,.

[11] T. Kaczorek, *Positive 1D and 2D Systems*, Springer-Verlag, London, 2002.

[12] T. Kaczorek, "Positive linear systems with different fractional orders", *Bull. Pol. Acad. Sci. Techn.*, vol. 58, no. 3, pp. 453–458, 2010.

[13] T. Kaczorek, "Positivity and stability of standard and fractional descriptor continuous-time linear and nonlinear systems", *Int. J. of Nonlinear Sciences and Num. Simul.*, 2018 (in press) DOI: https://doi.org/10.1515/ijnsns-2017-0049.

[14] T. Kaczorek, "Positive linear systems consisting of n subsystems with different fractional orders", *IEEE Trans. on Circuits and Systems*, vol. 58, no. 7, pp. 1203–1210, 2011.

[15] T. Kaczorek, "Positive fractional continuous-time linear systems with singular pencils", *Bull. Pol. Acad. Sci. Techn.*, vol. 60, no. 1, pp. 9–12, 2012.

[16] T. Kaczorek, "Positive singular discrete-time linear systems", *Bull. Pol. Acad. Sci. Tech.*, vol. 45, no. 4, pp. 619-631, 1997.

[17] T. Kaczorek, "Positivity and stability of discrete-time nonlinear systems", *IEEE 2nd International Conference on Cybernetics*, pp. 156–159, 2015.

[18] T. Kaczorek, "Stability of fractional positive nonlinear systems", *Archives of Control Sciences*, vol. 25, no. 4, pp. 491–496, 2015.

[19] T. Kaczorek, "Stability of interval positive continuous-time linear systems", *Bull. Pol. Acad. Sci. Techn.*, vol. 66, no. 1, 2018.

[20] T. Kaczorek, *Theory of Control and Systems*, PWN, Warszawa, 1993 (in Polish).

[21] V. L. Kharitonov, "Asymptotic stability of an equilibrium position of a family of systems of differential equations, *Differentsialnye uravneniya*, vol. 14, pp. 2086–2088, 1978.

[22] L. Sajewski, "Descriptor fractional discrete-time linear system and its solution – comparison of three different methods", *Challenges in Automation, Robotics and Measurement Techniques*, Advances in Intelligent Systems and Computing, vol. 440, pp. 37–50, 2016.

[23] L. Sajewski, "Descriptor fractional discrete-time linear system with two different fractional orders and its solution", *Bull. Pol. Acad. Sci. Tech.*, vol. 64, no. 1, pp. 15–20, 2016.

[24] H. Zhang, D. Xie, H. Zhang and G. Wang, "Stability analysis for discrete-time switched systems with unstable subsystems by a mode-dependent average dwell time approach", *ISA Transactions*, vol. 53, pp. 1081–1086, 2014.

[25] J. Zhang, Z. Han, H. Wu, and J. Hung, "Robust stabilization of discrete-time positive switched systems with uncertainties and average dwell time switching", *Circuits Syst. Signal Process.*, vol. 33, pp. 71–95, 2014.

[26] W. Xiang-Jun, W. Zheng-Mao, and L. Jun-Guo, "Stability analysis of a class of nonlinear fractional-order systems", *IEEE Trans. Circuits and Systems-II*, Express Briefs, vol. 55, no. 11, pp. 1178–1182, 2008.

ПОЗИТИВНІСТЬ ТА СТІЙКІСТЬ ДЕСКРИПТОРНИХ ЛІНІЙНИХ СИСТЕМ З ІНТЕРВАЛЬНИМИ МАТРИЦЯМИ СТАНУ

Тадеуш Качорек

Досліджено позитивність та асимптотичну стійкість дескрипторних лінійних часово неперервних та дискретних систем з інтервальними матрицями стану та інтервальними поліномами. Встановлено необхідні та достатні умови для позитивності лінійних часово неперервних та дискретних систем. Показано, що опукла лінійна комбінація поліномів позитивних лінійних систем також є поліномом Гурвіца. Поширено теорему Харітонова на позитивні дескрипторні лінійні системи з інтервальними матрицями стану. Також встановлено необхідні та достатні умови для асимптотичної стійкості дескрипторних позитивних лінійних систем. Розглянуті припущення проілюстровано за допомогою числових прикладів.

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