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ASSESSMENT OF FILTRATION WATERS SPREADING ON THE SURFACE OF WATERBODIES

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Abstract. The paper is devoted to the development of the nonlinear mathematical model which describes the spreading of the liquid fraction of solid waste decomposition products on the surface of waterbodies. The solution to the problem of a lighter liquid fraction spreading is divided into three stages – the gravitational and inertial stage, the stage of surface tension and viscosity, and the gravitational and viscous stage. The author assesses the impact of dynamic processes on the nature of waterbodies pollution.

Key words: filtration waters, liquid floaty fraction, waterbodies, spreading stages, dynamic processes.

1. Introduction

Municipal solid waste must be disposed in dumps and special landfills, which is the simplest and the cheapest way of disposal. However, apart from specially designated and equipped places for waste disposal, there is a large number of illegal dumps. The lack of engineering and environmental studies and non-compliance with technical and sanitary-epidemiological standards for waste disposal might lead to the formation of sources of danger for the environment and human health. The decomposition products of municipal solid waste hinder the safe use of water and land resources. Municipal solid waste spontaneously accumulates not only along roadsides and forest belts, but also along waterbodies.

Municipal solid waste contains from 1 to 10 % of the liquid fraction which is washed out from dumps with rain and spring floods in nearby reservoirs. The main characteristics of municipal solid waste filtration waters are: complex chemical composition which changes at each stage of the waste lifecycle; high content of toxic components; the presence of various groups of microorganisms, including pathogens, in the water; significant difference from industrial and municipal solid wastewater [1]. According to the studies of the composition of filtration waters [1–3], they are a liquid fraction which consists of highly concentrated hazardous components and solutions. Consequently, the results of the assessment of the filtration waters behavior on the surface of waterbodies are used as the informative material for carrying out measures to decontaminate water pollution.

The present work aims at mathematical evaluation of the process of filtration waters spreading on the surface of waterbodies.

2. Material and methods

Let's consider a spill of filtration waters with the mass m, volume V and density $r \approx 650-1000$ kg/m^3. The density of the liquid fraction is less than the density of water $r_w \approx 1000$ kg/m^3.

The study is based on the fundamental laws of hydrodynamics and self-similar methods of solving equations of hydrodynamics. The solution to the problem of the lighter liquid floaty fraction spreading on the surface of waterbodies is divided into three stages – the gravitational and inertial stage, the stage of surface tension and viscosity, and the gravitational and viscous stage. The mathematical model takes into account the density and viscosity of the floaty fraction of filtration water.

3. Results and discussion

The composition of the filtrate includes surface active substances and petroleum products [2-3], which form spots on the surface of waterbodies due to their physical and chemical properties. Density, viscosity values play an important role in the buoyancy process and spreading of the liquid fraction.

The initial spreading is preceded by the buoyant force F_A and the vertically directed water resistance force F_r. Under the action of gravity, the liquid begins to spread in the radial direction. Initially, the gravitational-inertial stage of the spreading takes place. As the height of the liquid layer and, therefore, the potential energy reserve E decrease, the surface tension force and the friction force, caused by the final viscosity of the liquid, become significant. In this case, the first stage is replaced by the stage of surface tension and viscosity. Hereafter, the surface tension force becomes unimportant. It is the third phase, called the gravitational and viscous stage. The subsequent stages of the liquid fraction movement are due to the influence of winds, currents in the waterbodies, surface waves, turbulent diffusion, etc. [4, 5].

The gravitational and inertial stage. The equation of the liquid mass movement in the quasi-steady state – in the vertical direction – has the form

$$F_A = mg + F_r, \tag{1}$$

where $m = rV$, V is the volume of liquid, $F_A = m_w g = r_w Vg$, m_w is the mass of the water, $F_r = \frac{C}{2} r_w v^2 S$, C is the coefficient of water dynamic resistance, $S = pr^2$, r is the spot radius, $v = r/t$ is the velocity of the liquid element, t is the time from the beginning of the liquid fraction spreading, g is acceleration of free fall. From the relation (1) we have

$$(r_w - r)Vg = \frac{C}{2} p r_w \frac{r^4}{t^2},$$

or

$$r_1 = \left(\frac{2}{Cp} dgVt^2 \right)^{1/4} = C_1 \left(dgVt^2 \right)^{1/4}, \tag{2}$$

where r_1 is the radius of the spreading zone at the gravitational and inertial stage, $d = (r_w - r)/r_w$, $C_1 = (2/Cp)^{1/4} \approx 1$. With an average value $\rho = 0.8 \cdot 10^3$ kg/m^3 and $r_w = 10^3$ kg/m^3 we get $\delta = 0.2$.

The stage of surface tension and viscosity. In this case, the time dependence of the radius of the liquid spot r_2 is determined by the coefficient of kinematic viscosity v, the spreading time of the spot t, the density of the liquid r and the effective coefficient of the surface tension of the film s. The latter depends on the surface tension coefficients at the interface between air–air, air–liquid and liquid–water [6, 7]. Using the method of dimensions, we can compose the following expression of these quantities for the spreading radius:

$$r_2 = C_2 \left(\frac{s^2 t^3}{r^2 v} \right)^{1/4}. \tag{3}$$

The dimensionless coefficient $C_2 = 2{,}3$ is determined from strict hydrodynamic calculation [5].

Gravitational and viscous stage. We find the dependence of the radius of the liquid spreading zone r_3 on time, taking into account the process of viscosity of the liquid. This radius is determined by the spreading time t, free fall acceleration g, liquid volume V, kinematic viscosity coefficient v and value d. Using the method of dimensions, we can compose the following expressions of these values for the spreading radius:

$$r_3 = C_3 \frac{g^{1/6} V^{1/3} d t^{1/4}}{v^{1/12}}. \tag{4}$$

The constant $C_3 = 1{,}5$ is determined from strict hydrodynamic calculations [5].

Ratios (2)–(4) are self-similar and, therefore, valid only for a point source of the liquid.

The results of the calculating the time dependence of the radii of the liquid spreading zones are presented in Table 1. The calculation is made for $r \approx 800$ kg/m^3, $V = 3 \cdot 10^5$ m^3, $v = 10^{-6}$ m^2/s and $\sigma = 0.04$ N/m. According to Table 1, the radius r_2 assumes great importance. In fact, the surface tension force, which is most pronounced in relatively small areas of liquid spreading, and the friction force hamper the spreading of the liquid. If time interval is longer than 10^3 s, $r_3 < r_2$, and the main force that prevents the liquid from spreading is the friction force.

The larger the initial potential energy of the liquid volume, the larger the distance of overcoming surface tension force, i.e. the larger the radius of the liquid spreading zone and the thicker its layer.

Table 1

The time dependence of the radii of the liquid floaty fraction spreading zones on the surface of waterbodies

t, s		10^3	$3\cdot10^3$	10^4	$3\cdot10^4$	10^5	$3\cdot10^5$	10^6
$V = 1$ m^3	r_1, m	37.6	65.2	119	206	376	652	1190
	r_2, m	90.7	206.7	510	1163	2868	65.37	16128
	r_3, m	7.8	10.3	13.9	18.2	24.6	32.4	43.8
$V = 3$ m^3	r_1, m	49.5	85.7	157	271	495	858	15.65
	r_2, m	90.7	206.7	510	1163	2868	65.37	16128
	r_3, m	11.2	14.8	20.0	26.2	35.4	46.7	63.1
$V = 5$ m^3	r_1, m	56	146	178	308	562	975	1780
	r_2, m	90.7	206.7	510	1163	2868	65.37	16128
	r_3, m	13.3	17.6	23.8	31.4	42.1	55.4	74.9
$V = 10$ m^3	r_1, m	66.7	116	211	365	667	11.56	21.15
	r_2, m	90.7	206.7	510	1163	2868	65.37	16128
	r_3, m	16.8	22.1	30.0	39.1	52.9	69.7	94.2
$V = 30$ m^3	r_1, m	87.8	152	278	481	878	1523	2783
	r_2, m	90.7	206.7	510	1163	2868	65.37	16128
	r_3, m	24.3	32,0	43.2	56,6	76.5	100.8	136.2
$V = 50$ m^3	r_1, m	100	173	316	547	1000	1731	3160
	r_2, m	90.7	206.7	510	1163	2868	65.37	16128
	r_3, m	28.7	37.9	51.2	67.0	90.5	119	161

The energy approach allows us to calculate the maximum sizes of the zones of liquid spreading in the absence of external forces.

The potential energy. Due to the fact that the density of the liquid r is less than the density of water r_w, a part of the volume of the liquid ΔV of the height Δh initially rises above the water. At the same time, according to the equality of the liquid gravity and the buoyant force

$$\Delta V = V\left(1 - \frac{r}{r_w}\right) = dV .$$

In this case, the center of gravity of the liquid volume is at the depth $h_1 = h/2 - \Delta h$, where $h = V/S$. Then $\Delta h = \Delta V/S = dV/S$, where S is the base of the liquid volume. The buoyant force gives the potential energy to the liquid volume:

$$E = rVg\left(\frac{h}{2} - \Delta h\right) = \frac{rV^2 g}{S}\left(\frac{1}{2} - d\right).$$

When $S = V^{2/3}$ we have:

$$E = rV^{4/3} g d_1 , \tag{5}$$

where $d_1 = 1/2 - d = 0,3$.

The effect of the surface tension force. In the absence of other forces, the surface tension force allows the liquid spot to spread over a maximum area:

$$S_m = \frac{E}{s} = \frac{d_1 r V^{4/3} g}{s} . \tag{6}$$

The radius of the spreading zone:

$$r_m = \sqrt{\frac{d_1 r V^{4/3} g}{p s}} . \tag{7}$$

The minimum thickness of the liquid layer:

$$h_{\min} = \frac{s}{d_1 r g V^{1/3}} , \tag{8}$$

or

$$h_{\min} = \frac{H^2}{V^{1/3}} ,$$

where $H^2 = \dfrac{s}{d_1 r g}$.

The effect of friction force. When the liquid spreads over the area S, the friction force acts:

$$F = hS\frac{du}{dh} ,$$

where $h = rv$ is the coefficient of dynamic viscosity, $S = pr^2$ is the area of the liquid spot radius r, u is the horizontal spreading rate, h is the vertical coordinate. Take into consideration that

$$\frac{du}{dh} = \frac{du}{dr}\frac{dr}{dh} .$$

For the cylindrical volume of the liquid $V = pr^2 h = const$,

$$\frac{dr}{dh} = -\frac{r}{2h} = -\frac{pr^3}{2V} .$$

According to the ratio (4):

$$r(t) = at^{1/4},$$

where $a = C_3 \dfrac{g^{1/6}V^{1/3}d}{v^{1/12}} = \dfrac{3}{2}\dfrac{g^{1/6}V^{1/3}d}{v^{1/12}}$.

Then

$$u = \frac{dr}{dt} = \frac{a}{4}t^{-3/4} = \frac{a^4}{4r^3}.$$

Therewith

$$\frac{du}{dr} = -\frac{3a^4}{4r^4},$$

$$\frac{du}{dh} = \frac{3pa^4}{8Vr} = \frac{b}{r},$$

where $b = \dfrac{3pa^4}{8V}$.

Then

$$F = hS\frac{du}{dh} = phbr.$$

The work of the friction force:

$$A = \int_0^{r_m} F(r)dr = \frac{p}{2}hbr_m^2, \qquad (9)$$

where r_m is the maximum radius of the liquid spreading zone, limited by the friction force. Equating the expression (5) to the relation (9), we get

$$S_m = pr_m^2 = \frac{2E}{hb} = \frac{256}{243p}\frac{g^{1/3}d_1}{v^{2/3}d^4}V,$$

$$r_m = \sqrt{\frac{2E}{phb}} = \frac{8}{9hd^2}\sqrt{\frac{g^{1/3}d_1}{3v^{2/3}}V}. \qquad (10)$$

Therewith, the minimum thickness of the liquid film

$$h_{\min} = \frac{243p}{256}\frac{v^{2/3}d^4}{g^{1/3}d_1}. \qquad (11)$$

The presented model is valid in the absence of external forces caused by winds, currents, surface waves and turbulent diffusion. Winds and currents contribute to the drift of the liquid film. Surface waves cause wave drifts, they make the liquid blend when the waves collapse. Turbulent diffusion also leads to the increase in the liquid spot area. In the process of turbulent diffusion, partial evaporation of the liquid film, its dissolution, biodegradation, and changes of chemical composition occur.

Conclusions

The non-linear mathematical model, which describes liquid floaty fraction spreading on the surface of waterbodies, has been developed in this paper.

The author has calculated the time dependence of the radii of the liquid floaty fraction spreading zones. According to the results of the calculation, in the absence of external forces, a large radius of hazardous substances spreading characterizes the stage of surface tension and viscosity. Dynamic processes lead to the increase in the size of the zones polluted by filtration waters.

The practical significance of the work is that it forms the database describing the extent of water pollution to support management decision-making in the field of environmental safety.

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