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EARTH'S FIGURE CHANGES – GEODYNAMIC FACTOR OF STRESSED-DEFORMED LITOSPHERE STATE

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Purpose. The purpose of this work is to show how redistribution of masses occurs as a result of gravity-rotational and endogenous forces in the evolutionary self-development of the planet, which leads to the transformation of the lithosphere from the sphere to the biaxial and then to triaxial ellipsoid, and vice versa; and changes in compression and the movement of the pole in geological time. Determine the deformation of the figure of the lithosphere due to the reorientation of the figure's pole. **Methodology**. The figure of the lithospheric surface is geometrically rotated relative to the figure of the geoid. The orientation of these figures and the parameters of the ellipsoids that approximate them, have changed during the geological time. Such placement of the lithospheric figure and of the geoid figure can create a stress aimed at bringing the distribution of the lithosphere masses into conformity with the figure of the geoid. The calculation of the parameters of biaxial and triaxial ellipsoids was performed based on the data of the digital Earth surface model ETOPO1. Data from the digital modeling of the paleoDEM relief, obtained in the work of K. Skotese and N. Wright have been used for modelling the transformation of the Earth's figure and in the estimation of the impact of its reorientation on the stress-strain state of the lithosphere in the ancient geological epochs. **Results** The parameters of biaxial and triaxial ellipsoids were calculated for fixed moments of geological time. A comparative analysis of the results of changes in the Earth's figure for paleoDEM and created on the basis of raster images of DSMs, built on palaeogeological data by R. Blakey and K. Skotese, were carried out. The formulas for calculation of displacements and deformations, which are related to the transformation of the figure and the orientation of the upper shell of the planet, are given. The interpretation of the research results of planetary dynamics of the Earth's lithosphere figure and the global deformation state are presented. **Scientific novelty.** The characteristics of the deformation state of the Earth's lithosphere according to modeling of geopaleo-reconstruction in geological time are obtained. Given is the interpretation of the role of gravity-rotational forces in the formation of the global field of stress and the transformation of the lithospheric figure. **Practical significance.** The results will be used in further researches aimed at studying the planetary characteristics of our planet, the dynamics of its changes in time, and the global tension.

Key words: biaxial and triaxial ellipsoid, digital model of the relief of the Earth's lithosphere surface, stress state of the lithosphere, dilatation, displacement deformation.

Introduction

Tasks of a planetary scale are increasingly relevant in the sciences of the Earth, to which the fundamental geodesy belongs. Modern geodesy has become planetary and physical science, that is why scientific methods and satellite technologies are used to solve geodynamic problems associated with the evolution of the Earth, studying the motion of lithospheric plates, moving the center of the mass and the axis of rotation, changing of the gravitational field and spatial coordinates of the figure in time, uneven rotation, and tidal deformations of the globe. Certainly the dynamics of the Earth, and its changes in shape and gravitational field with the flow of time, is the main task of planetary geodesy and geodynamics.

When considering the key problems of planetary geodynamics the figure of our planet is of considerable interest, since changes of its surface are inextricably linked with the geodynamic and tectonic processes, with the evolution of the Earth.

In the historical context, the solution of the natural sciences tasks of astronomical geodesy at the turn of the XIX–XX centuries filed in the famous works of J. Stokes, F. A. Sludskii, G. Bruns, A. Poincare, F. R. Helmert, in which examples are given of the close connection between astronomy, geodesy, mechanics, and mathematics.

New development of astronomical geodesy in the first half of the twentieth century associated with the works of F.N. Krasovskii [Krasovsky, 1947, 1955, Mashimsov, 1999], in which a new subject area of geodesy – physical geodesy – is defined as "the problem of studying the internal structure of the globe, the Earth's hard shell, disturbances of equilibrium in it, deformations and displacements of various parts of the lithosphere". Source of quote?

Fundamental importance in the geodesy also includes the works by M. S. Molodensky, [Molodensky 1945, 1958], in which the author formulated the statement of the problem of determining the physical surface of the Earth; if measurements of *g* (gravitational acceleration) were performed on it and the values of the potential of gravitation were calculated. In the framework of such a statement, the inverse gravimetric problem actually placed on the boundary surface, which needs to be determined. At the present time, when satellite technologies allowed to obtain with high accuracy the the heights of the physical surface relative to the reference ellipsoid, the inverse gravimetric problem formulated as the determination of the value of the potential of gravitation of the known surface of the Earth at measured values of g on it [Moritz, 1994, 2001, Hofmann-Wellenhof, and Moritz, 2007]. It should also be noted that the data about the gradients of the potential of gravitation on the Earth's surface provide the ability to determine the angles between the plumb lines and normal plumb lines to the physical surface of the Earth's surface, which leads to a rethinking of many problems within the Earth sciences, and in particular the problems of calculating planetary tensions [Rebetskii, 2016].

At the present stage of studying the figure of the Earth it is considered that its shape is close to the surface of reference of the potential of gravitation forces – the geoid, which is the sum of gravitational potential of attraction forces and the potential of centrifugal forces. Usually, under the figure of the Earth it is necessary to understand the form of physical surface of the solid part of the planet (lithosphere). However, the concept of "Earth's surface" is ambiguous and allows for different interpretations. For example, under the Earth's surface you can understand the surface of its solid shell, that is, the surface of the land and the bottom of the oceans or the physical surface on the continents and the surface of The World Ocean. Until recently (when there were no space technologies), determining the real Earth's surface was a very difficult problem. Now in the databases quite accurate information is collected about the topography of the Earth's surface both for the continents (results of measurements up to 1–2 cm), and for the oceanic and seabed (accuracy is an order of magnitude smaller). In this connection, it is possible to calculate the average of the Earth's surface in the form of an ellipsoid of rotation or a triaxal ellipsoid, which is best approximated to the real surface of the lithosphere [Tserklevych et al., 2016].

The Earth's rotation is the most important factor determining the parameters of the equilibrium shape of the planet. The peculiarities of the rotation motion provide information about the internal structure of the Earth, and the variations of the rotational regime (speed of rotation and movement of the pole) are a real source of energy for tectogenesis. The mathematical reasoning of the role of this factor in tectogenesis was realized in numerous works by mathematicians A. Verona, P. Appel, L. S. Leibenzon, geodesist M. V. Stovas and others.

In particular, in the works by M. V. Stovas [Stovas, 1975] were done estimates of the stresses caused by the change in polar compression due to the slowdown in the rotation of the planet. He relied on the data that over the past 2/500 years the day increased on 1.16×10^{-11} from the Earth's total turnover. According to these data, the decrease in the speed of rotation should lead to a corresponding reduction of the polar compression for the ellipsoid, which characterizes the average surface of reference of the potential of gravitation in the planetary scale, which, in turn, leads to the deformation of the Earth's figure. Wherein arises latitudinal, meridional and radial stresses in the lithosphere, which is the cause for the emergence of a particularly stressed state in latitudinal zones between 30–40° in both hemispheres – zones of change of the sign of the main stresses. However, calculations by M.V. Stovas at the change of polar compression $d\varepsilon = 10^{-7}$ showed that the level of meridional normal stresses in the crust lies in the range from –230 to 130 din/cm^2 (from –23 to 13 Pa). This is a very low level of stress, much smaller (in 4.0–4.5 times) of the value operating in seismic regions [Rebetsky and Marinin, 2006; Rebetskii, 2009; Rebetsky, 2015; Rebetsky et al., 2013].

The periodic change in speed leads to a change in the parameters of the planet's rotation. The most short-period from them have duration of 21, 40, 100, 400 Ka, and 1.2 Ma are reflected in the changes in the tilting of Earth's rotation axis, its precession and orbit [Khain, 2010]. They were called the Mylankovich's cycles and were used by this author to explain the periodicity of the glacial and interglacial epochs during the last glacial period in the history of the Earth [Mank and MacDonald, 1964].

In the late 1980's, A. Scheidegger [Scheidegger, 1987] in the chapter "Earth's rotation" of the monograph "Fundamentals of Geodynamics" gave a generalized conclusion about the possibility of deformation of the lithosphere under the influence of the age-old slowdown in the speed of rotation of our planet due to tidal friction from the Moon. For example, at the beginning of the Phanerozoic, the duration of the day was only 20.5 hours, and then slowly slowed down by 2 ms in 100 years and reached the present 24 hours. Discharge of stresses, which accumulate during a certain period of geological time, when reaching the critical values of the strength of the rocks of the tectonosphere, leads to tectonic activation of the Earth.

The axial rotation of the Earth relates the differential delimitation of the polar and equatorial region, which passes through a parallel 40º. The consequence of such a division is the opposite direction of transgression and regression of the ocean in these regions, increased seismicity of this zone, which is characterized by the most dismembered sections of the relief of our planet [Odeskyi, 2004]. The uneven rotation of the Earth, its periodic acceleration and deceleration has another very important effect. They cause a change in the shape of the Earth, its polar compression and, most importantly, cause strain in the lithosphere, which, in the opinion of most researchers, is the only reason for the formation of the so-called rhagmatic grid of faults, cracks, and lineaments, which are naturally oriented relative to the figure of the Earth, forming an orthogonal and diagonal grid on its surface [Khain, 2010]. It is natural to associate the formation of latitudinal transverse faults systems, which are crossing the spreading mid-ocean ridges in the meridional direction with the Earth's rotation. The meridional and oblique faults in the Indian Ocean may be related to the northern drift of the continents [Levin, 2001, Khain, 2010].

There is now an increasing interest in understanding the role and significance of rotational processes, since rotational forces are practically the main component in most geodynamic concepts. In recent decades, the influence of rotational forces is also considered within the framework of the concept of the shell structure of the Earth, which brings its specificity into the problem under consideration. The problem of formation of additional planetary stresses in the crust from the action of tangential inertial mass forces caused by the daily Earth's rotation is considered in the paper [Rebetsky, 2016]. According to the results of the research, it was established that three levels of stressed state of different geodynamic type are formed in the crust: horizontal stretching, mudslide and compression with the orientation of maximum compression in the meridional direction. The author concludes that these stresses influence the patterns of the formation of planetary fracturing.

Another article of the same author [Rebetsky, 2016] considers the role of tangential mass forces in the emergence of lateral movements of lithospheric plates. It is proposed to calculate the amplitudes of such tangential mass forces based on the difference between two global ellipsoids of rotation. The first of them averages the level surface of the gravity potential (reference ellipsoid), and the secondly – the Earth's physical surface separately in its continental and oceanic parts. It is shown that the dynamics of the Earth's compression is equal to 1/305.5 and obtained from satellite measurements, corresponds well to the average polar compression of two ellipsoids of rotation, which approximate the shape of the Earth's physical surface. Thus, the Earth's physical surface is described by an ellipsoid having less polar compression than the reference ellipsoid (1/298.25), which approximates the shape of the level surface of gravity (geoid). The angles of deviation of the vector of gravity from the Earth's physical surface normal, calculated from the data of these ellipsoids, have rather small values (the maximum value of 16.4" at latitude 45°), which defines the minimum values of tangential mass forces (2.15×10^{-4} g/cm^3 at a latitude of 45°). These small tangential forces can lead to the appearance of a tangential stress of 0.3 MPa at the bottom of the continental lithosphere (depths 120–150 km). Stresses of such level may create a horizontal flow in the asthenosphere, which provides the speed of lithospheric plates in the first centimeters per year. The performed estimates allow us to consider tangential mass forces as a possible source of lithospheric plates motion.

K. Tyapkin and M. Dovbnich [Tyapkin, Dovbnich, 2009] quantitatively substantiated that the Earth's rotation is the source of the forces and energy necessary for the formation of phenomena such as tectonic activation of the Earth, world transgressions and regressions, and even the generation of a magnetic field, namely variations of its rotational mode (changes of angular velocity and displacement of the rotation's axis), which are the result of the interaction of our planet with the surrounding space fields. According to their calculations it turned out that the main contribution to the field of rotational stresses is not the variation of the speed of rotation (1 %), but changes in the position of the axis of rotation in the body of the Earth (99 % of the required forces and energy for the emergence of tectonic activations of the Earth).

Consequently, a review of scientific publications provides generalized information on the role of rotational regime in the Earth's tectogenesis and provoke that this mode should serve as the starting moment and the basis of mathematical and physical modeling of any geodynamic process.

Purpose

The purpose of the research is to determine the displacement of points of the Earth's surface in geological time based on the results of geopaleoreconstructions using the paleo-digital model of relief (paleoDEM) [Scotes and Wright, 2018]. It is clear that it is best to estimate the global displacements of the lithospheric surface, generalizing it in the form of simple geometric shapes (biaxial and triaxial ellipsoids). Topological non homothetic of placing a figure of a lithosphere and a geoid figure can create tectonic stresses. They are aimed at bringing the distribution of the masses of the lithosphere in accordance with the figure of the geoid (according to the mechanism of gravitational forces and the principle of minimizing potential energy), and due to the discharge of stresses, there is a mechanism of approaching the polar axis of the generalizing lithosphere to the axis of rotation (the polar axis of inertia). In addition, the field of rotational stresses caused by the displacement of the axis of the generalizing lithosphere figure obviously will tend to movements in the Earth's lithosphere synchronously with the movement of the axis of the figure in the process of its evolution. In this regard, the article also considers one of the reasons of possible causes of the lithospheric stress associated with the change in the Earth's shape in the process of evolution, i.e. it is assumed that the generation of the field of stress is conditioned by the transformation of the topological surface from one ellipsoid to another. In this sense, the modelling of the stress state of the Earth's lithosphere may possibly allow us to come closer to the correct understanding of the influential factors and the role of driving forces in the concept of plates tectonics.

Formulation of the problem

According to the results of previous studies, it was found that the surface of the lithosphere is geometrically rotated relative to the figure of the geoid and in the geological time changed the orientation of these figures and the parameters of the approximating ellipsoids [Tserklevych et al., 2016]. For the modern era, the value of the turning angle between the smallest axis of the ellipsoid, which approximates the surface of the lithosphere, and the axis of Earth's rotation is 2,6° [Tserklevych and Shylo, 2018]. An interesting task is to track the change of this turning angle in the process of evolutionary self- development of the planet, as this factor accumulates a complex of dynamic mechanisms of the emergence and formation of the main elements of tectonic structures through endogenous processes and the rotary motion of the Earth. As a result of the gravitational-rotational and endogenous forces, a redistribution of masses occurs, which can lead to the transformation of the figure from the twoaxial ellipsoid to the triaxial and vice versa, changes in compression and the axial velocity of rotation and the displacement of the pole in geological epoch. The turn of the ellipsoid's figure describing the surface of the lithosphere relative to the figure of the ellipsoid representing the surface of the geoid is due to the global displacement of the physical surface of the Earth. The global displacement of the lithospheric surface is generated by the mechanism that provoke to move the tectonic plates. The genesis of such forces, among other factors, may be related to the Earth's rotation [Rebetsky, 2016] and the deviation of the ellipsoid's axis, which approximates the surface of the lithosphere, from the axis of rotation [Tserklevych et al., 2017].

Let's accept by primeval state of the Earth the ellipsoid, marked by a solid line (in Fig. 1 this is an ellipse in projection onto a plane) and by dashed line for the state after the displacement of the polar axis of the lithospheric figure on angle θ . This means that the primary state of the surface of the Earth's lithosphere is associated with the geoid (the figure is close to the hydrostatic state), and the axis of rotation and the geoid's axis always coincide with the unlimited ability of the Earth to adapt to the existing dynamic forces in accordance with the moment of movement. In the process of moving the lithospheric plates, each point of the Earth is shifted to a new position: the movement of each point on the surface is indicated by an arrow from T to T'. What conditions accompany the displacement of the figure of the lithosphere? Obviously, they are of two types – "kinematic" and "dynamic". Kinematic conditions determine the change in the topological form of the Earth from one ellipsoid to another (the determination of the parameters of these ellipsoids is also the purpose of this article), for dynamic conditions, it is required that the equilibrium state be saved before and after the displacement. See Fig. 1.

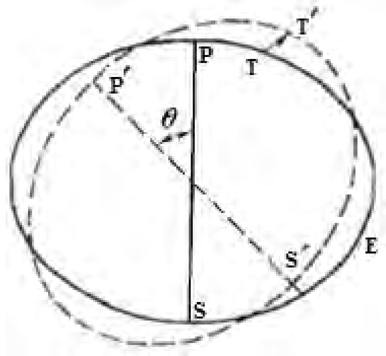


Fig. 1. Illustration for moving of the polar axis

Therefore, we shall consider in the future the kinematic conditions that lead to the difference between metrics of two ellipsoids surfaces, that is, the offset (displacement), which can be interpreted as deformation. The knowledge of the components of deformation allows using known expressions for stresses as a function of deformations in spherical coordinates and to determine the components of the stress tensor for a given rheological model.

Incoming data

To date, the most significant studies in the sense of geopaleo-constructions are performed by Christopher R. Scotese and N. Wright [Scotese and Wright, 2018]. They have built a paleo digital elevation model (paleoDEM) – a digital representation of palaeotopography and paleobathymetry, which was "reconstructed" in time. PaleoDEM accumulates the creation of 120 palaeogeographical reconstructions, which are part in the palaeographic atlas PALEOMAP. The basis for the construction of the paleographic atlas was laid 177 fragments of the paleo reconstructions of the Phanerozoic and three fragments of the Precambrian paleo reconstructions. Each PaleoDEM is an estimate of the heights of the surface of the land and the ocean basins' depths, defined in meters with a resolution of 1 ° × 1°.

The construction of digital models of paleotopography and paleobathymetry included several stages. Previously, a global tectonic paleostructure of plates placement was created, that is, paleogeographic maps that represented the ancient distribution of mountain ranges, lowlands, shallow seas and deep ocean basins. The first step was to compile a map of geological deposits that determined the ancient conditions for the accumulation of sedimentary formations, that is, an analysis of data on geological and tectonic history, paleomagnetism, linear magnetic anomalies, paleobiography, and paleoclimateology.

Paleogeography has been mapped for each time interval and corrections have been made to the topography and bathymetry. After that, this information has been converted into a digital display of paleotopography and paleobathymetry (i.e. PaleoDEM). Each high definition PaleoDEM consisted of more than 6 million grid nodes that integrated digital altitude information with a resolution of 10 km × 10 km horizontally and 40 meters vertically. This quantitative paleo-digital model of relief has allowed visualizing and analyzing the change of the Earth’s surface over time, using the GIS software and other computer modeling techniques. All these models are created with a 1° × 1° grid resolution.

The process of creating the paleoDEM began with digital topographic and bathymetric data sets of the modern world map, including Antarctica and the Arctic. These topographical and bathymetric data sets are combined into a global data set with a 1.8 m (6’) step. At the next stage, the individual nodes of the grid were corrected taking into account paleopositions based on the use of the global tectonic model of the PALEOMAP project plates.

At the final stage, modern digital topographic and bathymetric values of elevations, which were corrected and modified with the use of information about lithology and paleoecology, were subject to the elaboration. This was done using modern analogies for ancient geographic images and simple computer graphics techniques. At this stage, digital altitudes information turned into grayscale values, where the white color represented the highest elevations (+10000 m), and black – the deepest ocean hollows (-10000 m). Using 256 values of grayscale, it was possible to display topography and bathymetry with a resolution of 40 meters vertically. For high mountains and deep hollows, less gradient of gray is used, since these regions are only on a small part of the Earth’s surface.

After the completion of the reconstruction of the paleo-heights in shades of grayscale, these values were transformed again into digital altitude values. The resulting digital altitude file is “corrected” by a global paleotopographic and palaeobathymetric surface, or paleoDEM, representing the height of the surface of the land and the depth of the ocean basins at a certain geological time interval. After receiving the digital model, it was converted to a color-scale graduation, common to the average viewing.

It is clear that the problem of creating such maps is very complicated. These materials are a kind of degree of understanding of “reality” in past geological periods by modern science, but this understanding can constantly change depending on the achievements of future technologies and accumulated information, and accordingly, maps of geopaleoreconstruction will be improved.

Methodology

Approximation of the lithospheric surface by biaxial and triaxial ellipsoids

In accordance with the statement of the problem, it is first necessary to determine the size and orientation of such an ellipsoid, which most closely approximates the physical surface of the planet. The solution of this problem was considered on the example of the approximation of the Earth’s lithosphere heights by biaxial and triaxial ellipsoids in the article [Tserklevych et al., 2016].

Consider Fig. 2, where: P_L' – the physical surface of the Earth; G – geoid; E_L – ellipsoid which parameters must be determined; o – the center of the Earth’s masses; o' – the center of the sought-after ellipsoid; oZ – direction of the axis of the Earth’s rotation; $o'Z'$ – line parallel to the axis oZ; $o'Z''$ – the direction of the small axis of the ellipsoid E_L; κ, ς – two of Euler’s three corners; N – direction of the line of nodes; $\theta_0, \lambda_0, \rho_0$ – angles and distance, which determine the displacement of the center of the sought-after ellipsoid from the center of mass; x_0, y_0, z_0 – linear values that determine the displacement of the sought-after ellipsoid; T – a point on the physical surface of the Earth; $oT' = \rho'$; $o'o = \rho_0$; $oT = \rho$; $TT'' = h$; $o'T'' = r$; H – hight of a point T above sea level.

As for the approximation of a triaxial ellipsoid, we turn to Fig. 3, which illustrates the solution to this problem. TL – point on the physical surface of the Earth; T' – point on the surface of an ellipsoid whose parameters must be determined; XT, YT, ZT – coordinates of the point T in the base coordinate system; $X'T, Y'T, Z'T$ – coordinates in the system of the sought-after ellipsoid; x_0, y_0, z_0 – coordinates of the center of the sought-after ellipsoid relative to the base coordinate system; ax, ay, az – semiaxes; $\varepsilon, \psi, \omega$ – three angles of rotation (Euler angles).

The size and orientation of biaxial and triaxial ellipsoids are usually determined on condition $\sum h^2 \to \min$. This condition implies that the sum of the squares of distances between the surfaces of the sought-after ellipsoids and the surface of the lithosphere is minimal. See Fig. 2.

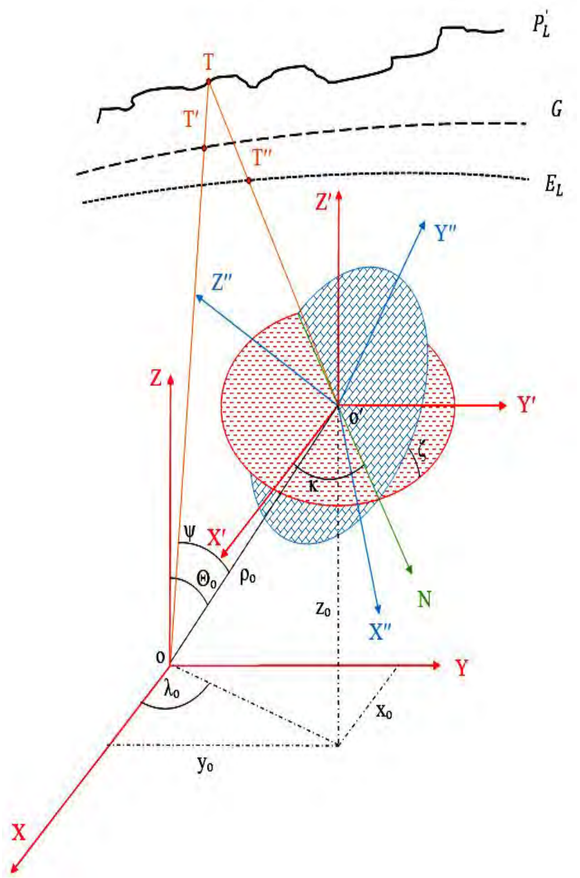


Fig. 2. Illustration of the approximation of the lithospheric surface by a biaxial ellipsoid

In Tables 1 and 2 the values of the biaxial and triaxial ellipsoids parameters are given. The calculations of the ellipsoid's parameters (seven for biaxial and nine for triaxial) were performed based on the data of the digital Earth's surface relief model ETOPO1 [Amante, Eakins, 2009]. This digital relief model covers the entire surface of the planet with a resolution of one minute represented in the geodetic coordinate system WGS84. For further processing of this digital model, the heights were averaged within the trapezes 5º×5º. As a result, 2592 values of the heights of the lithospheric surface h were obtained, which served as input data. See Fig. 3.

We draw attention to the fact that the poles of the smallest axes of the resulting ellipsoids do not coincide with the pole of the Earth, that is, the equatorial plane of the figure of the physical surface is inclined to the geoid's equatorial plane and the corresponding angle of inclination does not exceed 2.6° or ~ 250 km along the meridian arc.

The structural asymmetry of the northern and southern hemispheres is also clearly traced (columns 3 and 4 of Tables 1 and 2), which means that there is an extension of the latitudinal circles of the southern hemisphere and the shortening of the lengths of latitudinal circles of the northern hemisphere, that is, the presence of enlargement of the southern hemisphere and, accordingly, compression northern hemisphere relative to the equator of the Earth. The displacement of the center of mass relative to the center of the figure is also characteristic to the Earth.

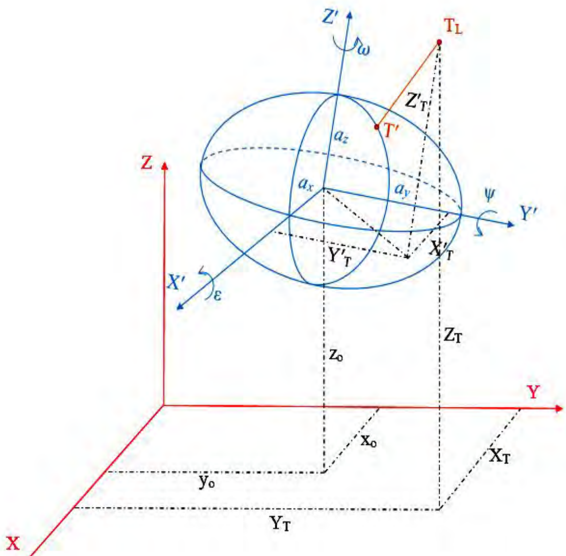


Fig. 3. Illustration of the approximation of the lithospheric surface by a trixial ellipsoid

Table 1

The parameters of biaxial ellipsoid

Parameters	All Planet	Northen hemisphere	Southern hemisphere
x_0, *m*	-741.89	-1170.64	-38.99
y_0, *m*	-491.57	-388.46	1.70
z_0, *m*	1421.79	1868.02	7287.17
a, m	6375117.01	6375114.92	6376300.87
b, m	6355640.09	6355772.53	6362552.67
1/f	327.31650	329.59304	463.79175
κ	133˚.774	159˚.308	110˚.029
ζ	2˚.612	2˚.610	2˚.774

Table 2

The parameters of triaxial ellipsoid

Parameters	All Planet	Northen hemisphere	Southern hemisphere
x_0, *m*	1048.11	1919.36	885.23
y_0, *m*	695.11	640.98	-40.43
z_0, *m*	1109.69	935.40	7333.25
a_x, *m*	6375959.84	6376764.42	6377391.18
a_y, *m*	6374317.51	6373521.24	6375286.65
a_z, *m*	6355605.19	6355769.29	6362642.67
ε, °	1.744	2.290	-0.761
ψ, °	-1.975	1.098	-2.592
ω, °	-84.835	-77.500	49.099
B_{ax}, °	-1.989	1.106	-2.611
L_{ax}, °	84.835	77.500	130.901
B_{az}, °	87.384	87.478	87.318
L_{az}, °	126.296	-38.102	147.260

Consequently, according to the results of studies of the lithospheric surface, this subsequent figure is non-homothetically placed relative to the figure of a geoid. Such placement of the figure of the lithosphere and the geoid figure creates tectonic strains in the lithosphere of the Earth, which is aimed at bringing the distribution of masses in line with the figure of the geoid.

Schematic illustration of the evolutionary formation of the Earth's figure is shown in Fig. 4. If you compare two positions of the outer Earth's shells, then you can observe the emergence of excite state due to changes in the speed of rotation and the position of the axis of lithospheric figure in relation to the axis of rotation. The discharge of these stresses at reaching critical values equal to the limit of the strength of the lithosphere rocks leads to tectonic activation of the Earth.

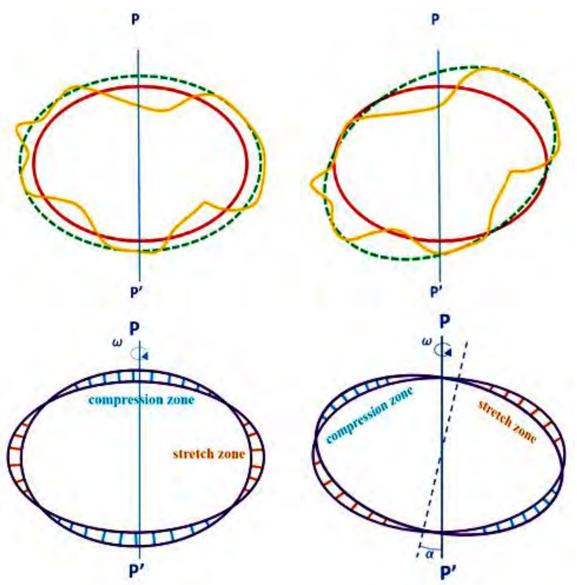


Fig. 4. Schematic illustration of the formation of the Earth's figure and the distribution of stress in the lithosphere:
____ surface of lithosphere, ____ an ellipsoid that is best suited to the surface of the lithosphere, ____ an ellipsoid that represents the Earth's surface in a distant geological epoch.

Determination of horizontal deformations and tense state due to the reorientation of the lithosphere's figure

Let us consider in more detail how it is possible to mathematically realize the kinematic conditions associated with the deformation of the lithosphere as a result of reorientation of the pole of the Earth's figure. It is well known that the difference in the elements of the length of the arc before and after the deformation is closely related to the deformation tensor [Rashevsky, 1967, Gadomska, B., & Teisseyre, 1984, Tadyeyev, 2017]. If mark through $ds^2 = g_{ij}dx^i dx^j$ – the metric of the surface to the deformation and accordingly through the $dS^2 = G_{ij}dx^i dx^j$ – the surface metric after the deformation, then:

$$dS^2 - ds^2 = (G_{ij} - g_{ij})dx^i dx^j,$$
$$dS^2 - ds^2 = 2\varepsilon_{ij}dx^i dx^j, \quad (1)$$

where ε_{ij} – deformation tensor equal to:

$$\varepsilon_{ij} = \frac{1}{2}(dS^2 - ds^2). \quad (2)$$

In this case, the identified metrics are assigned to the same variables x^i, but with different metric tensors g_{ij} and G_{ij}.

Consequently, the linear element ds identifies the metric form of the undeformed region Δ in the initial state, and the dS mapping of a linear element ds corresponding to the transformed region Δ' (the final state corresponding to a transformed figure, a biaxial, or triaxial ellipsoid).

We introduce the components x_i of the point T of the surface through spherical coordinates:

$$x_1 = r\sin\theta\cos\lambda, \quad x_2 = r\sin\theta\sin\lambda, \quad x_3 = r\cos\theta,$$

where r is the radius-vector from the center of the coordinate system to the surface, θ, λ – respectively, the polar distance and longitude.

For biaxial and triaxial ellipsoids, their radius-vectors, respectively, can be expressed by the following relations [Moritz, 1994]:

$$r = R\left(1 + \alpha\left(1/3 - \cos^2\theta\right)\right),$$
$$r = R\left(1 + \alpha\left(1/3 - \cos^2\theta\right) + 0{,}5\alpha'\sin^2\theta\cos 2\left(\lambda - \lambda_0\right)\right), \quad (3)$$

α and α' – average polar and equatorial compressions; R – the radius of the sphere of equal volume; λ_0 – the geographical longitude of the large axis.

The element of the arc length for the ellipsoid surface is as:

$$dS^2 = dx_1^2 + dx_2^2 + dx_3^2 = dM \cdot dM;$$
$$dS^2 = \left(\frac{\partial M}{\partial\theta}d\theta + \frac{\partial M}{\partial\lambda}d\lambda\right)\cdot\left(\frac{\partial M}{\partial\theta}d\theta + \frac{\partial M}{\partial\lambda}d\lambda\right);$$
$$dS^2 = \left(\frac{\partial M}{\partial\theta}\cdot\frac{\partial M}{\partial\theta}\right)d\theta^2 + 2\left(\frac{\partial M}{\partial\theta}\cdot\frac{\partial M}{\partial\lambda}\right)d\theta d\lambda + \quad (4)$$
$$+\left(\frac{\partial M}{\partial\lambda}\cdot\frac{\partial M}{\partial\lambda}\right)d\lambda^2.$$

For biaxial and triaxial ellipsoids, after simple transformations, we obtain respectively:

$$dS^2 = R^2\left[1 + 2\alpha(\frac{1}{3} - \cos^2\theta)\right]d\theta^2 +$$
$$+R^2\sin^2\theta\left[1 + 2\alpha(\frac{1}{3} - \cos^2\theta)\right]d\lambda^2. \quad (5)$$

$$dS^2 = R^2\left\{\begin{matrix}1 + 2\alpha\left(\frac{1}{3} - \cos^2\theta\right) + \\ +\alpha'\sin^2\theta\cos 2\left(\lambda - \lambda_0\right)\end{matrix}\right\}d\theta^2 +$$
$$+R^2\sin^2\theta\left\{\begin{matrix}1 + 2\alpha\left(\frac{1}{3} - \cos^2\theta\right) + \\ \alpha'\sin^2\theta\cos 2\left(\lambda - \lambda_0\right)\end{matrix}\right\}d\lambda^2. \quad (6)$$

For a spherical surface, the element of an arc expressed through the classical relation:

$$ds^2 = R^2 d\theta^2 + R^2 \sin^2\theta d\lambda^2. \quad (7)$$

By introducing (5), (6) and (7) into (2), we obtain deformations due to changes in form and surface, respectively:

– for a triaxial ellipsoid relative to the sphere:

$$\varepsilon_{\theta\theta} = \varepsilon_{\lambda\lambda} = \alpha\left(\frac{1}{3} - \cos^2\theta\right) + \frac{1}{2}\alpha'\sin^2\theta\cos 2(\lambda - \lambda_0); \quad (8)$$

– for a biaxial ellipsoid relative to the sphere:

$$\varepsilon_{\theta\theta} = \varepsilon_{\lambda\lambda} = \alpha\left(\frac{1}{3} - \cos^2\theta\right); \quad (9)$$

– for a triaxial ellipsoid relative to the biaxial:

$$\varepsilon_{\theta\theta} = \varepsilon_{\lambda\lambda} = \frac{1}{2}\alpha'\sin^2\theta\cos 2(\lambda - \lambda_0). \quad (10)$$

Hooke's law determines the classical connection between stress tensors and deformations in an elastic, homogeneous, isotropic medium [Zharkov & Trubitsyn, 1980]:

$$\sigma_{ij} = \lambda\delta_{ij}\varepsilon_{11} + 2\mu\varepsilon_{ij} \quad (11)$$

λ, μ – Lame's coefficients associated with the Young's modulus and the Poisson's ratio, respectively.

For two-dimensional space by analogy we obtain:

$$\sigma_{ij} = 2\mu\varepsilon_{ij} \quad (12)$$

The equation (12) allows us to connect the components of stresses with the deformations calculated in (8), (9) and (10):

$$\sigma_{\theta\theta} = \sigma_{\lambda\lambda} = 2\mu\alpha\left(\frac{1}{3} - \cos^2\theta\right) + \mu\alpha'\sin^2\theta\cos 2(\lambda - \lambda_0) \quad (13)$$

$$\sigma_{\theta\theta} = \sigma_{\lambda\lambda} = 2\mu\alpha\left(\frac{1}{3} - \cos^2\theta\right); \quad (14)$$

$$\sigma_{\theta\theta} = \sigma_{\lambda\lambda} = \mu\alpha'\sin^2\theta\cos 2(\lambda - \lambda_0). \quad (15)$$

An important characteristic for the overall estimation of compression and tension as a result of the transformation of the figure is dilatation. The dilatation is defined as a tensor of deformation and, accordingly, for different transformations of the surface of the Earth's lithosphere, we obtain:

$$\Delta = \varepsilon_{\theta\theta} + \varepsilon_{\lambda\lambda}. \quad (16)$$

– for a triaxial ellipsoid relative to the sphere:

$$\Delta = 2[\alpha\left(\frac{1}{3} - \cos^2\theta\right) + \alpha'\sin^2\theta\cos 2(\lambda - \lambda_0)]; \quad (17)$$

– for a biaxial ellipsoid relative to the sphere:

$$\Delta = 2\alpha\left(\frac{1}{3} - \cos^2\theta\right); \quad (18)$$

– for a triaxial ellipsoid relative to the biaxial:

$$\Delta = 2\alpha'\sin^2\theta\cos 2(\lambda - \lambda_0); \quad (19)$$

Let us now consider how it is possible to estimate deformations and stresses, which leads to a reorientation of the lithosphere's figure due to the drift of lithospheric plates. These deformations and stresses are to some extent the result of a change in the surface's curvature. The time required to move the lithospheric plates is not taken into account. Only the total change, caused by the transformation of the ellipsoid, that is, between its initial and final positions, is fixed. The difference between the designated conditionally "topological state 2" and "topological state 1" can be regarded as a field of deformations and stress arising as a result of transformation from the first ellipsoid into the second, or also as a field of deformations and stress due to changes in the ellipsoidal form. To obtain the necessary formulas we use the symbols shown in Fig. 5. From the spherical triangle (Fig. 5) we have:

$$\cos\gamma = \cos\theta\cos\theta_0 + \sin\theta\sin\theta_0\cos(\lambda - \lambda_0). \quad (20)$$

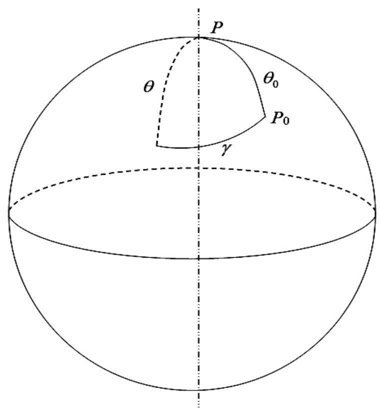


Fig. 5. Schematic illustration of the deviation of the pole of the figure: ***P*** – position of the pole in the initial state (1) of the figure of a biaxial ellipsoid; P_0 – the position of the pole in the final state (2) after the displacement of the figure of a biaxial ellipsoid

Taking into account the formulas (9) and (14) and the expression for the Legendre polynomial of the 2nd order $P_2(\cos\theta) = \frac{3\cos^2\theta - 1}{2}$, we obtain the general formulas for the deformations and stresses that arise as a result of the transformation of one ellipsoid into another in the original reference frame

$$\varepsilon_{\theta\theta} = \varepsilon_{\lambda\lambda} = -2\left(a_1 P_2(\cos\gamma) + a_2 P_2(\cos\theta)\right); \quad (21)$$

$$\sigma_{\theta\theta} = \sigma_{\lambda\lambda} = -2\mu\left(a_1 P_2(\cos\gamma) + a_2 P_2(\cos\theta)\right); \quad (22)$$

Assuming that the compression of the ellipsoids remained unchanged, then, respectively $a_1 = a_2$, we obtain:

$$\varepsilon_{\theta\theta} = \varepsilon_{\lambda\lambda} = 3\alpha_1\left(\frac{3\cos^2\theta - 1}{2} - \frac{1}{2}\sin^2\theta\cos 2\lambda\right); \quad (23)$$

$$\sigma_{\theta\theta} = \sigma_{\lambda\lambda} = 3\mu\alpha_1\left(\frac{3\cos^2\theta - 1}{2} - \frac{1}{2}\sin^2\theta\cos 2\lambda\right). \quad (24)$$

Consequently, as a result of the reorientation of the figure to a small value (say, 3 degrees), taking into account (21) and (22), we will have:

$$\varepsilon_{\theta\theta} = \varepsilon_{\lambda\lambda} = -2\alpha_1 \begin{pmatrix} \frac{3}{2}\left(\cos^2\theta_0 - 1\right)P_2\left(\cos\theta\right) + \\ +3\sin\theta\cos\theta\sin\theta_0 \times \\ \times\cos\theta_0\cos\left(\lambda-\lambda_0\right) + \\ +\frac{3}{4}\sin^2\theta\sin^2\theta_0\cos 2\left(\lambda-\lambda_0\right) \end{pmatrix}; \quad (25)$$

$$\sigma_{\theta\theta} = \sigma_{\lambda\lambda} = -2\mu\alpha_1 \begin{pmatrix} \frac{3}{2}\left(\cos^2\theta_0 - 1\right)P_2\left(\cos\theta\right) + \\ +3\sin\theta\cos\theta\sin\theta_0 \times \\ \times\cos\theta_0\cos\left(\lambda-\lambda_0\right) + \\ +\frac{3}{4}\sin^2\theta\sin^2\theta_0\cos 2\left(\lambda-\lambda_0\right) \end{pmatrix}. \quad (26)$$

To solve the problem of finding the vectors of moving the points on the surface of the figure approximate the Earth's lithosphere, as a result of transformation of one figure into another due to the displacement of the pole of the figures small axis, it is necessary to find the values of the displacement of points along the meridian dB_i and the parallels dL_i. Let's consider an example of one point A (Fig. 6). The figure shows the position of two points on the surface of the output ellipsoid: the point A with the input coordinates B, L, and the point A', which illustrates the displacement due to the reorientation of the ellipsoid. It is clear from the figure that dB and dL are not known values of moving points along latitude and longitude, that is, the projection of the segment AA' to the meridian and the parallel. Geodetic coordinates of point A are known in the coordinate system of the output ellipsoid, as well as the parameters of reorientation (coordinates of the center of the figure and angles of rotation around the corresponding axes).

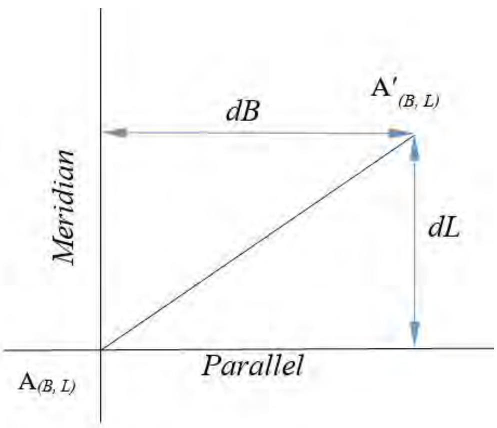


Fig. 6. Illustration to determine dB and dL

It is necessary to determine the changes of geodetic coordinates as a result of reorientation of the figure. The general course of the solution illustrates the expression:

$$B_A, L_A, H_A \rightarrow X_A, Y_A, Z_A \rightarrow X'_A, Y'_A, Z'_A \rightarrow B'_A, L'_A, H'_A. \quad (27)$$

That is, we first move to Cartesian geocentric coordinates by known formulas [Marchenko et al., 2013]:

$$X_A = (N+H)\cdot\cos B_A\cdot\cos L_A; \\ Y_A = (N+H)\cdot\cos B_A\cdot\sin L_A; \\ Z_A = \left[N\cdot\left(\frac{b^2}{a^2}\right) + H_A\right]\cdot\sin B_A. \quad (28)$$

The next step is to take into account the parameters of the reorientation of the figure. By known parameters of the transition between two rectangular geocentric coordinate systems, we perform a transition $\left(X_A, Y_A, Z_A \rightarrow X'_A, Y'_A, Z'_A\right)$, or in a matrix form:

$$\begin{vmatrix} X_A \\ Y_A \\ Z_A \end{vmatrix} = \begin{vmatrix} X'_A \\ Y'_A \\ Z'_A \end{vmatrix}\cdot R + \begin{vmatrix} x_0 \\ y_0 \\ z_0 \end{vmatrix}, \quad (29)$$

where R is the matrix of the turn, and x_0, y_0, z_0 is the coordinates of the center of the figure.

For the transition from the Cartesian coordinate system to a geodetic, we use the well-known formulas [Marchenko et al., 2013]

$$B'_A = arctg\left(\frac{Z'_A + N\cdot e^2\sin B'_A}{(N+H'_A)\cos B'_A}\right) \\ L'_A = arctg\left(\frac{Y'_A}{X'_A}\right) \\ H'_A = \frac{(N+H'_A)\cos B'_A}{\cos B'_A} - N \quad (30)$$

Differences of ellipsoidal coordinates, brought to one figure of attribution, we will calculate according to the formulas:

$$dB = B'_A - B_A; \\ dL = L'_A - L_A; \\ dH = H'_A - H_A. \quad (31)$$

Projections dB, dL are expressed in kilometers on the surface of the output ellipsoid by the following formulas:

$$dx = dB\cdot M; \\ dy = dL\cdot N\cdot\cos L; \\ dz = dH. \quad (32)$$

To solve the problem, a grid was set with a step of 10 degrees (from -80 ° to + 80 ° latitude and from – 180 ° to + 180 ° longitude).

The results obtained and discussion

According to the approximation of biaxial and triaxial ellipsoids of the heights of the Earth's physical surface, results were obtained that characterize their parameters in the modern period (see Tables 1 and 2), and for various past geological epochs, according to the data obtained by K. Scotese and N. Wright [Scotese and Wright, 2018]. In the article [Tserklevych at al., 2017] similar studies of geoevolutionary changes in the figure of the Earth were carried out on the basis of two variants of raster maps of the paleoreconstruction of the continents placement and the water surface, which were independently created by R. Blakey [Blakey, 2016] and K. Scothese [Scotese, 2017]. Since all raster images were created consistently in certain colors, by finding a connection between the image of the altitudes of the model ETOPO1 and bitmap maps of the paleoreconstruction, with a certain approach for transition to the digital surface topography model (DSM) of the Earth's lithosphere for geological epochs that were tied to corresponding maps showing the continents and the water surface. Previous sentence is far to unnecessary long. It would be advisable to show the relation between the results of previous studies and the results of approximation by biaxial and triaxial ellipsoids of the heights of the paleo-digital model of relief (paleoDEM).

Fig. 7 shows the graphs of the change of the minor and major semi-axes of the biaxial ellipsoid, as well as the mean radius during geological time. The blue color shows a curve that corresponds to the approximation of the modeled values for raster paleoreconstruction by R. Blakey. The red curve represents the modeled values of K. Scotese. Green represents the results obtained by the input data, which are covered in this paper. As we see, changes in all quantities reflect such tendencies in geological time, although some differences are present. Such a division demonstrates the reliability of the results obtained, since different inputs data were used.

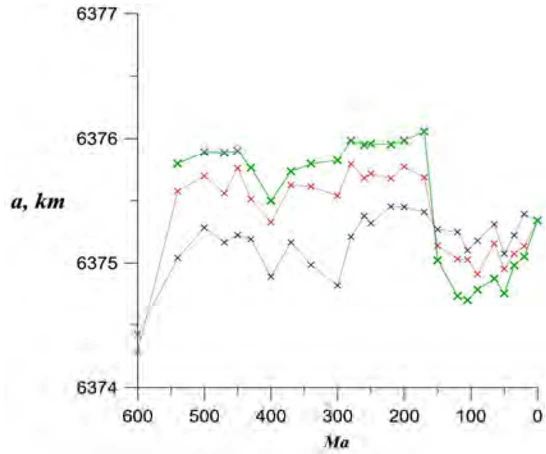
Interesting for understanding is Fig. 8, which shows the positions of the pole of the minor axis of the figure of a biaxial ellipsoid, depending on latitude. As we see here, there is no relative displacement, obviously, because the angular values, unlike the sizes of axes and compression, are independent of the scale of the ellipsoid's models. We also note that in the position of the pole there are clearly two major deviations from the axis of rotation. This is in the modern period and nearly 400 Ma. In the time interval of 100–300 Ma, the deflection of the pole was within 1 °. Proceeding from the figure, the results according to K. Skotese and R. Blakey are well coordinated. Consequently, the significant movement of the pole of the figure, if given to it semantic geological meaning, may indicate the activation of tectonic processes in these periods of geological time.

Note, that in the results obtained from the approximation of the heights of the paleo-relief, there are clear regularity in the compression of two ellipsoids (biaxial and triaxial), which are shown in Fig. 9. An orange color reflects the compression of the figure of a biaxial ellipsoid, while black is a compression of the triaxial ellipsoid in the plane of axes a_x , a_z :

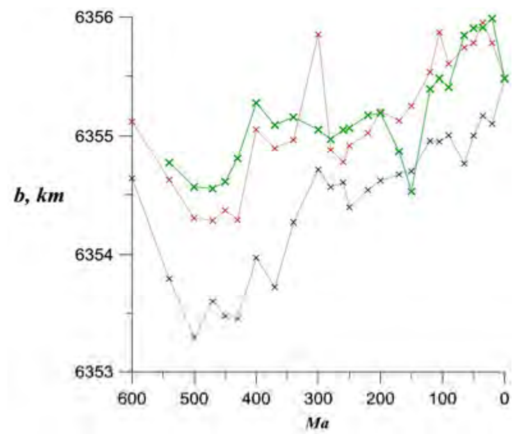
$$\alpha = \frac{a_x - a_z}{a_x} \quad (33)$$

Yellow – in the plane of axis ax, az:

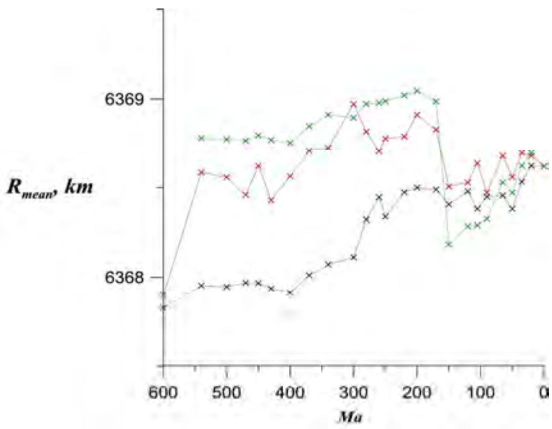
$$\alpha = \frac{a_y - a_z}{a_y} \quad (34)$$



a



b



c

Fig. 7. Changes in the parameters of a biaxial ellipsoid: a – a major semi-axis, b – a minor semi-axis, c – an average radius during geological time

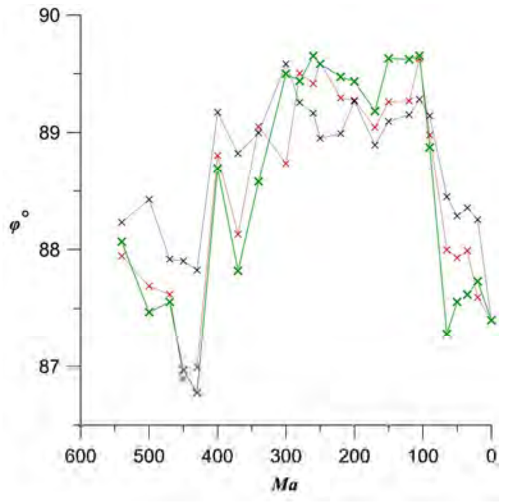


Fig. 8. Deviation of the pole of a biaxial ellipsoid during geological time

Purple – polar compression:

$$\alpha = \frac{\sqrt{a_x a_y} - a_z}{\sqrt{a_x a_y}} \qquad (35)$$

Also, in this graph, two periods in the history of the Earth (470 Ma and 160 Ma) can be traced (according to the biaxial ellipsoid), during which the compression was maximal, we note that the third period with the greatest compression of the planet takes place at the present time. The difference between the ellipsoid semi-axes is about 20 km.

The opposite trend is observed in the period of 65 Ma, when the planet had less compression, and the difference between the semi-axes was about 17 km. The difference in the compressions of ellipsoids (size of 3 km) transformed the area of the figure which is the most significant in the polar zones has caused tectonic activation.

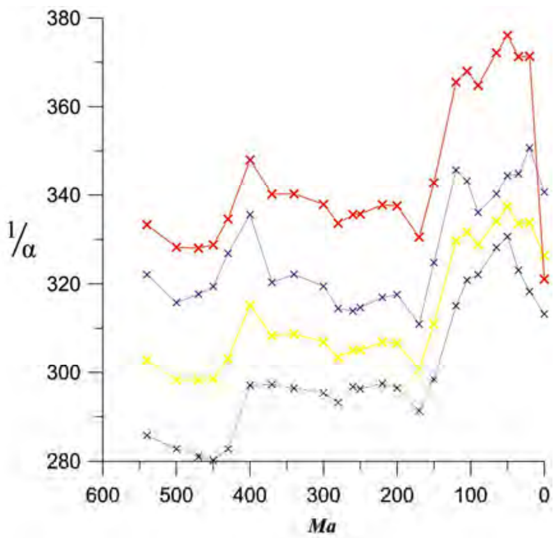


Fig. 9. Changes in compression of a biaxial and triaxial ellipsoids during geological time

Interesting are the graphs for changing the difference between the major and minor axes of a triaxial ellipsoid in the plane of the equator, which are shown in Fig. 10. They show a clear transformation of the two-dimensional ellipsoid into a trivial one, which occurred 400 million years later. In that period of the geological time, the continental and oceanic surfaces of the lithosphere were best described by a triaxial ellipsoid with a difference in axes in the equatorial plane of ~3.2 km, and 50 million years later, a similar indicator reached only 0.5 km. At the present time, this indicator is 1.5 km.

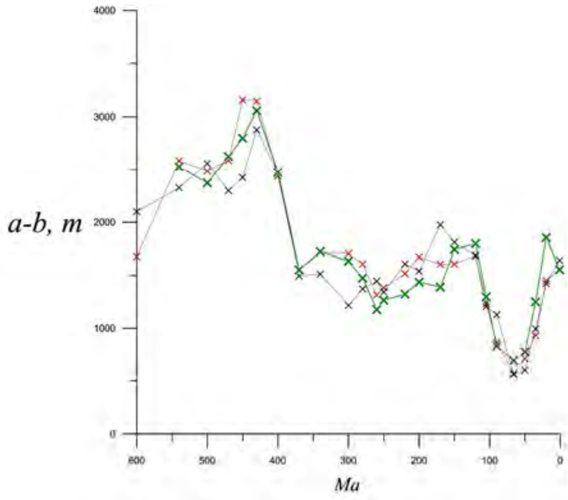


Fig. 10. The graphs of the difference between the major and minor axes of the triaxial ellipsoid in the plane of the equator by the results of the modelling

The general system of displacements is a system with two vortices whose focuses are located in the region of equator, and the landslide deformation parameters in them are minimum about 20 km (Fig. 11). As deviations from these epicentres deformation parameters increase. From the figure we see that the "main" lines of deformation are located along the meridians, with these strips at a width of about 50 degrees in length. Their maximum value is about 320 km in the period of 470 Ma. Also, maximum displacement is observed in the polar regions. Directions of displacement are opposite in these vortices, and are absolutely symmetric. As the parameters of the reorientation of the figures change, the epicenter moves somewhat. It is logical that this should cause a constant change in the stresses in the cortex and, accordingly, a change in the tense state of the lithospheric shell. Therefore, there must be a moment of critical stresses that provoke irreversible deformation processes that give rise to linear structures and faults.

Let's go now from landslide to dilatation, which arises as a result of changing the figure. For example, from a sphere to a biaxial ellipsoid (Fig. 12, *b*). The values that appear on it are calculated by the expression (18). As we know, dilatation is a change in square. So, on the chart, blue shows the areas of the figure where the square is decreasing, and the red – where increases.

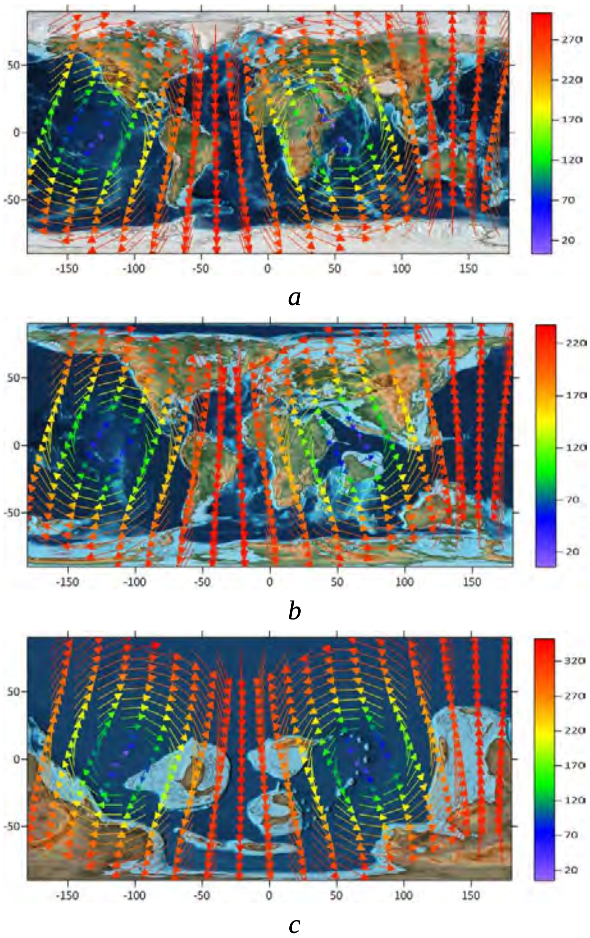


Fig. 11. Moving the points on the surface of the ellipsoid (km): a – the modern era, b – 65 Ma, c – 470 Ma

As we see in the graph, the zone of zero deformation passes in the region of the 50th parallel in the northern and southern hemispheres. The positive values of the dilatation coefficient are concentrated around the equator, and negative in the polar regions. The maximum and minimum values are respectively at the equator and poles.

Fig. 12, a is built from the data calculated by the expression (17). The picture is somewhat different from Fig. 12, b. The contour of the equatorial zone with positive coefficients has become the shape of a strip that periodically narrows and expands. In the places of expansion there are 4 maximums of dilatation. Negative coefficients formed oval zones at poles.

Fig. 12, c illustrates the transformation of a triaxial ellipsoid into a biaxial. Here there is an absolute symmetry, through 0 and 180th meridians pass 6 oval zones with maximums, and through -90° and +90 ° symmetrical ovals with minimums.

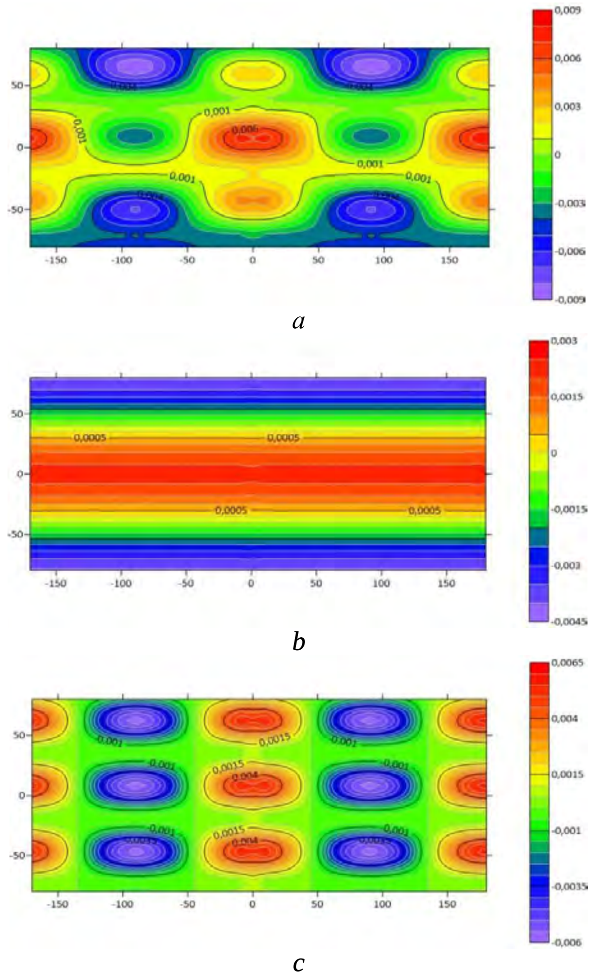


Fig. 12. The dilatation: a – transformation of the sphere to the triaxial ellipsoid, b – transformation of the sphere to the biaxial ellipsoid, c – transformation of the biaxial ellipsoid to the triaxial.

In Fig. 13, the stresses that occur in an elastic shell – an equivalent lithosphere with a reorientation by about three degrees [Tserklevych et al., 2017] – are presented.

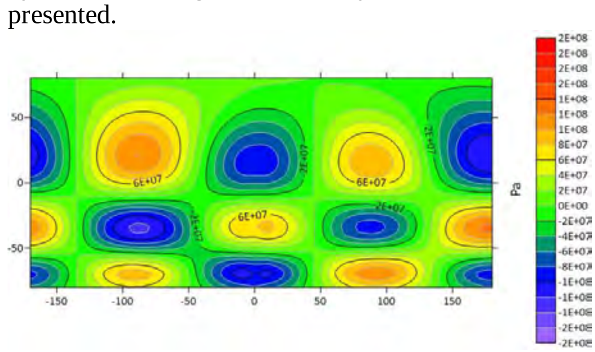


Fig. 13. The stress state of the lithosphere due to the reorientation of the figure

As you can see from the figure, the maximum stress is $\pm 2 \cdot 10^8$ Pa. The constructed mapping on the basis of equation (26), in addition to the specified maximum stresses, allows us to consider the lines of isostresses as the limiting values of the destruction of the lithosphere. However, we must take into account the fact that it may be difficult to directly apply the indicated formulas obtained for an absolutely elastic event to phenomena occurring during geological time scales. We also note that the value of stress leads to the corresponding deformations, and Fig. 13 will be similar, only in the corresponding scale of deformation values.

Scientific novelty and practical significance

Obtained characteristics of the stressed-deformed state of the Earth's lithosphere based on modeling of geopaleoreconstructures in geological time. The interpretation of the role of gravity-rotational forces in the formation of the global field of deformations and stresses as a consequence of the transformation of the surface of the surface of the Earth's lithosphere was given. The results will be used in further researches aimed at studying the planetary characteristics of our planet, the dynamics of their changes in time and the global stressed-deforme state.

Conclusions

A brief retrospective review of the study of the Earth's figure in the context of the development of studies of a stressed-deforme state was carried out in this work from the boundary of the XIX–XX centuries to the present day.

The mathematical transition of the transformation of the planet from the geosphere to the biaxial and triaxial ellipsoids is described in detail through the determination of elements of deformations and stresses. The quantitative characteristic of these transformations are given in the form of figures which show dynamics of changing parameters of figures that approximates the lithosphere and reorientation of these figures. Interesting were the projections of the minor axis of a biaxial ellipsoid on the surface of the lithosphere in different geological epochs. Thus, the maximum deviation of the poles of the minor semi-axis from the north pole of the Earth does not exceed ~3.2° for the 26 ellipsoids which approximate the surface of the lithosphere for different geological epochs, received according to the data of paleoDEM [Scotese and Wright, 2018]. This may indicate the critical magnitude of the deviation of the geographic pole from the axis of Earth's rotation for any configuration of the placement of continents and the water surface.

Also are illustrated change of two parameters of a stress-strain state, which associate it with these transformations of the figure (dilatation and deformation of the shift). Their mathematical relations and figures with the results of calculations are given.

Research of landslide deformations that arise as a result of the reorientation of our planet's thin solid shell showed that a field of landslides is formed on its surface. Vectors of landslides are located in the form of two vortices, and their two focuses are located near the equator. After analyzing them, it can be noted that the values of the vortex centers are minimal, and at the borders – the maximum. The axes of this vortex system are movable, as illustrated by the modelling (Fig. 11).

Due to such deviations of the two main Earth's figures which approximates surface of lithosphere and geoid, individual blocks of the crust are in a permanent state of motion, and accordingly will change the field of deformations and stresses.. In our opinion, this is one of the possible factors of the process which provokes global movements of lithospheric blocks.

As a result, there is a transformation of the lithosphere, which is characterized by a change in the size of the ellipsoid axes, which describes the surface of the lithosphere, and their orientation. In this case, the Earth can be consider as a dynamic system, in which there is a regular change in volume, with the general tendency to increase it. The mass of the Earth remains constant, and the Earth itself cyclically expands

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ЗМІНИ ФІГУРИ ЗЕМЛІ – ГЕОДИНАМІЧНИЙ ФАКТОР НАПРУЖЕНО-ДЕФОРМОВАНОГО СТАНУ ЛІТОСФЕРИ

Мета. Мета цієї роботи – показати, як у процесі еволюційного саморозвитку планети в результаті дії гравітаційно-ротаційних та ендогенних сил відбувається перерозподіл мас, що приводить до трансформації фігури літосфери від сфери до двовісного та тривісного еліпсоїдів і навпаки, зміни сплющеності та переміщення полюса в геологічному часі. Визначити деформації фігури літосфери внаслідок переорієнтації полюса фігури. **Методика.** Фігура поверхні літосфери геометрично повернута відносно фігури геоїда і в геологічному часі орієнтація цих фігур і параметри еліпсоїдів, які їх апроксимують, змінювались. Таке розміщення фігури літосфери і фігури геоїда може створювати напруження, яке направлене на приведення розподілу мас літосфери у відповідність з фігурою геоїда. Обчислення параметрів двовісного і тривісного еліпсоїдів виконувалося на основі даних цифрової моделі поверхні Землі ETOPO1. Для моделювання трансформації фігури Землі і оцінки впливу її переорієнтації на напружено-деформований стан літосфери в далекі геологічні епохи використані дані цифрового моделювання рельєфу paleoDEM, отримані в роботі К. Скотези і Н. Врайта. **Результати.** Обчислені параметри двовісного і тривісного еліпсоїдів на фіксовані моменти геологічного часу. Проведений порівняльний аналіз результатів зміни фігури Землі за paleoDEM та створеними на основі растрових зображень ЦМР, побудованими за палеогеологічними даними Р. Блекі і К. Скотези. Наведені формули для обчислення зміщень і деформацій, які пов’язані з трансформацією фігури і орієнтацією верхньої оболонки планети. Приведена інтерпретація отриманих результатів досліджень планетарної динаміки фігури літосфери Землі та глобального деформаційного стану. **Наукова новизна.** Отримані характеристики напружено-деформаційного стану літосфери Землі за даними моделювання геопалеореконструкцій в геологічному часі. Така інтерпретація ролі гравітаційно-ротаційних сил у формуванні глобального поля деформацій і напружень як наслідок трансформації фігури поверхні літосфери Землі. **Практична значущість.** Подані результати будуть використовуватись у подальших дослідженнях, які спрямовані на вивчення планетарних характеристик нашої планети, динаміки їх змін у часі та глобального напружено-деформованого стану.

Ключові слова: двовісний і тривісний еліпсоїд, цифрова модель рельєфу поверхні літосфери Землі, напружений стан літосфери, дилатація, деформація зсуву.

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