GEODESY

UDC 550.831:528.11:519.281

P. DVULIT[1], J. DZHUN[2]

[1] Department of Higher Geodesy and Astronomy, Lviv Polytechnic National University, 12, S. Bandery Str., Lviv, 79013, Ukraine, e-mail: dvupet@ukr.net.
[2] Department of Mathematical Modeling, International University of Economics and Humanities named after Academician Stepan Demianchuk, 4, S. Demianchuk Str., Rivne, 33027, Ukraine, e-mail: iosif-june@rambler.ru.

https://doi.org/10.23939/jgd2019.01.005

DIAGNOSTICS OF THE HIGH-PRECISE BALLISTIC MEASURED GRAVITY ACCELERATION BY METHODS OF NON-CLASSICAL ERRORS THEORY

The purpose of the investigation is to show the necessity of using modern ideas about the law of error distribution for observations involved in the categories of the "Non-classical error theory of measurements" (NETM) in the process of performing high-precision ballistic definitions of gravitational acceleration. These definitions are characterized by large volumes, which according to the H. Jeffreys' theory, professor at the University of Cambridge, automatically takes them beyond the bounds of the classical concepts about the errors of measurements law. These outdated views about the distribution law of errors of large volume measurements are the main obstacles to improve the methodology of these highly precise and important definitions. **The research methodology** is provided by the NETM-procedures that was designed to control the probabilistic from of the statistical distribution of absolute high-precise ballistic measurements g with large sample volumes based on H. Jeffreys' recommendations and on the principles of hypothesis testing theory. **The main result** of the research is to carry out NETM-diagnostics of a metrological situation with the ballistic gravimeter FG-5 after some improvements of the program of the observation. This method of diagnostics is based on the use confidence intervals to the estimates of asymmetry and kurtosis of the obtained samples of measurements g with the following application of the Pearson's χ^2-test to determine the significance of the deviations of its distribution from the established norms. In accordance with the categories of the NETM, such norms are the Gauss's and Person-Jeffreys's laws, since only they ensure the non-singularity of the weight function of the sample, and therefore the possibility of obtaining non generate estimates g during the mathematical processing of measurements. **Scientific novelty:** using the possibilities of the new important tool in the field "Data analysis" using the NETM to improve the technique of the high-precise measurements g, which are performed in a complicated metrological situation with the necessity of taking into account a number of non-stationary sources of systematic errors. **The practical significance** of the research is in use of NETM-diagnostics of the probabilistic form of the distribution of measurements g in order to improve the methodology of these highly precise determinations. The investigation seeks reasons for the deviations of errors distributions from established norms providing metrological literacy of the high-precise large-scale measurements.

Key words: laws of errors Gauss and Pearson-Jeffreys, absolute measurements gravity acceleration, non-classical errors theory.

Introduction

Absolute, high-precise ballistic measurements of g are performed in a complicated metrological situation, which is constantly changing under the influence of various factors. Such measurement conditions cause the need to perform statistical control of the metrological situation after the ending of the observations. Method of such control is the analysis of the general form of the obtained error distribution of measurements, which was also recommended by Pearson [Pearson, 1902].

If the form of the distribution of these errors is insignificantly different from the Gauss' law, now in accordance with classical notions, then this is a proof of the uncorrelatedness of the measurement results, that is, the errors are purely random and do not contain any information. However, if the form of the distribution of the measurement was significantly different from the Gauss' law, then it was a mark of instrumental disruption or the effect of unaccounted systematic errors. The introduction of such approaches, was initiated by academician A. N. Kolmogorov and first carried out by N. A. Borodachev [Borodachev, 1950].

Over time, the classical ideas about the law of errors have undergone evolution, the main stages of which we are going to consider later in the process of

© *P. Dvulit, J. Dzhun*

Lviv Polytechnic National University Institutional Repository http://ena.lp.edu.ua

justifying the NETM procedures. However, the main stimulus of this evolution was the action of Hampel-paradox [Hampel et al., 1986], or, which is the same v the Elyasberg-Hampel paradox [Dzhun, 2012], according to which any hypothesis about the type of probability distribution will be rejected for a sufficiently large number of members of the statistical series. Since the considered measurements g require the generation of large samples, usually with volumes n > 500, then, in accordance with H. Jeffreys' conclusions, the normality hypothesis in this case is usually untenable [Jeffreys, 1938, 1939, 1998]. According to Jeffreys, completely random errors follow the Pearson distribution of type VII with a diagonal information matrix and exponent m within the bounds: $3 \leq m \leq 5$, if there no any systematic influences. Then we will consider in detail how these bounds were obtained. Now, finishing the introduction, we will form the main concept that follows from the principles of the NETM: the Gaussian character of the errors of multiple high-precise measurements g with n > 500 is evidence of non-excluded systematic errors.

Purpose of the study

Our main goal is to justify the practicability of using modern ideas for the error distribution of multiple large-scale observations, outlined in the NETM, in the process of conducting high-precise g definitions. According to the Jeffreys' theory [Jeffreys, 1998], samples of volume n > 500 are not Gaussian, even if measurements are performed under homogeneous measurement conditions. The assumption that such measurements should follow the normal law is the main obstacle to improving the program of highly precise g measurement. The fact that these measurements are performed in a complicated metrological situation that is non-stationary and continuously violated, according to Jeffreys, should cause significant positive excesses of errors, but not their normality in any way, if only the systematic influences are correctly eliminated. The gravimeter with such high measurement accuracy is fed completely unperceived by the observer effects: microseisms, tides, gravitational effects of the atmosphere and other factors. Therefore, the main task of our research is to develop such a method for diagnosing the probabilistic form of measurement errors g on the basis of the NETM concepts, which would indicate that these errors are completely random without any systematic influences.

Methodology for the study of distributions

In general, this methodology is provided by the NETM procedures that were designed to control the probabilistic form of the distribution of high-precision measurements g with large volumes of data based on the principles of the Neumann-Pearson hypothesis testing theory.

The problem of improving the methods of observation is one of the most important and includes two aspects:

– sufficiently complete elimination of systematic errors from measurement results;

– g lead to the errors distribution of measurements to such norms that are advanced by the non-classical errors theory measurements.

The success in solving the problem of eliminating systematic errors from the results of observations depends on the degree of development of the theory of the measured phenomenon and the completeness of the gravimeter's study.

As for the requirements of the error theory, they were first formulated in the classical version by K. F. Gauss in his famous treatises [Gauss, 1809, 1823]. This requirement in the first of them was somewhat veiled and was reduced to the observance of the condition for the weight functions:

$$-\frac{f'(x)}{x \cdot f(x)} = const, \qquad (1)$$

where *f(x)* – probability density of measurement errors x. It is easy to prove that the requirement (1) means that *f(x)* is normality of distribution [Dvulit & Dzhun, 2017].

The measurements only with the errors normality have the same weights σ^{-2} and we can calculate the arithmetic mean from the results of measurements. We find the lower bounds for the dispersions σ_a^2 and σ_σ^2 of the effective estimates of the parameters a and σ of the function *f(x)* using the Rao-Cramer inequality (otherwise the information inequality of G. Darmais and M. Frechet), well-known formulas:

$$\sigma_a^2 = \frac{\sigma^2}{n};\ \sigma_\sigma^2 = \frac{\sigma^2}{2n}. \qquad (2)$$

We should note, that expressions (2) can be used only under the normal error law.

A fundamental expression for the weight function of measurement errors

Substituting into formula (1), the differential form of the family of the Pearson`s curves [Bolshev & Smirnov, 1983]:

$$\frac{f'(x)}{f(x)} = -\frac{x+c_1}{c_0+c_1x+c_2x^2}, \qquad (3)$$

we obtain the main relation of the theory of weight functions of the measurement results:

$$\mathrm{P}(x) = \frac{f'(x)}{x*f(x)} = \frac{x+c_1}{x(c_0+c_1x+c_2x^2)} \qquad (4)$$

$$= \frac{1}{c_0 + c_1x + c_2x^2} + \frac{c_1}{x(c_0 + c_1x + c_2x^2)}$$

where the start point for x is the mean and the constants are:

$$c_0 = \left[s^2(4b_2 - 3b_1)\right]/b;$$

$$c_1 = \left[s\sqrt{b_1(b_2+3)}\right]/b; \qquad (5)$$

$$c_2 = (2b_2 - 3b_1 - 6)/b;$$

Lviv Polytechnic National University Institutional Repository http://ena.lp.edu.ua

$$b = 2(5\beta_2 - 6\beta_1 - 9);\ \sigma^2 = \mu_2;$$

$$\beta_1 = \mu_3^2 / \mu_2^3; \quad \beta_2 = \mu_4 / \mu_2^2; \tag{6}$$

$$\mu_r = \int_{l_1}^{l_2} x^r f(x)dx;\ r = 2,3,4,...;$$

$$\mu_0 = 1; \quad \mu_1 = a = 0; \quad x = \vartheta; \tag{7}$$

the values l_1, l_2 – bounds of the natural range of density variation *f(x).*

The classical error theory is based on the law of normal distribution with the additional assumption of the absence of systematic errors in the results of measurements [Gauss, 1823].

Evolution of the concept of error distribution

The methods of the classical error theory of measurements (CETM) have been massively and successfully applied for more than two hundred years and up to the present. Many researchers believe: if these methods have been proving themselves so well for such a long time, then it is the uncontested evidence of the correctness and inviolability of those laws (axioms) that are based on them. There was the illusion that we do not have to check the adequacy of these laws, if they are confirmed by so many years of practice. The view about the universality of the Gauss' law as a "law of errors" began to change only in 1886, when a famous mathematician and astronomer S. Newcomb [Newcomb, 1886] first encroached on the monopoly of the normal distribution on the basis of an analysis of astronomical observation errors. [Ogorodnikov, 1928]. He proposed real non-Gaussian errors as a "mixture" of several normal distributions with a common center, but with different variances. However, when the work [Hulme & Syms, 1939] performed by astronomers in Greenwich was published, only then, it became obvious, that it is necessary to change the fundamental concept of the error distribution. H. R. Hulme and L. S. T. Syms analyzed two series of observations of latitude in Greenwich in the period 1927–1931 and 1932–1936, which have volumes of 4540 and 4982. The number of errors $\varepsilon > 3\sigma$ in the first row was 357 (7.86 %), that is in 30 times more than it should be by Gauss, and the second row had already 453 (9.09 %), which is 35 times more than it would be expected by Gauss. The non-Gaussian character of the errors was demonstrated in the works of other well-known researchers [Eddington, 1933; Doolittle, 1910, 1912; Student, 1927; Tukey, 1960, 1962]. However, the author of a new fundamental concept of the law of distribution of random errors of observations was H. Jeffreys [Jeffreys, 1938]. He does not completely abandon the Gauss' concept. In §5.7 of the work [Jeffreys, 1998], he states: "... usually the distribution of observation errors follows the normal law quite closely". But the merit of Jeffreys is precisely that he first answered the question: when "usually" and when "closely". "Usually" and "closely" when there are no more than 500 observations. Having analyzed the Pearson's data [Pearson, 1902], he showed in the work [Jeffreys, 1939], that the normal law becomes practically and theoretically baseless with a sample volume $n > 500$. In this case, he suggests using the Pearson distribution of type VII, but not its classical form. The form transformed by him, has a diagonal information matrix, since it is known that the independence of estimates of mathematical expectation and dispersion occurs only for a normal population [Geary, 1947; Lucacs, 1942]. The classical Pearson's curve of type VII does not have this property. We will call the Jeffreys' form of the Pearson's curve with a diagonal information matrix as the Pearson-Jeffreys distribution (law), abbreviated *PJVII*-distribution with the purpose to avoid confusion, and it has the form [Jeffreys, 1938]:

$$f(x) = \frac{c}{\sigma}\left[1 + \frac{0{,}5}{M}\left(\frac{x-\lambda}{\sigma}\right)^2\right]^{-m}, \tag{8}$$

where $c = \frac{\Gamma(m+1)}{\sqrt{2\pi(m-0{,}5)}\cdot\Gamma(m+0{,}5)}$;

$\Gamma(m)$ – gama – function;

$$M = (m - 0.5)^3 m^{-2};\ \lambda,$$

σ – respectively, the parameters of position and scattering; m – a key parameter of the distribution (8) depending on the kurtosis and, thus, shows the degree of deviation of the *PJVII*-distribution from the Gauss' law.

In fact, the form (8) is a generalization of the Gauss and Student distributions: with $m = \infty$ (8) it is a normal law and with $m < \infty$ (8) - a t-distribution for discrete values of the degrees of freedom $\nu = 2m - 1$. Jeffreys suggesting using the form (8) for $n > 500$ in the work [Jeffreys, 1938], subjected this proposal to such a deep mathematical analysis, that it actually made it possible to put it into the foundation of the NETM. The concept of Jeffreys' errors (8) turned out to be as perfect mathematically as the concept of Gauss' normality, but unlike the latter, it is substantially more adequate to the real form of error distributions in large samples of $n > 500$. Jeffreys in the work [Jeffreys, 1939] illustrates the method of constructing effective estimates of the parameters of the ***PJVII***-distribution and for the first time made it for the example of the analysis of latitude observations in Greenwich. In the same paper, Jeffreys analyzes six series of approximately 500 observations found by Pearson [Pearson, 1902], which were obtained under controlled homogeneous observation conditions. Jeffreys, by adjusting the *PJVII*-distribution to these data, found that the four series have longer "tails" and two series – more short than the normal distribution. Jeffreys, also as Pearson did, discovered significant serial data correlations for the series with short "tails". He found for these six series of Pearson, see Table 1, such relationships between the exponent $\mu = m^{-1}$ of VII and II types of

Lviv Polytechnic National University Institutional Repository http://ena.lp.edu.ua

symmetric Pearson's curves and the coefficient of serial correlation of these observations.

The value $\mu < 0$ corresponds to the exponent of the Pearson's curve of type II. Assuming the dependence of μ on r in the form: $\mu = a + br$, Jeffreys finds the solution: $a = 0.273 \pm 0.093$; $b = 0.62 \pm 0.22$. If there is an absence of a serial correlation of errors ($r = 0$), then we have $m = 3.66$ with such limits according to the standard 2.73–5.56. Jeffreys, according to these data, concludes that truly independent errors under homogeneous observation conditions should follow the *PJVII*-distribution with $3 \leq m \leq 5$.

Table 1

Jeffreys analyzes six series of approximately 500 observations by Pearson

$\mu = m^{-1}$	r	$M_{calculated}$	*O-C*
+0.230	+0.16	+0.173	+0.057
+0.163	+0.24	+0.123	+0.040
+0.111	+0.23	+0.129	-0.018
+0.040	+0.57	-0.083	-0.063
-0.080	+0.32	-0.073	-0.053
-0.225	+0.72	-0.018	-0.049

Jeffreys' conclusions were at variance with the existing ideas about the law of error distribution of observations. This was illustrated with a fundamental test. It was implemented in the Academy of Sciences of Ukraine on the initiative of Academician E. P. Fedorov, a world-famous specialist in celestial mechanics and the movement of the Earth's poles [Fedorov, 1963]. The series of high precision observations of the highest quality, beginning with the historical series of F. W. Bessel and ending with modern astronomical, space, gravimetric and other observations in various branches of science were used during the test. There were considered 69 series with a total number of observations of 190178. The results of this test are shown in the figure, where the graph is used as the working field with the purpose to identify the types of Pearson distributions [Bolshev & Smirnov, 1983]. Fig. 1 shows that each empirical distribution is characterized by three coordinates: kurtosis e, squared asymmetry and sample volume n in thousands of vertical direction. A single point e the coordinate origin corresponds to normal distribution on this field. It can be seen from the Fig. that only a small part of the series of errors, mostly of small volume, is grouped around zero. The main mass of error series is shifted to the right from zero and has significant excesses $e > 0$. The extent of the excesses is from -0.20 to + 6.0; the excesses $e < 0$ turned out to be insignificant. As a result, one can draw such an important conclusion**: *the most characteristic feature of the errors of observations of a large volume is their zero asymmetry and a positive kurtosis of high significance*.**

The Russian scientist N. I. Idelson in 1947 wrote: "As far as we know, there were no series of errors with negative kurtosis" [Idelson, 1947]. However, neither experts in the field of the error theory, nor mathematics attached special significance to this conclusion.

Basic principles of the NETM and their importance for improving the method of highly-precise measurements *g*

The results obtained in [Dzhun, 1992] allowed to start the development of the NETM, which was completed in 2015 [Dzhun, 2015]. We will outline the fundamental categories of this theory, because our further conclusions are based on them.

Let's formulate the first fundamental principle of the NETM, which is essentially the proposal of H. Jeffreys formulated in [Jeffreys, 1939]:

for a large number of multiple observations (n > 500), their random independent errors follow the Pearson-Jeffreys law of type VII with an exponent m within the limits:

$$3 \leq m \leq 5 \tag{9}$$

that corresponds to the excess:

$$6 \geq e \geq 1{,}2 \tag{10}$$

or to such degrees of freedom ν of t-distribution:

$$5 \leq \nu \leq 9 \tag{11}$$

None of the researchers who use non-Gaussian distributions, besides H. Jeffreys, considered that it was necessary to check their information matrices for diagonality. At the same time, exact independence of the distribution parameters provided the greatest simplicity of obtaining their estimates. Therefore, the use of the law (8) in the error theory had no alternative.

Assuming the distribution (8) as the non-classical error law, we violate condition (1), which takes place only for the Gauss' law. It becomes necessary to generalize condition (1), which leads to the definition of the second fundamental principle of the NETM:

individual weights of observations that obey the Pearson-Jeffreys distribution of type VII are characterized by their weight function adapted to this distribution.

Lviv Polytechnic National University Institutional Repository http://ena.lp.edu.ua

Differentiating the formula (8) and using expression (1), we obtain the weight function of the Pearson-Jeffreys error law:

$$P(x) = \left[\frac{(m-0.5)^3}{m}\sigma^2 + \frac{\vartheta^2}{2m}\right]^{-1}, \qquad (12)$$

where the error of observation $\varepsilon = x - \lambda$, λ, σ, m – estimates of the parameters of the probability density law (8), which are determined by the method of maximum likelihood (MML) from the measurement results.

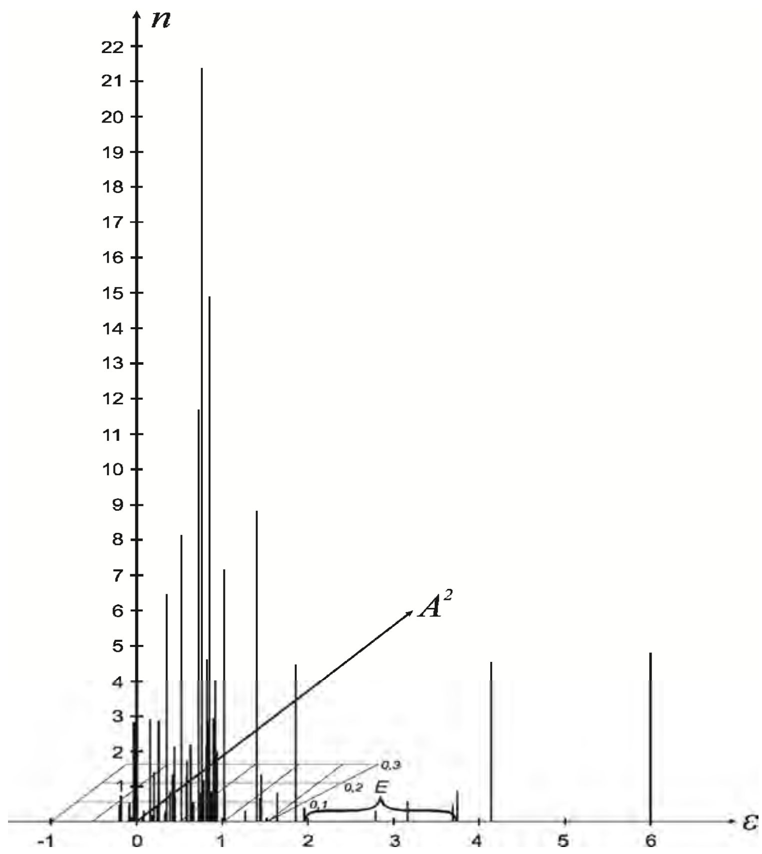

Fig. 1. Location of empirical error distributions of astronomical, space, gravimetric, geodetic, economic series (Domain E) on the graph for determining the type of the Pearson's curve as a function of A[2] and ε. The axis n indicates the sample objects in thousands. The Pearson distribution of type VII corresponds to a line with excess: $0 < \varepsilon < 1.5$

The physical meaning of *P(x)* is the follows: the weight *P(x)* is the inverse dispersion of the observation *x*, which has the error $\varepsilon = x - \lambda$. Thus, the formula (12) allows *m* to determine the weight of each individual gravimetric observation, even anomalous. Naturally, the weight of the latter will be very small, because it is inversely proportional to the square of the error ε. It is also easy to see that for $m = \infty$ (Gauss' law) $P(x) = \sigma^{-2} = const$.

Since the *PJVII*-distribution satisfies all the conditions for the existence of the bounds of the Rao-Cramer inequality (also obtained by G. Darmais and M. Frechet and called the information inequality), the lower bounds for the variances σ_λ^2, σ_σ^2, σ_m^2 of effective estimates of its parameters λ, σ, m, will be as follows [Dzhun, 1992]

$$\sigma_\lambda^2 \ge \frac{\sigma^2}{n}\frac{(m-0.5)^2(m+1)}{m^3}; \quad \sigma_\sigma^2 \ge \frac{\sigma^2}{2n}\frac{(m+1)}{(m-0.5)}; \qquad (13)$$

$$\sigma_m^2 \ge \left\{n\left[\psi'(m-0.5) - \psi'(m) - \frac{m+1}{2m^2(m-0.5)}\right]\right\}^{-1}, \qquad (14)$$

where $\psi'(m-0.5)$, $\psi'(m)$ are trigamma functions from m.

The bounds of the Rao-Cramer inequality (13) for the Gauss' law ($m = \infty$) are identical to the relations (2).

Analyzing the weight function (4), we see that with a significant asymmetry of the error distribution ($c_1 \neq 0$) and with $x = 0$, and also with $\varepsilon < 0$, it acquires a degenerate (singular) character. Consequently, the only evaluation area in which (4) is nonsingular,

Lviv Polytechnic National University Institutional Repository http://ena.lp.edu.ua

corresponds to the Jeffreys' errors of the form (11) with the excesses $0 \le \varepsilon < \infty$.

Since the Jeffreys' errors are completely random, without any systematic influence, they follow the law (8) with the value *m* within (9), then it is possible to formulate the third fundamental principle of the NETM, which is also the criterion for the absence of a significant effect of systematic errors:

the influence of weak, non-excluded, correlated errors in the results of observations can be neglected only when the weight function of the error distribution of measurements is nonsingular, and m is within the bounds 3 ≤ m ≤ 5.

Criteria for nonsingular weight function

It is reasonable to use estimates of asymmetry and kurtosis as criteria for the nonsingularity of the weight function, because they exactly determine its character. They can be obtained on the basis of unbiased moment estimates [Cramer, 1946]:

$$A = \frac{\sqrt{n(n-1)}}{n-2}\frac{m_3}{m_2^{1.5}}; \qquad (15)$$

$$\varepsilon = \frac{(n-1)(n^2-2n+3)}{n(n-2)(n-3)}\frac{m_4}{m_2^2} - \frac{3(n-1)(2n-3)}{n(n-2)(n-3)} - 3, \qquad (16)$$

where m_r – sampling central moments of order *r* calculated by the results of measurements x_i:

$$m_r = n^{-1}\sum (x_i - \bar{x})^r; \quad \bar{x} = n^{-1}\sum x_i; \quad \bar{x} = g. \qquad (17)$$

The standard errors for *A* and ε are obtained from the formulas [Cramer, 1946]:

$$\sigma_A = \sqrt{\frac{4\mu_2^2\mu_6 - 12\mu_2\mu_3\mu_5 - 24\mu_2^3\mu_4 + 9\mu_3^2\mu_4 + 35\mu_2^2\mu_3^2 + 36\mu_2^5}{4\mu_2^5 n}}; \qquad (18)$$

$$\sigma_\varepsilon = \sqrt{\frac{\mu_2^2\mu_8 - 4\mu_2\mu_4\mu_6 - 8\mu_2^2\mu_3\mu_5 + 4\mu_4^3 - \mu_2^2\mu_4^2 + 16\mu_2\mu_3^2\mu_4 + 16\mu_2^3\mu_3^2}{\mu_2^6 n}}, \qquad (19)$$

where μ_r – central moments of order *r*.

The moments μ_r in (18)–(19) with the values $n > 500$, can be replaced by the sampling moments m_r calculated by the formula (17), and, in this case, the displacement of the moments m_r with such volumes of samples does not particularly affect the estimates σ_A and σ_ε.

Having obtained the values *A*, ε, σ_A, σ_ε by the formulas (15)–(19), we determine the confidential intervals for *A* and ε:

$$A \pm t_\alpha \cdot \sigma_A; \quad \varepsilon \pm t_\alpha \cdot \sigma_\varepsilon \qquad (20)$$

where t_α – quantile, determined by the Laplace function for the significance level α; σ_A and σ_ε are calculated by the formulas (18–19).

In accordance with the theory of testing hypothesis of Neumann-Pearson, if the confidential intervals (20) cover zero, it is a necessary and, as a rule, a sufficient sign of the normality of measurement errors. If even one confidential interval does not cover zero, then we turn to Table 1. with a purpose to solve the problem of nonsingularity or singularity of the weight function, remembering that only the laws of Gauss and Pearson-Jeffreys provide the possibility of obtaining nondegenerate estimates in mathematical data processing. Table 1. is a program for metrological diagnostics of highly precise measurements *g*.

Results

We used the results of absolute gravimetric measurements at the points: Borowa Gora, Josefoslaw, Ksiaz with a purpose to implement the algorithm of metrological diagnostics of high precision *g*-definitions considered in Table 2 given to us Marcin Barlik from Politechnika Warshawska. The numerical characteristics of these measurements and their histograms are given in Table 3 and Table 4.

Now, let's consider these measurements from two points of view, one of which will be based on the principles of the CETM, the second on the NETM positions.

If we consider these measurements from the point of view of the CETM, we should say that they have been performed well: the asymmetry in all cases is insignificant, since the confidential intervals for *A* cover zero and within α = 10 %, α = 5 % (lines 5 and 6 of the Table 3). According to the kurtosis, the most favorable situation is observed at the points Borowa Gora and Josefoslaw, and the worst, but admissible situation is at the point Ksiaq, (lines 8 and 9 of the Table 3). The normality testing of these measurements also showed good results: the probability P_{χ^2} of the fact, that the measurements at the points Borowa Gora, Josefoslaw and Ksiaq are samples from the normal general complex, that, respectively, are equal to 15.22 %; 49.17 %; 20.00 %, that is, they are far from their critical bounds (the top row of the Table 4, besides the probabilities P_{χ^2}, shows the number of degrees of freedom for *r* of the $\chi 2$-criterion). It means that the measurement system on this gravimeter was worked out by the creators of this tool in good faith, but within the framework of outdated classical ideas about the law of error distribution of measurements of a larger volume.

Now consider the results of observations at the same points from the outlook of the NETM requirements.

We see that Ksiaq has the best metrological situation (line 3, Table 3): the accuracy of measurements at this point is significantly higher than at other points. In addition, the left bound of the confidential interval for ε on Ksiaq with a risk level α = 10 % covers zero, and is so close to it that it practically confirms its positive significance. The error distribution at this point deviates from the normality in a good direction, $\varepsilon < 0$ namely, it is the closest to the perfect form of Jeffreys' errors (8).

The confidential interval for ε at the Borowa Gora, Ksiaq, Josefoslaw (Table 3, lines 8, 9) cover zero. According to the concepts of the NETM, the asymmetry is insignificant for this point, we can make the conclusion, about the correctness of the classical data processing method applied at this point taking into account, the data in Table 3.

Lviv Polytechnic National University Institutional Repository http://ena.lp.edu.ua

Table 2

Diagnostics of results of gravimetric measurements based on the construction of confidential zones for asymmetry and kurtosis

Results	Diagnostics of results
Confidential intervals for asymmetry *A* and excess *ε* cover zero, that means the confirmation of hypotheses: $A = 0$; $\varepsilon = 0$	The weight function is in the nonsingular domain of classical estimation. The obtained results should be considered as final. Although, the distribution of errors is not ideal, that's why, there is no need to apply the NETM
The confidential interval for *A* covers zero, and for *ε* – covers or touches the most favorable zone (10) for excess, that means the confirmation of hypotheses: $A = 0$; $1.2 \le \varepsilon \le 6$	Every experimenter should dream about such case: the weight function is not only nonsingular, but also provides an effective estimation in the next necessary approximation, realized by methods of the NETM
The confidential interval for *A* covers zero, for *ε* – is inside the zone (10), without touching its edges, that means the confirmation of hypotheses: $A = 0$; $0 < \varepsilon < 1.2$	The weight function is non-singular, i.e. the estimation is permissible, but the distribution of errors is not ideal, because the action of weak, not excluded systematic errors is confirmed. To improve the quality of estimates, the following approximation is necessary in order to estimate the parameters of a mathematical model using the methods of the NETM.
The confidential interval for *A* covers zero, and the whole confidential interval for *ε* is in the negative domain, that means the confirmation of the hypotheses: $A = 0$; $\varepsilon < 0$	The weight function of distribution of errors is singular in this case, but the estimation is possible if none of the errors is equal or greater than the value $\left[2\mu_2\beta_2/\lvert\varepsilon\rvert\right]^{0.5}$. If this condition is met, then with the purpose of more objective estimation, which does not exaggerate reliability, the following approximation, after applying classical methods, is necessary to evaluate the parameters of the mathematical model of the NETM using the weight function of the Pearson-Jeffreys distribution of type II. [Dzhun, 2015]
The confirmation of the hypotheses: 1) $A < 0$; $\varepsilon = 0$. 2) $A > 0$; $\varepsilon = 0$. 3) $A < 0$; $\varepsilon < 0$. 4) $A > 0$; $\varepsilon > 0$. 5) $A > 0$; $\varepsilon < 0$. 6) $A < 0$; $\varepsilon > 0$	These are pathological cases of evaluation. In all these cases, the weight function is singular, falling into inadmissible domains of estimation, that is impossible due to the irregularity of the weight function

Table 3

Characteristics of samples of III series absolute ballistic measurements of Galilean acceleration at points: Borowa Gora, Josefoslaw, Ksiaq.

No.	Sample characteristics	Points of observations		
		Borowa Gora	Josefoslaw	Ksiaq
1	2	3	4	5
1	The measurement result and its standard: $g \pm \sigma_g$, date	981250155.20±0.17 15.08.2016. 21:57:24	981213788.32±0.47 01.11.2011. 14:05:02	981056794.22±0.17 17.04.2008. 17:51:15
2	Volume of sample *n*	1083	516	822
3	Root mean square error of measurement and its standard: $\sigma \pm \sigma_\sigma$	5.68±0.12	10.79±0.34	4.86±0.12
4	Asymmetry and its standard: $A \pm \sigma_A$	0.016±0.076	-0.018±0.118	-0.039±0.097

Lviv Polytechnic National University Institutional Repository http://ena.lp.edu.ua

Continued of Table 3

1	2	3	4	5
5	Confidential interval for *A*, *a* = 10 %	-0.109+0.141	-0.112+0.176	-0.199+0.121
6	*a* = 5 %	-0.133+0.165	-0.249+0.213	-0.223+0.145
7	Kurtosis and its standard: $\sigma \pm \sigma\varepsilon$	-0.002+0.151	+0.020+0.241	+0.240+0.167
8	Confidential interval for ε, *a* = 10%	-0.250+0.246	-0.376+0.411	-0.035+0.515
9	*a* = 5%	-0.298+0.294	-0.452+0.492	-0.087+0.567
	Sampling moment			
10	m_2	32.222	116.383	23.60
11	m_3	2.853	-22.232	-4.42
12	m_4	3112.982	41 147.381	1811.30
13	m_5	603.675	-128 8514.322	-1667.56
14	m_6	509 897.214	26 008 872.567	240 843.45
15	m_8	117 241 423.602	23 426 330 151.996	43 989 697.36

Table 4

Histograms of the III series errors of the absolute ballistic measurements of Galilean acceleration at the points: Borowa Gora, Josefoslav and Ksiaz.

№	Borowa Gora, P_{c^2} = 15.22 %, *r* =12				Josefoslaw, P_{c^2} = 49.17 %, *r* = 8				Ksiaq, P_{c^2} = 20.00 %, *r* = 11			
	Intervals	Frequency n_i	Gaussian frequencies n'_i	$n_i - n'_i$	Intervals	Frequency n_i	Gaussian frequencies n'_i	$n_i - n'_i$	Intervals	Frequency n_i	Gaussian frequencies n'_i	$n_i - n'_i$
1	37.1-39.4	5	2.16	+2.84	49.300–55.305	2			776.80–779.04	2		
2	39.4–41.7	5	6.51	-1.51	55.305–61.293	3	3.08	+1.92	779.04–781.26	1	3.00	0.00
3	41.7–44.0	14	16.83	-2.83	61.293–67.280	5	10.04	-5.04	781.26–783.48	10	7.98	+2.02
4	44.0–46.3	38	37.03	+0.97	67.280–73.268	28	28.85	-0.85	783.48–785.69	22	21.39	+0.61
5	46.3–48.6	68	69.28	-1.28	73.268–79.255	64	61.34	+2.66	785.69–787.91	38	47.21	-9.21
6	48.6–50.9	99	110.59	-11.59	79.255–85.242	107	96.68	+10.32	787.91–790.13	80	84.60	-4.60
7	50.9–53.2	169	149.34	+19.66	85.242–91.230	100	112.80	-12.8	790.13–792.35	141	123.48	+17.52
8	53.2–55.5	179	171.89	+7.11	91.230–97.217	96	97.47	-1.47	792.35–794.56	133	146.14	-13.14
9	55.5–57.8	151	168.29	-17.29	797.217–	67	62.38	+4.62	794.56–796.78	148	142.21	+5.79
10	57.8–60.1	136	140.15	-4.15	803.205	30	29.59	+0.41	796.78–799.00	122	112.24	+9.76
11	60.1–62.4	102	99.28	+2.72	03.205–09.192	10	10.38	-0.38	799.00–801.21	66	71.91	-5.91
12	62.4–64.7	64	59.93	+4.07	09.192–15.180	4	2.70	+1.30	801.21–803.43	39	37.89	+1.11
13	64.7–67.0	40	30.50	+9.50	15.180–21.167				803.43–805.65	9	16.17	-7.17
14	67.0–69.3	7	13.43	-6.43					805.65–807.86	7	6.81	+0.19
15	69.3–71.6	2							807.86–810.08	4	1.60	+2.4
16	71.6–73.9	3	6.92	-0.92								
17	73.9–76.2	1										

The worst metrological situation occurred at Josefoslaw, since here is observe the most $\sigma = 10{,}79$ μ Gal (Table 3, line 3).

Table 3 shows the results of the NETM diagnostics of the 3 observations series at Borowa Gora, Jozefoslaw and Ksiaz.

The NETM diagnostics of the 1st series from our observations showed a significant negative excess in Borowa Gora at risk of α = 10 %, and a practically significant asymmetry in Ksiaz. Therefore, the results of this diagnosis were not published due to the singularity of the weight function for these two distributions.

Lviv Polytechnic National University Institutional Repository http://ena.lp.edu.ua

The NETM diagnostics of the 2nd series of measurements g on the same three points was published in the work [Dvulit & Dzhun; 2017]. It showed an improvement in the shape of distributions and the fact that their processing can be carried out using the CETM methods. It is interesting to compare the results of the x^2 -test of the normalization hypothesis in the work [Dvulit & Dzhun; 2017] to Table 4 of this paper (see Table 5).

Table 5

The comparison of the results of the normalization hypothesis for the II and III series of observations g using χ^2 - criterion of Pearson

No. series / Name of the point	The probability of a normalization hypothesis P_x^2 .		
	Borowa Gora	Jozefoslaw	Ksiaz
II series	31.38 %	79.36 %	52.80 %
III series	15.23 %	49.17 %	20.00 %

According to the CETM, observations of the II series are more perfect, since they have on an average have about twice as much probability of $P\chi^2$. In the NETM categories, the observations of the III series are better because they are closer to the ideal form of the non-gaussian, Jeffreys errors (8). But these measurements are far to the norms (9–11).

There are rows of the highest quality in gravimetry, the errors of which are close to the Jeffreys' norm (9). For example, the series obtained at the International Gravimetric Station No. 5035, performed by a GABL gravimeter (Moscow Region Test Base IFZ AN SSSR) [Dzhun, at al., 1984]. For this series m = 6.67 ± 1.37. Confidence interval at α = 10 %: 4.42 < 6.67 < 8.92. Its right part m = 4.42 covers the interval (9), which confirms the insignificance of the deviation of m = 6.67 from the Jeffreys' norm. Let P_G and P_J – χ^2 – are the probabilities that the series in Ledovo is a sample of Gaussian and Jeffreys general sets. Then P_G = 1.5 %, P_J = 46.6 % [Dzhun, at al., 1984], that is, e $P_J > P_G$ is 31 times, which is natural since m = 6.67 is significantly less than $m = \infty$, that is typical for the Gauss law.

The fact that high quality observations on the GABL have been confirmed by the International Bureau of Measures and Weights (IBMW, Paris, Sevr), where there is one of the most accurate stationary for measuring units g [Sakuma, 1973] and as a result of its comparison with the gravimeter of the National Bureau of Standards USA [Hammond & Faller, 1971; Arnautov et al., 1982]. In the upper part of the measurement distributions g in Jozefoslaw and Ksiaz, instead of the distribution peak, a platform is observed. This is evidence of the lack of proper protection of the FG-5 gravity meter from the effects of microseisms, as evidenced in the work [Arnautov, et al., 1982, p. 18, Fig. 5].

Thus, due to the NETM, the results of measurements at all three points are still far from perfect and should cause serious suspicion with regard to the effect of non-excluded systematic errors. Exactly these errors keep error distributions on these points in the grip of normality. Completely random measurement errors with n > 500 should have the form of Jeffreys' errors (8) with norms (9–11).

Scientific novelty and practical importance of the research

In the conducted research, the methods of the NETM are used to improve the technique of measurements g which are performed in a complicated metrological environment with the need to take into account a number of non-stationary sources of systematic errors.

The practical significance of the research is the development of an algorithm for controlling the probabilistic form of the distribution of samples of measurements g (Table 2) in order to improve their methodology. The study of the reasons for the deviations of error distributions from established norms has long been a necessary element of the theory of production accuracy [Borodachev, 1950], the process of monitoring the normative operation of aggregates and measuring instruments. Implementation of such approaches which were involved by Kolmogorov and his school for the first time and then realised in the NETM, has long been the main strategy that provides metrological literacy for large-scale measurements.

Conclusions

1. In accordance with the principles of the CETM, the results of absolute high-precise measurements g at Borowa Gora, Josefoslaw and Ksizq, should be classified as effective and consistent estimates. It should always be remembered that any measurement experiment is in the domain of acceptable estimation only in the case when the confidential interval for asymmetry covers zero, and the confidenctial interval for the kurtosis also covers zero or is in the positive region.

2. The point Ksiaq has the lowest value of the P_{χ^2} – probability, which is 20.00 %, but it does not mean that these observations are that bad. On the contrary, the distribution of errors at this point deviates from the Gauss' law, according to the

Lviv Polytechnic National University Institutional Repository http://ena.lp.edu.ua

principles of the NETM and in a good direction: it has a positive and almost significant excess, it means, that it is the closest to Jeffreys' errors of the form (8). The best metrological situation has been achieved at Ksiaq point and the evidence of this fact is that the standard measurement error is substantially less than at the remaining points and is 4.86 ± 0.12 (Table 3). The weight of one measurement at the points Josefoslaw and Borowa Gora in 1.4 times less.

3. According to the NETM categories, the proximity to the normality of the measurement error distributions for sample sizes $n > 500$ is an outdated expediency far from the ideal (8) with the norms (9–11). In other words, the normality of errors with $n > 500$ is only half way towards the above-mentioned Jeffreys' standards. The essence of the strategy of continuous improvement of high-precise definitions g is to bring the distribution of their errors to the ideal (8) with the norm (9). In addition, the errors of measurements of the absolute gravimeter GABL created by the Institute of Automation and Electrometry of the Siberian Branch of the Russian Academy of Sciences correspond to the Pearson-Jeffreys law (8) with $m = 6.67 \pm 1.37$ [Dzhun, 1983; Dzhun at al., 1984], that is, it almost reaches the norm (9).

4. The normality of errors of high-precision measurements g with $n > 500$ means only that the weight function (4) of such observations allows estimation, leaving the errors correlated (non-random). This correlation can be attributed to the noise field (ignore it) only when the measurement errors follow form (8) with the norms (9–11). It is achieved through a deeper study of the sources of systematic errors with subsequent exclusion from the results of observations. It should always be remembered that the normality of measurement errors with $n > 500$ is not the norm, but a reason for serious concern regarding to the non-excluded systematic influences. Exactly they rigidly hold the empirical distribution of errors in the embrace of normality. The causes of systematic errors include the influence of the Moon, the trends of energy and frequency of microseisms, the gravitational influence of the atmosphere, its pressure, temperature, and the peculiarities of the place of observation. Gravimetrists await ahead of purposeful work in order to relieve the definitions g from systematic errors and lead their distributions to the ideal (8), and the kurtosis to the norm (10). The successes of this work will facilitate the acquaintance with the fundamental works of H. Jeffreys [Jeffreys, 1938, 1939, 1998], as well as with the works [Dzhun, 1983; Dzhun at al., 1984; Dzhun, 1992, 2015, 2017].

5. Is it possible not to respond to modern developments in the field of the error theory and data analysis, ignore the conclusions of H. Jeffreys and the concept of the NETM? There is, however, a huge risk to remain forever in the arms of routine obscurantism, thus, closing the prospects for qualitative changes in the way of improving observations when studying one of the main mysteries of modern science – the mysteries of the gravity. Also, it should be remembered that the NETM does not disprove the Gaussian CETM, it retains it as its necessary element and stands on its shoulders, overcoming the Elyasberg-Hampel paradox with $n > 500$. The CETM methods are always implemented before the NETM procedures are applied, since there is nothing like them in their simplicity and clarity.

References

Arnautov, G. P., Koronkevich, V. P., & Stus, Yu. F., (1982). The Interferometer of the absolute lazers ballistic gravimeter. Institut avtomatici i elektrometrii SO AN USSR, Novosibirsk, Preprint 196. 37 p.

Bessel, F. W. (1818). Fundamenta astronomiae. Konigsberg.

Bessel, F. W. (1838). Untersuhungen uber die Wahr-scheinlichkeit der Beobachtungs-fehle. Astronomische Nachrichten, b. 15, 369.

Bolshev, L. N., & Smirnov, N. V. (1983). Tables of Mathematical Statistics. Moscow: Science. (in Russian).

Borodachev, N. A. (1950). The Main Questions of the accuracy of the Theory of Manufacture. Editor A. N. Kolmogorov. Moscow – Leningrad: AS USSR Publ., 360 p, [In Russian].

Bruevich, N. G. (Editor). (1973). Production Accuracy in the Mechanic and Instrument engineering.

Cramér, H. (1946). Mathematical methods of statistics. 1946. *Department of Mathematical SU.*

Doolittle, C. L. (1910). Results of Observations with the zenith telescope and the Wharton reflex zenith tube. *The Astronomical Journal*, XXVI, 608, Albany.

Doolittle, C. L. (1912). Results of observation with the zenith telescope and the Wharton reflex zenith tube. *The Astronomical Journal*, *27*, 133–138.

Dvulit, P., & Dzhun, I. (2017). Application of methods of the non-classical error theory in absolute measurements of Galilean acceleration. *Geodynamics*, (22), 7–15.

Dzhun, I. V. (1969). Pearson Distribution of type VII in the errors of Observations of Latitude Variations. Astrom. Astrofiz. 2, 101115.

Dzhun, I. V. (1974). Analysis of parallel Latitudinal Observations performed under the general program. Extended abstract of Cand. Degree of Phis. – Math. Sci.: spec. 01.03.01 "Astrometry and Celectial Mechanics". Kyiv: Institute of mathematics of AS USSR.

Dzhun, I. V. (1983). Fluctuations in Weight of Individual Measurements of the Gravity Acceleration and the Way of their Account for ballistic Observations Processing. In Repeated Gravity Observations: Theory and Results.

Lviv Polytechnic National University Institutional Repository http://ena.lp.edu.ua

Moscow: MGK Prezidiume AS USSR, Neftegeofizika Publ., 46–52.
Dzhun, I. V., Arnautov G. P., Stus Yu. F., Shcheglov S. N. (1984). Feature of the Dis-tribution Law for the Results of Ballistic Measurement of the Gravity Acceleration. Repeat Gravimetric Observations: Theory and Results. Moscow: MGK Prezidiume AS USSR, Neftegeofizika Publ., 87–100.
Dzhun, I. V. (1992). Mathematical Treatment of Astronomical and Space-Based Information in non-Gaussian Observation Errors. Extended Abstract of Doctoral Dissertation in Physics and Mathematics. Main Astronomical Observatory of the National Academy of Sciences of Ukraine, Kyiv.
Dzhun, I. V. (2012). Distribution of errors in multiple large-volume observations. *Measurement Techniques*, *55*, 393–396., Springer.
Dzhun, I. V. (2015). The Non-classical Errors Theory of Measurements. Rivne: Estero Publ., 168 [in Russian].
Dzhun, J. V. (2017). A new importnat tool in the field of intelligent data analysis. Alcide De Gasperi University of Euroregional Economy in Jozefow. Intercultural Communication, 1/2, 162–175.
Eddington, A. S. (1933). Notes on the method of least squares. *Proceedings of the Physical Society*, *45*(2), 271.
Fedorov, E. P. (1963). Nutation and forced motion of the Earth's pole from the data of latitude observations. *Oxford, New York, Pergamon Press.*
Gauss, C. F. (1809). *Theoria motus corporum coelestium in sectionibus conicis solem ambientium* (Vol. 7). Perthes et Besser.
Gauss, C. F. (1823). *Theoria combinationis observationum erroribus minimis obnoxiae* (Vol. 1). Henricus Dieterich.
Geary, R. C. (1947). Testing for Normality. Biometrika, 34, 209–242.
Hammond, J. A., & Faller, J. E. (1971). A laser-interferometer system for the absolute determination of the acceleration due to gravity. *Precision Measurement and Fundamental Constants; Proceedings*, *343*, 457.
Hampel, F. R., Ronchetti, E. M., Rousseeuw, P. J., & Stahel, W. A. (1986). *Robust statistics* (pp. 29-30). New York: Wiley.
Hulme, H. R., & Symms, L. S. T. (1939). The law of error and the combination of observations. *Monthly Notices of the Royal Astronomical Society*, *99*, 642.
Idelson, N. I. (1947). Method of Least Squares and the Theory of Math. Treatment of Observations). [In Russian]. Geodezizdat. Moscow – Leningrad.
Jeffreys, H. (1938). The law of error and the combination of observations. *Philosophical Transactions of the Royal Society of London. Series A, Mathematical and Physical Sciences*, *237*(777), 231–271.
Jeffreys, H. (1939). The law of error in the Greenwich variation of latitude observations. *Monthly Notices of the Royal Astronomical Society*, *99*, 703.
Jeffreys, H. (1998). *The theory of probability*. OUP Oxford.
Lucacs, E. A. (1942). A Characterization of the normal Distribution. *Annals of Mathematical Statistics*. 13, 91–93.
Newcomb, S. (1886). A generalized theory of the combination of observations so as to obtain the best result. *American journal of Mathematics*, 343–366.
Ogorodnikov, K. F. (1928). Procedure for Reducing Observations by introducing Mean Weights in application to Statistical Study of Stellar Motions, Astron., Jurn., *5*(1), 1–21.
Pearson, K. (1902). On the Mathematical Theory of Errors of Judgment with special Reference to the Personal Equation. *Philosophical Transactions of the Royal Society of London*. Ser. A., 198, 235–296.
Sakuma, A. (1973). A permanent station for the absolute determination of gravity approaching one microgal accurace. Proc. Symposium on Earth's gravitational field and secular variations in position. University of N. S. W., Sidney. p. 674–684.
Student. (1927). Errors of routine analysis. *Biometrika*, 151–164.
Tukey, J. W. (1960). A survey of sampling from contaminated distributions. *Contributions to probability and statistics*, 448–485.
Tukey, J. W. (1962). The future of data analysis. *The annals of mathematical statistics*, *33*(1), 1–67.

П. Д. ДВУЛІТ[1], Й. В. ДЖУНЬ[2]

[1] Кафедра вищої геодезії та астрономії, Національний університет "Львівська політехніка", вул. С. Бандери, 12, Львів, 79013, Україна, ел. пошта: dvupet@ukr.net.
[2] Кафедра математичного моделювання, Міжнародний економіко-гуманітарний університет ім. акад. С. Дем'янчука, вул. С. Дем'янчука, 4, Рівне, 33027, Україна, ел. пошта: iosif-june@rambler.ru.

ДІАГНОСТИКА ВИСОКОТОЧНИХ БАЛІСТИЧНИХ ВИМІРІВ ГРАВІТАЦІЙНОГО ПРИСКОРЕННЯ МЕТОДАМИ НЕКЛАСИЧНОЇ ТЕОРІЇ ПОХИБОК

Мета дослідження: показати необхідність використання сучасних уявлень про закон розподілу похибок спостережень, задіяних в категоріях "Некласичної теорії вимірів" (НТПВ) при проведенні високоточних балістичних визначень гравітаційного прискорення. Ці визначення характеризуються

Lviv Polytechnic National University Institutional Repository http://ena.lp.edu.ua

великими обсягами, що, відповідно до теорії професора Кембриджського університету Г. Джеффріса, автоматично виводить їх за межі дії класичних уявлень про закон похибок вимірів. Ці застарілі уявлення про закон розподілу похибок вимірів великого обсягу є головною перешкодою на шляху вдосконалення методики цих дуже важливих визначень. **Методика дослідження** забезпечується процедурами НТПВ, які розроблені з метою контролю ймовірнісної форми статистичних розподілів високоточних абсолютних балістичних вимірів із великими обсягами вибірок на основі рекомендацій Г. Джеффріса і на принципах теорії перевірки гіпотез. **Основним результатом** дослідження є проведення НТПВ-діагностики метрологічної ситуації високоточних вимірів балістичним гравіметром FG-5, виконаних після деяких удосконалень програми спостережень. Цей метод діагностики ґрунтується на використанні довірчих інтервалів для оцінок асиметрії і ексцесу отриманої вибірки вимірів g з наступним застосуванням χ^2-тесту Пірсона для визначення значимості відхилень їх розподілів від встановлених норм. У відповідності з категоріями НТПВ такими нормами є закони Гауса і Пірсона-Джеффріса, оскільки саме вони забезпечують несингулярність вагової функції вибірки і можливість отримання невироджених оцінок g при математичній обробці вимірів. **Наукова новизна:** задіяні можливості нового інструмента в області “Data Analysis” – НТПВ з метою вдосконалення методики високоточних вимірів g, які виконуються в складній метрологічній ситуації і необхідністю врахування ряду нестаціонарних джерел систематичних похибок. **Практична значущість** дослідження полягає в застосуванні НТПВ – діагностики ймовірнісної форми розподілу вимірів g з метою вдосконалення методики цих високоточних визначень. Дослідження причин відхилень розподілів похибок від встановлених норм забезпечує метрологічну грамотність проведення високоточних вимірів великого обсягу.

Ключові слова: закони похибок: Гауса, Пірсона–Джеффріса; абсолютні виміри гравітаційного прискорення; некласична теорія похибок вимірів.

Received 15.05.2019

Lviv Polytechnic National University Institutional Repository http://ena.lp.edu.ua