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CHANGE IN THE ZONAL HARMONIC COEFFICIENT C_{20}, EARTH'S POLAR FLATTENING, AND DYNAMICAL ELLIPTICITY FROM SLR DATA

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Purpose. We examine the change in the Earth's second degree zonal harmonic coefficient derived from UTCSR SRL time series of $\bar{C}_{20}(t)$ given (a) for the period from 1976 to 2017 as monthly solutions of the zonal coefficient $\bar{C}_{20}$ and (b) for the period from 1992 to 2017 as weekly solutions of the zonal coefficient $\bar{A}_{20}$ obtained via the eigenvalue-eigenvector problem and related to the principal axes system. The mean difference between the coefficients $\bar{C}_{20}$ or $\bar{A}_{20}$ given in various systems consists of the value $\approx 10^{-15}$ which is smaller than time variations in the coefficients $\bar{C}_{20}$ or $\bar{A}_{20}$. These time series of $\bar{C}_{20}$ were modeled by polynomials' different degrees simultaneously with Fourier series with seasonal signals (for annual, semiannual, and quarter-year periods). Final representation was chosen at the epoch J2000 by means of the polynomial of second degree. Then the models for the time-dependent astronomical dynamical ellipticity H_D and the precession constant p_A with respect to the common value $p_A = 50.2879225''/\mathrm{yr}$ were constructed using the model for the zonal coefficient $\bar{A}_{20}$ for the time-interval of about 25 yr. As the third step these time series of $\bar{A}_{20}(t)$ were applied to determine a basic trend and periodic variations of the time-dependent Earth's polar flattening from 1992 to 2017. A variation of the global dynamical and geometrical figure of the Earth was investigated and some important quantitative results were found: the polar flattening f_p is increasing within the considered 25 year time-interval. Therefore, this study aims to derive the variation of the global geometrical figure of the Earth, represented by the second-degree coefficients of time-series and the astronomical dynamical ellipticity H_D. As a result, a special attention was given to the study of temporally varying components including seasonal variations of some fundamental parameters of the Earth.

Key words. SLR, change in the zonal harmonic coefficient $\bar{C}_{20}$, Earth's polar flattening, dynamical ellipticity.

Introduction

First consistent estimate of temporal change in the zonal harmonic coefficient $\bar{C}_{20}(t) = -J_2(t)/\sqrt{5}$ of the Earth's gravitational potential was obtained by Yoder (1983) in the form of the so-called linear model for $\bar{C}_{20}(t) = \bar{C}_{20} + D\bar{C}_{20} = \bar{C}_{20} + \dot{\bar{C}}_{20}(t - t_0)$, where $\bar{C}_{20}$ is the time independent part given at epoch t_0 and $\dot{\bar{C}}_{20}$ is the secular variation of $\bar{C}_{20}$. Now we have a new development with the launch in 2002 yr of the mission GRACE (satellite-satellite observations) together with the traditional Satellite Laser Ranging (SLR) LAGEOS-1 and LAGEOS-2 (Laser Geodynamic Satellite) which were launched in 1976 yr and 1992 yr respectively (where LAGEOS-1 can be considered as a main tool of the NASA Crustal Dynamics Program). In general, basic estimates of different solutions of $\bar{C}_{2m}(t)$, $\bar{S}_{2m}(t)$ are obtained from the analysis of SLR observations of five geodetic satellites: LAGEOS-1 and 2, Starlette, Stella, and Ajisai [Ries, 2017]. It should be noted that the accuracy of satellite observations became more precise after the launch of the Italian LARES mission designed to test General Relativity in the weak gravity field of the Earth, including geodetic data.

In the last decades these satellites together with astronomical data allow to determine with higher accuracy the Earth's fundamental constants by means of different approaches [Marchenko & Schwintzer, 2003; Groten, 2004; Petit & Luzum, 2010; Chen & Shen 2010; Chen et al., 2015; Cheng et al, 2011; Cheng, et al, 2013; etc]. Generally speaking due to unstable determination of the precession constant (before 2002) or the dynamical ellipticity H_D, the consistency of fundamental parameters have led in particular to the additional adjustment at epoch of different sets of the second degree gravitational harmonic coefficients $\bar{C}_{2m}$, $\bar{S}_{2m}$ of the Earth's gravity models and various values of the dynamical ellipticity H_D [Marchenko & Schwintzer, 2003]. Most recent gravity field models give more accurate resolution of the time-dependent coefficients $\bar{C}_{2m}(t)$, $\bar{S}_{2m}(t)$ from SRL and GRACE observations.

In addition, latest determinations of the dynamical ellipticity H_D derived from the precession constant through VLBI are based on the non-rigid Earth's rotation theory [Mathews et al, 2002; Bourda & Capitaine, 2003; Capitaine et al. 2003; Fukushima, 2003; Liu & Capitaine, 2017] which was adopted by IAU resolutions at the epoch J2000 (Capitaine et al, 2009). It has to be pointed out that new value of H_D already include the time-dependence of secular variation of $\bar{C}_{20}$ [Marchenko & Schwintzer, 2003; Bourda & Capitaine, 2004; Capitaine et al, 2009] in the frame of linear model.

The consistency of the accurate modeling of the time evolution of the gravity field model solutions is of great importance for the interpretation of physically significant temporal gravity field variations. For example according to Cheng, et al. (2013) the above mentioned linear model should be replaced by a more precise one with the inclusion of a quadratic term at least. Therefore, this study aims to derive (a) the model for the zonal harmonic coefficient; (b) the appropriate model of the astronomical dynamical ellipticity H_D for the evolution with time and further solution of eigenvalue-eigenvector problem; (c) the time-evolving model of the Earth's polar flattening.

Hence the following three sections will focus on the modeling the time-dependent zonal harmonic coefficient $\bar{C}_{20}(t) = -J_2(t)/\sqrt{5}$, astronomical dynamical ellipticity $H_D(t)$ as a function of time t (fixed at the epoch J2000), and the time-dependent polar flattening of the Earth.

Modelling the time-dependence zonal harmonic coefficient C_{20}

Hereafter the coefficient $\bar{C}_{20} \approx \bar{A}_{20}$ will be considered as $\bar{A}_{20}$ related to the principal axes ($\bar{A}, \bar{B}, \bar{C}$) system of the values of the principal moments of inertia (*A*, *B*, *C*). In the first step we will verify a differences between $\bar{C}_{20}$ and $\bar{A}_{20}$ using the spherical Lambek formulas [Marchenko, 2009]. Let us express the vector $\mathrm{g} = \left[\bar{C}_{20}; \bar{C}_{21}; \bar{S}_{21}; \bar{C}_{22}; \bar{S}_{22}\right]^{\mathrm{T}}$ of the harmonic coefficients, adopted in the frame XYZ, via the vector $\mathrm{g}_{Z'} = \left[\bar{A}_{20}; \bar{A}_{21}; \bar{B}_{21}; \bar{A}_{22}; \bar{B}_{22}\right]^{\mathrm{T}}$ given in the coordinate system X′Y′Z′ with a small angle between the axes Z and Z', expressed by the mean pole coordinates:

$$\tan x_P = \tan q_p \cos l_p , \quad \tan y_P = -\tan q_p \sin l_p . \qquad (1)$$

These relationships give exact expressions for the polar coordinates q_p, l_p considered at the unit sphere. Thus, to avoid uncertainty in the deviatoric part of inertia tensor in the case of different sequences of finite rotations we will use on the contrary to usual approach a commutative rotation about the nodes line of XYZ and X′Y′Z′ systems, with the following transformation of the coordinate vector $\mathrm{r}' = \mathrm{Q} \cdot \mathrm{r}$ from XYZ to X′Y′Z′ frame, where according to Marchenko (2009) $\mathrm{Q} = \mathrm{R}_3(-l_P)\mathrm{R}_2(q_P)\mathrm{R}_3(l_P)$ is the rotation matrix dependent on the polar coordinates (1) of the axis Z′ in the system XYZ. The corresponding relationship between the degree *n* zonal harmonic coefficients $\bar{C}_{20}$ and $\bar{A}_{20}$ reads:

$$\bar{C}_{n0} = \sum_{m=0}^{n} (-1)^m \left(\bar{A}_{nm} \cos m l_P + \bar{B}_{nm} \sin m l_P\right) \cdot \tilde{P}_{nm}(\cos q_P) , \qquad (2)$$

where $\tilde{P}_{nm}(\cos q_P)$ are the A. Schmidt's quasi-normalized by the factor $\sqrt{(2-d_{m0})\frac{(n-m)!}{(n+m)!}}$ associated Legendre functions of the first kind (d_{m0} is the Kronecker delta). For the order *m*=0 these functions coincide with $P_{nm}(\cos q_P)$. When the order $m > 0$ we have the following simple relationship: $\bar{P}_{nm}(\cos\theta_P) = \sqrt{2n+1}\tilde{P}_{nm}(\cos\theta_P)$.

If we apply the transformation of the coefficients $\mathrm{g} = \left[\bar{C}_{20}; \bar{C}_{21}; \bar{S}_{21}; \bar{C}_{22}; \bar{S}_{22}\right]^{\mathrm{T}}$, defined in the Earth's fixed geocentric coordinate system XYZ, to the vector $\tilde{\mathrm{g}} = \left[\bar{A}_{20},\ 0,\ 0,\ \bar{A}_{22},\ 0\right]^{\mathrm{T}}$ of the two nonzero harmonic coefficients $\bar{A}_{20}$, $\bar{A}_{22}$ in the coordinate system of the Earth's principal axes of inertia $\bar{A}$, $\bar{B}$, $\bar{C}$ [Marchenko, 1998; Marchenko, & Schwintzer, 2003; Marchenko, 2009] we can determine $\bar{A}_{20}$ at epoch and the differences between $\bar{C}_{20}$ and $\bar{A}_{20}$. In this case such differences can be based on weekly solutions UTCSR of the time-dependent coefficients $\bar{C}_{2m}(t)$, $\bar{S}_{2m}(t)$ from 1992 to 2017. After transforming $\bar{C}_{2m}(t)$, $\bar{S}_{2m}(t)$ to the coefficients $\bar{A}_{20}(t)$, $\bar{A}_{22}(t)$ we got average difference between $\bar{C}_{20}$ and $\bar{A}_{20}$ $\approx 10^{-15}$ which is smaller than a mean difference for UTCSR monthly solution via formula (2). A value of difference $\approx 10^{-15}$ corresponds to the magnitude of non-zero invariant $I_1 \approx 10^{-15}$ according to the typical Lambek's formulas (Lambek, 1971).

Here one starts from the standard model considered in [Cheng, et al, 2011; Cheng et al, 2013] representing the $\bar{A}_{20}$ change:

$$\bar{A}_{20} = \bar{A}_{20}{}^{0} + \dot{\bar{A}}_{20}(t-t_0) + \ddot{\bar{A}}_{20}(t-t_0)^2 + \dddot{\bar{A}}_{20}(t-t_0)^3 + \sum_{i=1}^{3} A_i \cos\left(\frac{2p}{P_i}(t-t_0) - \{f_i\}\right), \qquad (3)$$

where $\bar{A}_{20}{}^{0}$ are the mean value of $\bar{A}_{20}$ at some reference epoch t_0; $\dot{\bar{A}}_{20}$, $\ddot{\bar{A}}_{20}$, $\dddot{\bar{A}}_{20}$ are the parameters of secular variations in $\bar{A}_{20}$, which are valid in the vicinity of t_0; (A_i, f_i) are the components of an oscillation for the annual, semiannual, and quarter-year P_i periods respectively.

Fig. 1 (1976 to 2017) and Fig. 2 (1996 to 2017) illustrate time series of $\bar{A}_{20}$ UTCSR monthly and weekly solutions respectively which were modeled by polynomials up to the 3rd degree simultaneously with Fourier series using the adopted usual seasonal signals for the annual, semiannual, and quarter-year periods. It has to be noted that the second solution is based on the corresponding fully normalized coefficients $\bar{C}_{2m}$, $\bar{S}_{2m}$ which are selected from the weekly or second UTCSR gravity field model. Final models were based only on the polynomial of second degree (blue line) and annual period (green line) at the epoch J2000.

Table 1

Coefficients for the polynomial representation of the long-term trend for $\bar{A}_{20}$ (blue line) in the form $\bar{A}_{20} = \bar{A}_{20}{}^{0} + \dot{\bar{A}}_{20}(t-t_0) + \ddot{\bar{A}}_{20}(t-t_0)^2$ at epoch J2000 (see formula (3)

Version	$\bar{A}_{20}{}^{0}$	$\dot{\bar{A}}_{20}$	$\ddot{\bar{A}}_{20}$
I	-484.1694554194E-06	0.1166E-11	-0.4844E-12
II	-484.1695458067E-06	-0.1001E-10	0.3659E-12

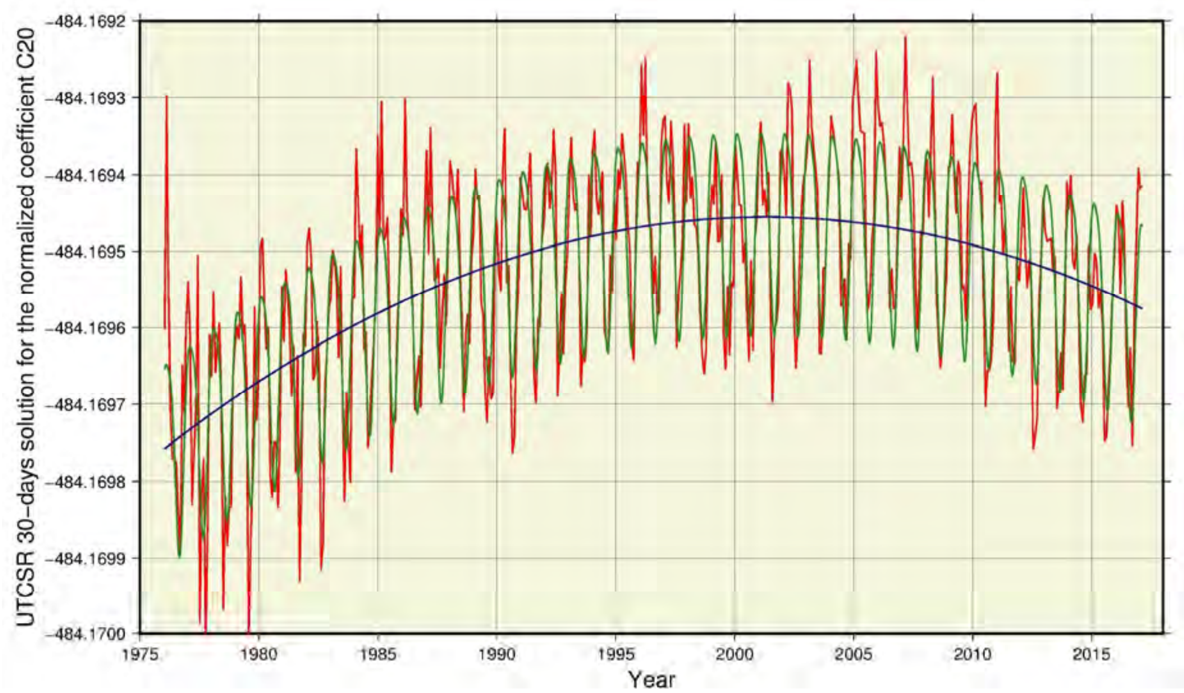

Fig. 1. Time series of $\bar{C}_{20}$ (UTCSR monthly solutions – red line) modeled by polynomials up to the 3rd degree simultaneously with the Fourier series using seasonal signals for annual, semiannual, and quarter-year periods. Final representation includes only the polynomial of second degree (blue line) and annual period (green line) at the epoch J2000

In addition a second model is demonstrated in Fig. 2 and Table 1 because the coefficient $\bar{A}_{20}$ was obtained via rigorous solution of the eigenvalue-eigenvector problem [Marchenko, & Schwintzer, 2003] and the LAGEOS-2 satellite SLR observations were provided additionally for the time-interval from 1992 to 2017 as LAGEOS-1 was launched at the end of 1976.

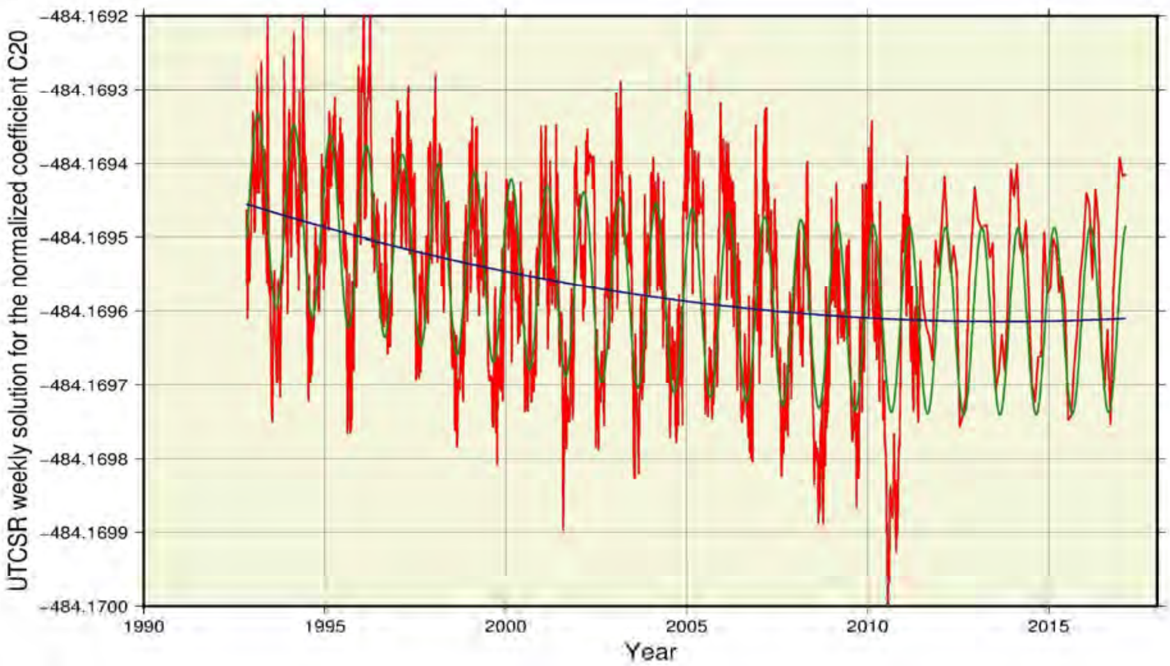

Fig. 2. Time series of $\bar{A}_{20}$ (UTCSR weekly solutions – red line) modeled by polynomials up to the 3rd degree simultaneously with Fourier series using seasonal signals for the annual, semiannual, and quarter-year periods. Final representation includes only the polynomial of second degree (blue line) and annual period (green line) at the epoch J2000

Modelling the Earth's time-dependence dynamical ellipticity

If the vector **g** of $\bar{C}_{2m}$ and $\bar{S}_{2m}$ is given, the solution of the eigenvalue problem Marchenko and Schwintzer (2003) provides the computation of the corresponding $\bar{A}_{20}$, $\bar{A}_{22}$ in the principal axes system via the eigenvalues of the quadratic form the corresponding degree 2 potential. By involving the astronomical dynamical ellipticity H_D we can determine the normalized by the factor $1/Ma^2$ principal moments of inertia A, B, and C:

$$H_D = (2C - A - B)/2C \iff C = -\sqrt{5}\bar{A}_{20}/H_D, \quad (4a)$$

$$A = \sqrt{5}\bar{A}_{20}(1 - 1/H_D) - \sqrt{15}\bar{A}_{22}/3, \quad (4b)$$

$$B = \sqrt{5}\bar{A}_{20}(1 - 1/H_D) + \sqrt{15}\bar{A}_{22}/3. \quad (4c)$$

Then the orientation of the principal axes in the XYZ frame is based on the exact solution of eigenvector problem, using $\bar{C}_{2m}$, $\bar{S}_{2m}$ only without the dynamical ellipticity H_D. Table 2 summarizes some estimates of H_D and the precession constant p_A. The first H_D was discussed in Williams (1994) as obtained for the nutation theory, taking into account secular variation $\dot{\bar{A}}_{20}$. The IAU H_D was adopted in the MHB2000 non-rigid Earth's nutation theory [Mathews, et. al., 2002]. The values of H_D given in Table 2 have small differences between the adopted H_D according to IAU2000/2006 Precession-Nutation model (see, Petit, Luzum, 2010, IERS Conventions 2010) contrary to previous determinations [Dehant, et al., 1998]. To transform the associated quantities from different p_A to the common value $p_A = 50.2879225''/\text{yr}$ the relationship $dH_D = 6.4947 \cdot 10^{-7} d\, p_A$ of Souchay and Kinoshita (1996) was applied to compute the values H_D given in brackets in Table 2. These new H_D have a much better accordance to the IAU 2000/2006 H_D than determinations before 2002.

Table 2

Determinations of the dynamical ellipticity H_D. Transformed H_D-values to the common MHB2000 precession constant p_A =50.2879225″/yr are given in brackets

Reference	Precession constant p_A [″/yr], epoch = J2000	H_D
Williams, 1994	50.287700 (50.2879225)	0.0032737634 (0.0032737779)
Mathews et al., 2002 (MHB2000)	50.2879225	0.0032737949
Capitane et al., 2003 (IERS Conventions 2010)	50.28796195 (50.2879225)	0.00327379448 (0.00327379450)
Fukushima, 2003	50.287955 (50.2879225)	0.0032737804 (0.0032737783)

With H_D, $\bar{A}_{20}$, $\bar{A}_{22}$, the computation of A, B, C, and the trace $\mathrm{Tr(I)} = A+B+C = \sqrt{5}\bar{A}_{20}(2-3/H_D) = 3I_m$ of the Earth's inertia tensor according to Eq. (4) are straightforward. By this we get a direct dependence of A, B, C, $\mathrm{Tr(I)}$, and the mean moment I_m of inertia on the adopted gravity field model and on the treatment of the permanent tide in the $\bar{C}_{20} \approx \bar{A}_{20}$ harmonic coefficient. It is assumed that the H_D values are related to the zero-frequency tide system [Groten, 2000].

Let us derive the variations of the dynamical flattening H_D from $\bar{A}_{20}$, $\bar{A}_{22}$ time-series and IAU2000/2006 dynamical ellipticity $H_D = 0.0032737945$ fixed at the epoch J2000. If we consider H_D as a constant parameter, the expression (4a) is valid in the following form:

$$H_D = -\frac{\sqrt{5}\bar{A}_{20}}{C} . \quad (5)$$

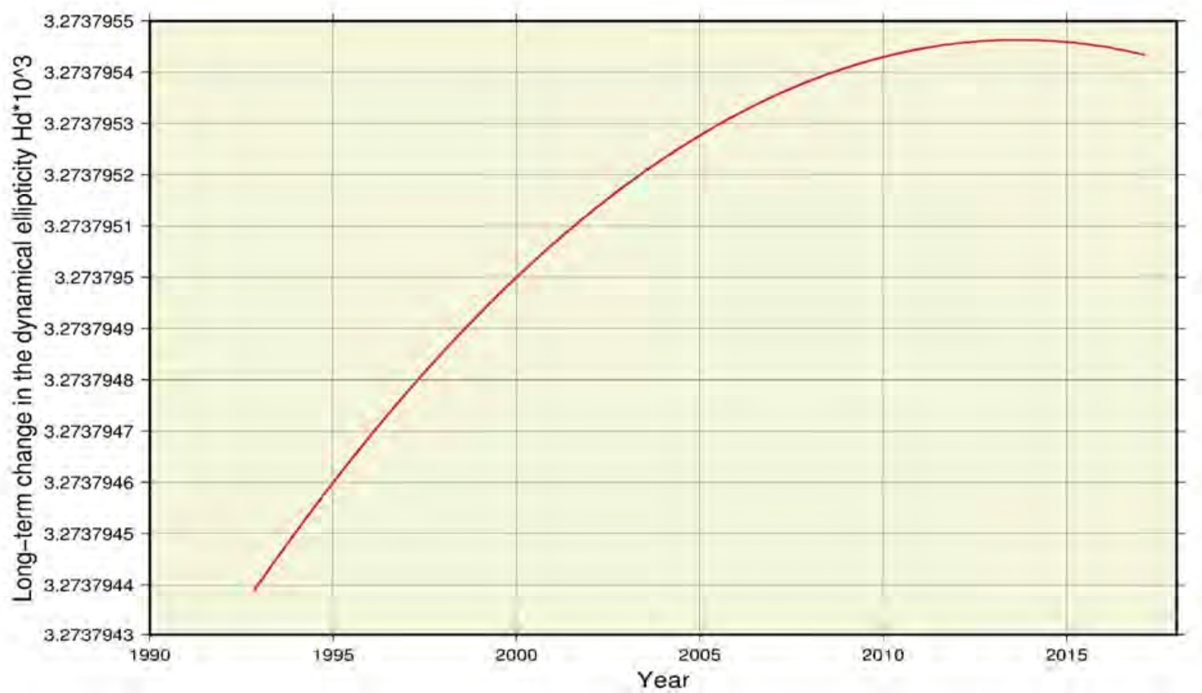

Fig. 3. Change in the dynamical ellipticity H_D modeled by polynomials of second degree (red line) with t_0 = J2000

If the celestial body is elastic and gravity field is variable we need to add small variations of all parameters in formula (5). From Eq. (5), taking into account, that the non-tidal variation dc in the dimensionless moment of inertia C is a function of $\bar{C}_{20}(t)$, and the condition Tr(**I**)=constant [Rochester, & Smylie, 1974] "as zonal forces do not change the revolution shape of the body" [Melchior, 1978], one gets for the secular change in H_D:

$$H_D(t) = -\frac{\sqrt{5}\bar{A}_{20}(t)}{C(t)} = -\frac{\sqrt{5}\bar{A}_{20}(t)}{C} \cdot \frac{1}{1+dc/C}, \qquad C(t) = C + dc. \tag{6}$$

where dc is the variable part of C.

This formula was used for the Taylor expansion [Bourda, & Capitaine, 2004] written as:

$$H_D(t) = H_D\big|_{t=t_0} + dH_D = H_D\big|_{t=t_0} - \frac{\sqrt{5}d\bar{A}_{20}(t)}{C}, \tag{7}$$

where $d\bar{A}_{20}(t)$ is nothing else but the change of the transformed to the principal axes system of the time-dependent zonal harmonic coefficient.

The final solution is given at the epoch t_0 = 2000 based on the following relationship:

$$H_D(t) = H_D\big|_{t=t_0} - \frac{\sqrt{5}}{C} \cdot \left(\dot{\bar{A}}_{20}(t-t_0) + \ddot{\bar{A}}_{20}(t-t_0)^2 \right), \tag{8}$$

where $\dot{\bar{A}}_{20}$ and $\ddot{\bar{A}}_{20}$ are taken from the Table 1 (version II) and shown in Fig. 3.

If H_D depends on the time t according to (8) the change of the precession constant p_A with respect to the common value $p_A = 50.2879225''/\mathrm{yr}$ is also possible by means of the following relationship:

$$p_A(t) = p_A\big|_{t=t_0} + \frac{H_D(t) - H_D\big|_{t=t_0}}{6.4947 \cdot 10^{-7}}, \tag{9}$$

based on the expression $dH_D = 6.4947 \cdot 10^{-7} d\,p_A$ of Souchay and Kinoshita (1996) and applied here to solve the inverse problem. After computations based on the model (8) we get the following similar to the Fig. 3 dependence of the long-term change in the precession constant shown in Fig. 4.

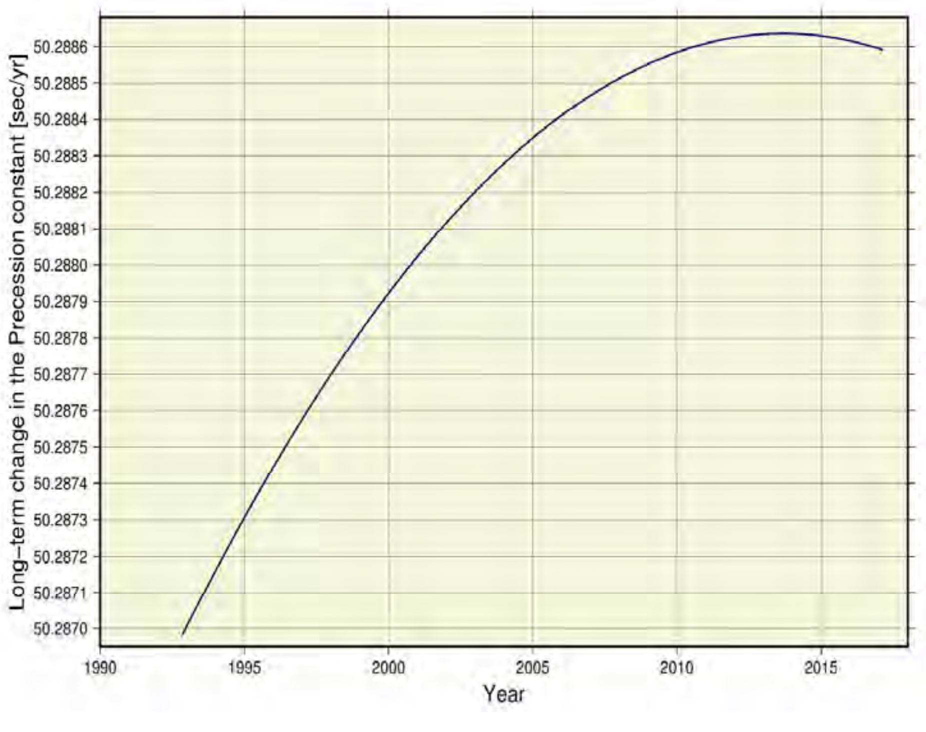

Fig. 4. Change in the precession constant p_A with respect to the common value $p_A = 50.2879225''/\mathrm{yr}$ modeled by means of the formula (9) (blue line) with respect to the epoch t_0 = J2000 and the corresponding model for H_D

Modelling the Earth's time-dependence polar flattening

With the obtained model for H_D the solution of eigenvalue-eigenvector problem and computation of polar flattening and other constants in the system of principal axes of inertia is straightforward. It should be noted that the secular change in the 2nd-degree zonal coefficient $\dot{\bar{A}}_{20}$ has opposite sign in I and II versions of the model.

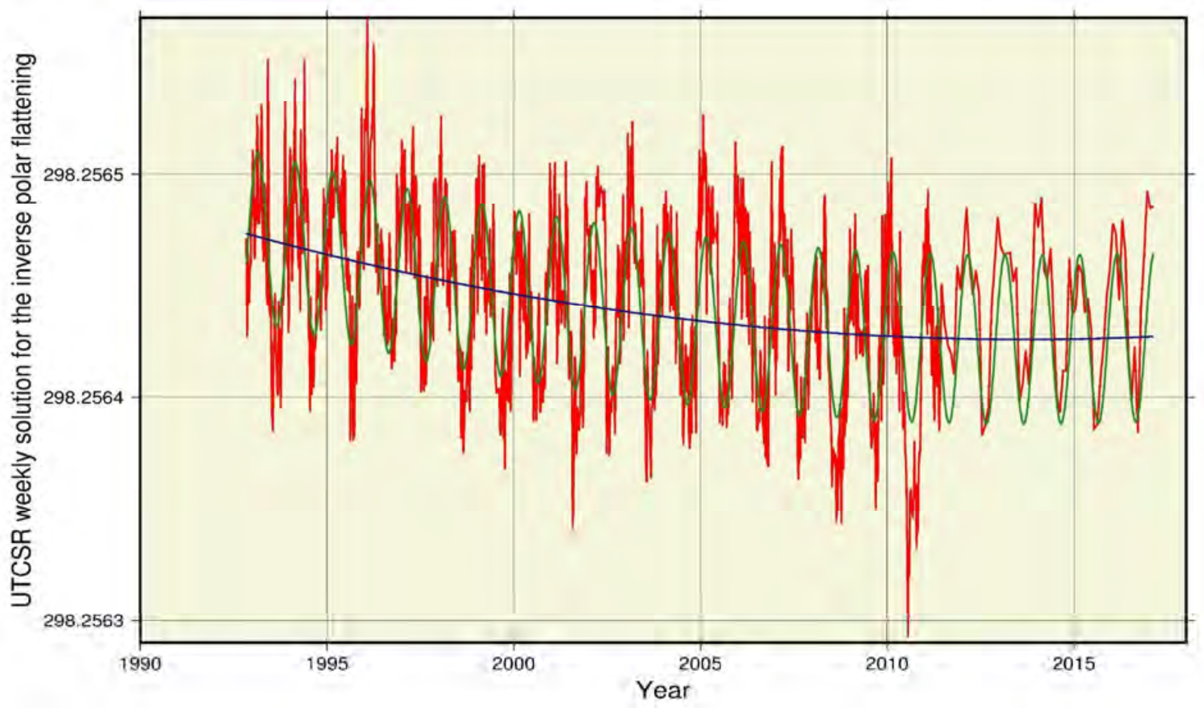

Fig. 5. Time series of inverse polar flattening $1/f_p$ obtained from $\bar{A}_{20}$ for each current epoch (UTCSR weekly solutions – red line) modeled by polynomials simultaneously with Fourier series using seasonal signals. Final model includes only the polynomial of second degree (blue line) and annual period (green line) at the epoch J2000

The second model is close to the simple linear model for $\dot{\bar{A}}_{20}$ because $\ddot{\bar{A}}_{20}$ is two orders smaller. The time variable coefficients in these solutions are referred to one epoch J2000. Modelling the Earth's time-dependence polar flattening was obtained based on the solution II. Fig.4 illustrates the increasing of the polar flattening f_p within the considered time-interval because the inverse flattening $1/f_p$ (as denominator) is decreasing. This fact is in contradiction to our previous views on the change in the Earth's polar flattening [Yoder, et al., 1983].

Conclusions

In summary we can conclude. The time-dependent change of the Earth's zonal harmonic coefficient was derived from the UTCSR SRL time series of $\bar{C}_{20}(t)$ for the period from 1976 to 2017 (as monthly solutions in the first step) and for the period from 1992 to 2017 (as weekly solutions for $\bar{C}_{2m}$, $\bar{S}_{2m}$ in the second step) and the obtained weekly zonal coefficients $\bar{A}_{20}$ via the rigorous eigenvalue-eigenvector problem. The difference between the coefficients $\bar{C}_{20}$ or $\bar{A}_{20}$ given in various system consists of the value $\approx 10^{-15}$ which is smaller then time variations in the coefficients $\bar{C}_{20}$ or $\bar{A}_{20}$.

These time series of $\bar{C}_{20}$ or $\bar{A}_{20}$ were modeled by polynomials of different degrees simultaneously with the Fourier series by means of standard seasonal signals (with annual, semiannual, and quarter-year periods). Final approximation was chosen at the epoch J2000 by means of the polynomials of second degree only. Then the model for the time-dependent astronomical dynamical ellipticity H_D and the precession constant p_A with respect to the common value $p_A = 50.2879225''/\text{yr}$ were constructed using the model for the zonal coefficient $\bar{A}_{20}$ for the 25 year time-interval. This model allows determinations of the Earth's time-dependent fundamental parameters, including polar flattening. Therefore these time series of $\bar{A}_{20}(t)$ were applied to determine a basic trend and annual periodic variations in the Earth's polar flattening from 1992 to 2017. The Earth's polar flattening f_p is increasing within this time-interval although Yoder (1983) and other authors from SRL LAGEOS-1 observations have obtained the decreasing of f_p. Fig.1 demonstrates this fact where the long-term change of harmonic coefficient $\bar{C}_{20}$ before approximate epoch J2000 has a variation $\dot{\bar{C}}_{20}$ with sign (+) and after this epoch we get a variation $\dot{\bar{C}}_{20}$ with sign (-).

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References

Bourda, G., N. Capitaine (2004). Precession, nutation, and space geodetic determination of the Earth's variable gravity field. *Astronomy & Astrophysics*, *428*(2), 691–702, doi: 10.1051/0004-6361:20041533

Capitaine N., Wallace, P. T., & Chapront J. (2003). Expressions for IAU 2000 precession quantities. *Astronomy & Astrophysics, 412*(2), 567–586, doi: 10.1051/0004-6361:20031539

Capitaine N, Mathews, P. M., Dehant, V., Wallace, P. T., & Lambert, S. B. (2009). On the IAU 2000/2006 precession–nutation and comparison with other models and VLBI observations. *Celestial Mechanics and Dynamical Astronomy*, *103*(2), 179–190, DOI 10.1007/s10569-008-9179-9

Chen, W., & Shen, W. B. (2010). New estimates of the inertia tensor and rotation of the triaxial nonrigid Earth. *Journal of Geophysical Research, 115*: B12419. doi: 10.1029/2009JB007094.

Chen, W., Li, J. C., Ray, J., Shen, W. B., & Huang, C. L. (2015). Consistent estimates of the dynamic figure parameters of the earth. *Journal of Geodesy*, *89*(2), 179–188, doi: 10.1007/s00190-014-0768-y

Cheng, M., Ries, J. C., & Tapley, B. D. (2011). Variations of the Earth's figure axis from satellite laser ranging and GRACE, *Journal of Geophysical Research: Solid Earth, 116,* B01409, doi: 10.1029/2010JB000850.

Cheng, M., Tapley, B. D., & Ries, J. C. (2013). Deceleration in the Earth's oblateness, *Journal of Geophysical Research: Solid Earth, 118(2)*, 740–747, doi:10.1002/jgrb.50058.

Dehant, V. Arias, F., Bizouard, C., Bretagnon, P., Brzezinski, A., Buffett, B., Capitaine, N., & Zhu, S. (1998). Considerations concerning the non-rigid Earth nutation theory. *Celestial Mechanics and Dynamical Astronomy*, 72, 245–309.

Fukushima, T. (2003). A New Precession Formula. *The Astronomical Journal*, *126*(1):494–534.

Groten, E. (2004). Fundamental parameters and current (2004) best estimates of the parameters of common relevance to astronomy, geodesy, and geodynamics. *Journal of Geodesy*, 77, 724–797, doi:10.1007/s00190-003-0373-y

Lambeck, K. (1971). Determination of the Earth's pole of rotation from laser range observations to satellites. *Bulletin Géodésique*, *101*(1), 263–281.

Liu, J. C., & Capitaine, N. (2017). Evaluation of a possible upgrade of the IAU2006 precession. *Astronomy & Astrophysics,* 597, A83 (2017), doi: 10.1051/0004-6361/201628717

Marchenko, A. N. (1998). *Parameterization of the Earth's gravity field. Point and line singularities*. Lviv Astronomical and Geodetic Society, Lviv, 1998.

Marchenko, A. N. (2009). Current estimation of the Earth's mechanical and geometrical parameters. In: M.G. Sideris (ed.), *Observing our Changing Earth,* (pp. 473-481) International Association of Geodesy Symposia 133. Springer, Berlin, Heidelberg.

Marchenko, A. N., & Schwintzer, P. (2003) Estimation of the Earth's tensor of inertia from recent global gravity field solutions. *Journal of Geodesy*, *76*(9-10), 495–509.

Mathews, P. M., Herring, T. A., & Buffet, B. A. (2002). Modeling of nutation-precession: New nutation series for nonrigid Earth, and insights into the Earth's interior, *Journal of Geophysical Research*, *107*(B4), doi: 10.1029/2001JB000390.

Melchior P. (1978). *The tides of the planet Earth*. Pergamon.

Petit, G, & Luzum, B. (eds) (2010). IERS conventions (2010). IERS Technical Notes 36. Observatoire de Paris, Paris

Ries, J. C. (2017). ftp://ftp.csr.utexas.edu/ pub/ slr/ degree_2/ (Private communication).

Rochester, M. G., & Smylie, D. E. (1974). On changes in the trace of the Earth's inertial tensor. *Journal of Geophysical Research*, *79*(32), 4948–4951.

Williams, J. G. (1994) Contributions to the Earth's obliquity rate, precession and nutation, *Astronomical Journal*, 108, 711–724.

Yoder, C. F., Williams, J. G., Dickey, J. O., Schutz, B. E., Eanes, R. J., & Tapley, B. D. (1983). Secular variation of earth's gravitational harmonic J2 coefficient from Lageos and nontidal acceleration of earth rotation. *Nature*, *303*(5920), 757–762.

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ЗМІНА ЗОНАЛЬНОГО ГАРМОНІЧНОГО КОЕФІЦІЄНТА C_{20}, ПОЛЯРНОГО ТА ДИНАМІЧНОГО СТИСНЕННЯ ЗЕМЛІ ЗА ДАНИМИ СУПУТНИКОВОГО ЛАЗЕРНОГО ДІАПАЗОНУ

Досліджено зміну коефіцієнта зональної гармоніки другого ступеня Землі, отриманого з UTCSR SRL часових рядів $\bar{C}_{20}(t)$ даних (a) для інтервалу з 1976 р. по 2017 р. як місячні рішення зонального коефіцієнта $\bar{C}_{20}$ та (b) для інтервалу з 1992 р. по 2017 р. як тижневі рішення зонального коефіцієнта $\bar{A}_{20}$ отриманого за допомогою задачі власних значень – власних векторів і пов'язаного з системою головних осей інерції. Середня різниця між коефіцієнтами $\bar{C}_{20}$ та $\bar{A}_{20}$ в різних системах оцінюється $\approx 10^{-15}$, що є меншим, ніж часові варіації коефіцієнтів $\bar{C}_{20}$ та $\bar{A}_{20}$. Ці часові ряди $\bar{C}_{20}$ моделювалися поліномами різних ступенів сумісно з рядами Фур'є (з річними, піврічними та квартальними періодами). Остаточну модель обрано на епоху J2000 за допомогою полінома другого ступеня. На наступному кроці, використовуючи модель для зонального коефіцієнта $\bar{A}_{20}$ з інтервалом часу близько 25 років, побудовано залежні від часу моделі астрономічного динамічного стиснення H_D та постійної прецесії p_A з фіксацією значення $p_A = 50.2879225''/\text{yr}$ IAU 2000 на епоху J2000. На третьому кроці часові ряди $\bar{A}_{20}(t)$ застосовано для визначення основного тренду та періодичних варіацій залежного від часу полярного стиснення Землі з 1992 року по 2017 року. Досліджено варіацію глобальної динамічної та геометричної фігур Землі та знайдено деякі важливі кількісні результати: полярне стиснення f_p збільшується в межах розглянутого інтервалу часу, який становить близько 25 років, що суперечить попереднім дослідженням. Тому метою цієї роботи є визначення варіацій глобальної геометричної фігури Землі, представлених через гармонічні коефіцієнти другого ступеня часових рядів і астрономічного динамічного стиснення H_D. Як результат, особливу увагу приділено вивченню залежних від часу компонентів, включаючи сезонні варіації деяких фундаментальних параметрів Землі.

Ключові слова: SLR, зміна коефіцієнта зональної гармоніки, полярне стиснення Землі, астрономічне динамічне стиснення

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ИЗМЕНЕНИЕ ЗОНАЛЬНОГО ГАРМОНИЧНОГО КОЭФФИЦИЕНТА C20, ПОЛЯРНОГО И ДИНАМИЧЕСКОГО СЖАТИЯ ЗЕМЛИ ПО ДАННЫМ СПУТНИКОВОГО ЛАЗЕРНОГО ДИАПАЗОНА

Исследовано изменение коэффициента зональной гармоники Земли второй степени $\bar{C}_{20}(t)$, полученного из UTCSR SLR временных рядов данных (a) для интервала с 1976 г. по 2017 г. как месячные решения для зонального коэффициента $\bar{C}_{20}$ и (b) для интервала с 1992 г. по 2017 г. как недельные решения зонального коэффициента $\bar{A}_{20}$, полученного с помощью задачи на собственные значения – обственные векторы и связанного с системой главных осей инерции. Средняя разница между коэффициентами $\bar{C}_{20}$ и $\bar{A}_{20}$ в разных системах оценивается как $\approx 10^{-15}$, что меньше, чем временные вариации коэффициентов $\bar{C}_{20}$ и $\bar{A}_{20}$. Эти временные ряды $\bar{C}_{20}$ моделировались полиномами различных степеней совместно с рядами сезонных сигналов Фурье (с годовым, полугодовым и квартальным периодам). Окончательная модель выбрана на эпоху J2000 с помощью полинома второй степени. На следующем этапе, используя модель для зонального коэффициента $\bar{A}_{20}$ с интервалом времени около 25 лет, построены зависящие от времени модели для астрономического динамического сжатия H_D и постоянной прецессии p_A с фиксацией значения $p_A = 50.2879225''/\mathrm{yr}$ IAU 2000 в эпоху J2000. На третьем этапе временные ряды $\bar{A}_{20}(t)$ применены для определения основного тренда и периодических вариаций зависимого от времени полярного сжатия Земли с 1992 г. по 2017 г. Исследована вариация глобальной динамической и геометрической фигуры Земли и найдены некоторые важные количественные результаты: полярное сжатие f_p увеличивается в пределах рассматриваемого интервала времени, составляющего около 25 лет находится в противоречии с предыдущими исследованиями. Поэтому, целью данного исследования является определение вариаций глобальной геометрической фигуры Земли, представленных через гармоничные коэффициенты второй степени временных рядов и астрономического динамического сжатия H_D . Как результат, особое внимание уделено изучению зависимых от времени компонентов, включая сезонные вариации некоторых фундаментальных параметров Земли.

Ключевые слова: SLR, изменение коэффициента зональной гармоники, полярное сжатие Земли, астрономическое динамическое сжатие..

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