Definition of the Path with Minimum Total Length on Base the Flows between Nodes of the Network

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INTRODUCTION

Significant costs for the construction and repair of modern high-speed transport networks necessitate (in the design process) the provision of transport communication of the given basic nodes of the network while minimising its total length. Therefore, the task of definition the path of the minimum total length, which connects all basic nodes of the network, taking into account the flows existing between them, is solved in this work. In this case, in contrast to the classical problem of searching for the shortest path on a given vertices of the weighted graph [1], we propose a method based on the introduction of the additional nodes, through which the path of the minimum total length is passed. For determine the co-ordinates of this additional nodes does not apply network's graph model. Since the specified additional nodes do not coincide with the basic nodes and are not on the arcs that connect them, then the solvable problem is not reduced to the classical problem of placement [1].

The input data for the solvable problem are the co-ordinates of the basic nodes of the network, which must be connected by paths of minimum total length. In this case, the length of the paths between the pairs of basic nodes is minimised, if possible, taking into account the intensity of the flows that exist between these nodes.

METHOD FOR DEFINITION OF THE PATH WITH MINIMUM TOTAL LENGTH

The essence of the proposed approach to network design is presented in a demonstration example (Fig. 1).

Let the basic nodes of the network *(1,2,3)* be located at the vertices of an equilateral triangle with a length of the side *L (i, j)* = 60 km. In this case, the total length of the paths that directly connect all the basic nodes is

Ls = 180 km (variant 1). In the second variant, the distance between the basic nodes (for example (1, 2)) through the additional node 4 is *D(1,2)* = *D(1,4)* + *D(4,2)* = 69,3 km. The total length of the paths that connecting all basic nodes through an additional node 4 is *Ds* = 104 km (variant 2). The coefficient of reduction of the total length of the path is equal *Kz* = 180/104 = 1.7. The coefficient of lengthening of the path between the basic nodes (1, 2) through node 4 is *Ku* = 69.3/60 = 1.16. Thus, the method provides a significant reduction in the total length of the path between the basic nodes with a slight increase in the path between pairs of basic nodes through an additional node *(Ku < Kz)*. In case of significant costs for network construction (for example, for high-speed railways at speeds of 250 km / h, the cost of a kilometer is € 20 million) variant 2 is more promising.

At different flows between basic nodes, the additional node is shifted towards the nodes with more intensive flows to reduce the path length through node 4 (is denoted by a wavy line).

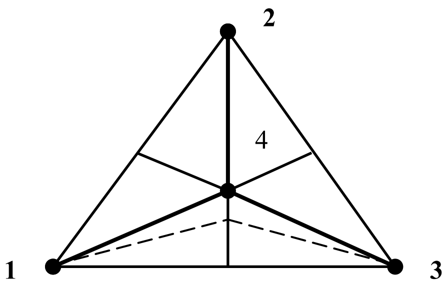

Fig. 1 The demonstration variant of the network (example 1).

Program implementation of the method at given coordinates of the basic nodes of the network and the intensity of flows between them consists of the following steps:

1. The most remote nodes *(i, j)* of the network, which connected directly to each other (for example, (1, 2)) is determined *(x (i) = xmin, x (j) = xmax, y (i) = ymin, y (j) = ymax, Δx = x (j) - x (i), Δy = y (j) -y (i), Δx < Δy or Δy < Δx)*.

2. The initial coordinates of the additional nodes that located at the intersection points of the straight line between the most distant nodes and the straight lines perpendicular to it and passing through the rest of the basic nodes of the network are determined. These initial coordinates will be changed during the iterative search for an optimal network variant.

3. The distance between all the basic nodes of the network is determined

$$L(i,\ j)=SQRT((x(j)-x(i))^2+(y(j)-y(i))^2) \qquad (1)$$

where: $i = 1, N, j = 1, N, i \neq j, N$ – is the number of basic nodes of the network.

4. The distance between each of the additional nodes m and adjacent basic nodes and additional nodes is determined

$$D(m, j)=SQRT((x(j)-x(m))^2+(y(j)-y(m))^2) \quad (2)$$

5. For each pair of basic nodes (i, j), that adjacent to the additional node m, the coefficient of elongation is determined

$$Kp(i, j)=(D(m, i)+D(m, j))/L(i, j) \quad (3)$$

6. The generalized elongation factor is determined

$$Kq(i, j)=Kp(i, j)*Ss(i, j) \quad (4)$$

where: $Ss\ (i, j)$ - is the total flow between the basic nodes (i, j).

7. For all pairs of basic nodes (i, j) the optimization index is determined

$$\Delta Kq(i, j)=(Kq(i, j)-Kqmin) \quad (5)$$

where: $Kqmin = min\ (Kq\ (i, j)), i = 1, M, j = 1, M, i \neq j, M$ - the number of basic nodes adjacent to the additional node m.

8. By changes of the additional nodes coordinates $(x + \Delta x, y + \Delta y)$ of the node m (nine variants in accordance with Table 1) minimizes the value

$$Q=\Sigma(\Delta Kq(i, j))^2 \quad (6)$$

Where: $i=1,M, j=1,M, i\neq j$.

TABLE 1

VARIANTS FOR CHANGING THE COORDINATES OF ADDITIONAL NODES

V	1	2	3	4	5	6	7	8	9
Δx	0	0	1	1	1	0	-1	-1	-1
Δy	0	1	1	0	-1	-1	-1	0	1

9. In the presence of several additional nodes, a joint change in their coordinates is made. That is, all variants for the second node at each of the variants of the first is analyzed (combinatorial optimization). In this case, the computational complexity for the analyze of the variants for each iteration is proportional to $(9)^M$.

10. Search stops when variants 2-9 do not lead to a decrease in Q.

The results of determining the coordinates of additional nodes at different variants of network (that shown in Figs. 2 and 3) are presented in Table II and III.

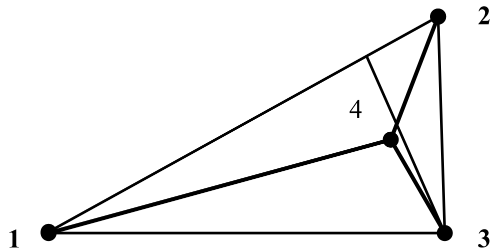

Fig. 2 The demonstration variant of the network (example 2).

TABLE 2
THE RESULT OF THE PATH DEFINITION (EXAMPLE 2)

	n	*x(n)*	*y(n)*	*i*	*j*	*L(i,j)*	*m*	*j*	*D(m,j)*
Introdused	1	0	30	1	2	224	4	1	166
	2	200	100	1	3	180	4	2	80
	3	180	30	2	3	102	4	3	32
Determined	4	40	38	Total		506	Total		278

TABLE 2 (CONTINUATION)

i	*j*	*Path*	*D(i,j)*	*Ku(i,j)*	*Kz*
1	2	1, 4, 2	246	1.1	1.82
1	3	1, 4, 3	198	1.1	
2	3	2, 4, 3	112	1.1	

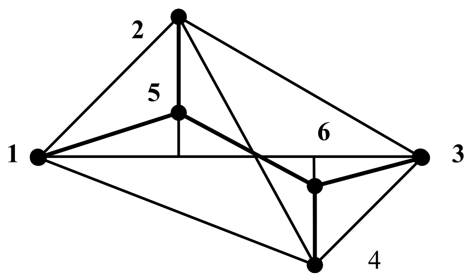

Fig. 3 The demonstration variant of the network (example 3).

TABLE 3
THE RESULT OF THE PATH DEFINITION (EXAMPLE 3)

	n	*x(n)*	*y(n)*	*i*	*j*	*L(i,j)*	*m*	*j*	*D(m,j)*
Introdused	1	0	30	1	2	57	5	1	41
	2	40	70	1	3	100	5	2	32
	3	100	30	1	4	76	5	6	33
	4	70	0	2	3	72	6	3	30
Determined	5	40	38	2	4	76	6	4	25
	6	70	25	3	4	42	Total		161

TABLE 3 (CONTINUATION)

i	*j*	Path	*D(i,j)*	*Ku(i,j)*	*Kz*
1	2	1,5,2	73	1.28	2.63
1	3	1,5,6,3	104	1.04	
1	4	1,5,6,4	99	1.3	
2	3	2,5,6,3	95	1.32	
2	4	2,5,6,4	90	1.18	
3	4	3,6,4	55	1.31	

CONCLUSIONS

The proposed method provides a definition of the path of minimum total length with an insignificant lengthening of paths, that connect the basic nodes of the network, taking into account the flow between them. The method can be applied in the computer-aided design of automobile road and railways, as well as transmission and communication lines.

REFERENCES

[1] E. Minieka. *Optimization Algorithms for Networks and Graphs.* Mir, M: 1981.