Electrical Model of Vibratory Micro Angular Velocity Sensor

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INTRODUCTION

Angular velocity sensor – gyroscope - is a device which is commonly used to measure the rotation rate of a specified solid object with respect to an inertial frame of reference. “Commonly” means that this kind of device is used commercially in equipment many people used. Microgyroscopes are one of the most important types of silicon-based sensors in MEMS world [5]. Because of small dimensions of this sensor application potential continuously grows and this device finds its appropriate place in navigation, automotive safety systems, rollover detection, computer input devices, sports and medical equipment, platform stabilization of heavy machinerymilitary, platform stabilization of avionics, unmanned air or land vehicles, inertial measurement units for inertial navigation, automatics and robotics and more.

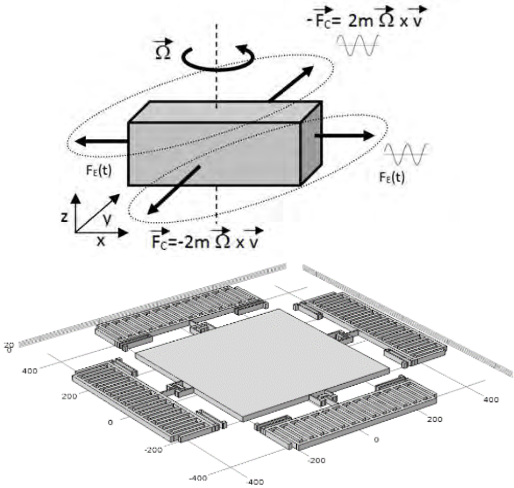

Fig. 1 General gyroscope and MEMS vibrating gyroscope operation principal and model of decoupled gyroscope considered in this article.

A gyroscope (also accelerometers) behaves as a damped mass on a spring [1-2]. Principle of gyroscope operation is not complex, it uses Coriolis effect (Coriolis force) which appears in rotating objects (fig.1). When object rotates around *z* axis and moves along *x* axis with linear velocity *v* (which is orthogonal to rotating axis) additional force along *y* axis appears. Coriolis phenomena causes that in case of clockwise rotation, Coriolis force acts along with negative part of *y* axis to the left of the object motion. In case of anticlockwise rotation, the force acts along with positive part of *y* axis. This force is equals [3,4]:

$$F_C = -2m(\vec{\Omega} \times \vec{v}) \qquad (1)$$

where Ω – is angular velocity, *v* – linear velocity, *m* – mass of object.

Vibrating MEMS gyroscope can be described with second order differential equations:

$$m\frac{d^2x}{dt^2} + b_x\frac{dx}{dt} + k_x x = F_D \sin(\omega t) \qquad (2)$$

$$m\frac{d^2y}{dt^2} + b_y\frac{dy}{dt} + k_y y = -2m\frac{dx}{dt}\Omega \qquad (3)$$

where *m* – vibrating mass, b_x, b_y – damping coefficient, k_x, k_y – spring coefficient, *W* – measured angular velocity, F_D – force generated by comb drive actuator.

These equation reflects behavior inertial mass motion along two direction: one in drive direction (actuator) and one in sense direction (sensor). Spring constant is calculated using following formulas, which allows to obtain single beam spring constant, parallel and serial spring connection:

$$k_b = \frac{Ewt^3}{4L^3},\; k_p = \sum_i k_{pi}\,, \frac{1}{k_s} = \sum_i \frac{1}{k_{si}} \qquad (4)$$

Because both drive and sense directions includes comb drive set, it is incredibly important to minimize or avoid any unnecessary mutual motion leverage which appear during rotation. This is why model of gyroscope should be strongly decoupled and has separated springs motions in both directions. For drive direction four edge springs with comb drive and two central springs on sense side are coupled (fig. 1). Similarly is for sense direction. Based on gyroscope mass *m* and stiffness resonant frequency □$_0$ and quality factor can be calculated applying following equation:

$$\omega_0 = \sqrt{\frac{k}{m}} = \sqrt{\frac{k_0(1+\alpha T)}{m}} \qquad (5)$$

$$Q = \frac{m\omega_0}{b} \qquad (6)$$

Q factor describes how gyroscope behaves in meaning of damping. High Q factors reflects oscillators with low damping – they vibrates longer. Q factor value of 0.5 is threshold which changes meaningfully damping during vibrations.

ELECTRICAL EQUIVALENT MODEL OF MEMS GYROSCOPE

Whole above theory which is related to mechanical behavior of gyroscope sensor can be successfully transferred to other domain and described by equivalent physical phenomena. Well-known motion equation (2) has its counterpart in other physical domains. Model created in Matlab/SIMULINK was based on electrical analogies of mechanical and electrical physical quantities. In MEMS modelling world electrical equivalent models are very popular and one can find many examples of different sensors and actuators. Popularity comes from fact, one can use software like PSPICE for modeling and simulations. Transferring gyroscope model from mechanical domain to electrical one can be easy achieved with transferring appropriate physical quantities from one domain to other. Table 1. includes some mechanical-physical analogies. One can easy transform force, velocity, mass, spring constant and damping coefficient to electrical domain by substituting quantities in set of two equations (2) and (3).

TABLE 1
LIST OF ELECTRICAL ANALOGIES FOR MODEL.

Mechanical Quantity	**Electrical Quantity**
v - velocity	V - voltage
F – force	I - current
m - mass	C - capacitance

Transformed motion equation is presented in (4).

$$I = C\frac{dV}{dt} + \frac{1}{R}V + \frac{1}{L}\int V dt \qquad (7)$$

$$I = C\frac{d^2\phi}{dt} + \frac{1}{R}\frac{d\phi}{dt} + \frac{1}{L}\phi \qquad (8)$$

where: ϕ - flux in electrical domain (equivalence of displacement x).

In fig. 2 there is electrical equivalent model of MEMS gyroscope created by authors with use these eq. 6 and 7 describing dynamics of electrical circuit. Fig. 2 also presents subschema for mass calculation (in electrical equivalent this reflects to capacitance). This subschema calculates total mass based on sum of masses particular parts: inertial mass, edge and center springs.

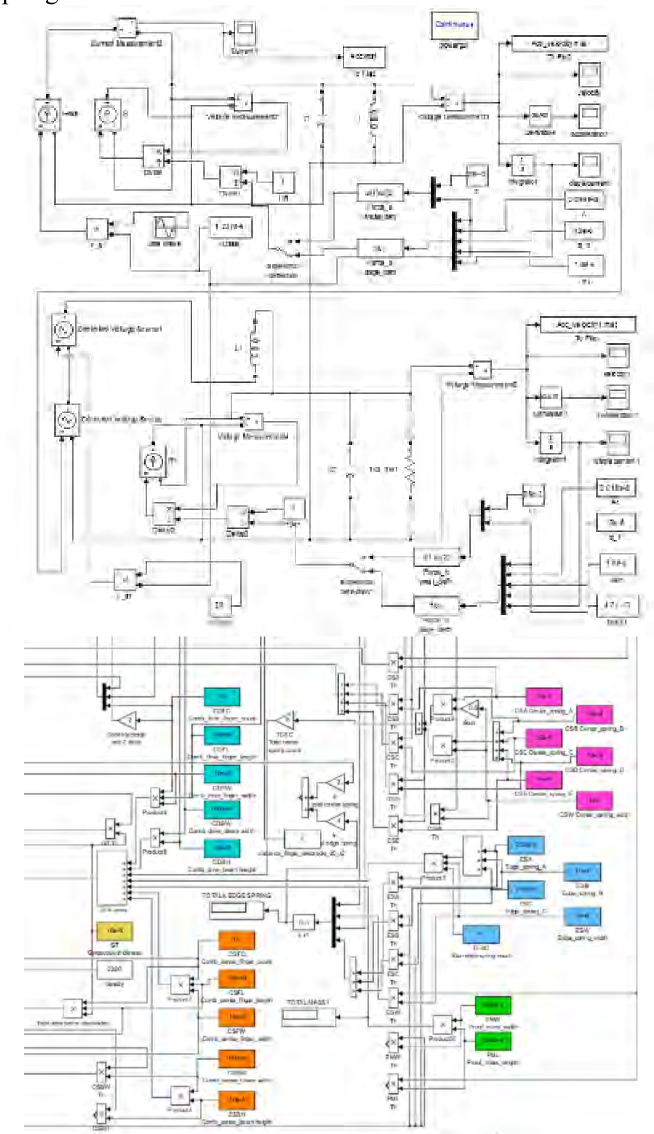

Fig.2 General schema of the presented model: electrical equivalent model of gyroscope and mass calculation.

Results obtained from this model were compared with results came from mathematical and physical models implemented with standard and SimScape Toolboxes. Displacements, velocities and accelerations along drive and sense directions were very similar.

According to mentioned earlier it is incredibly important to get maximum displacement along y direction caused by Coriolis force. Because this system is vibrating, maximum displacement can be achieved when external frequency applied along x direction is equal to eigenfrequency.

RESULTS OF SIMULATIONS

We performed simulation to test how model behaves when above resonance condition is meet. First, these tests required to calculate spring coefficient, mass and then – resonant frequency. Values taken for simulation were as following: W_m=450·10^{-6}m, L_m=450·10^{-6}m, k=0.3461N/m, m=1.189·10^{-9}m, □$_0$=17060 1/s. Tests were performed with □=5rad/s applied. Results of simulations are presented in frequency domain to show optimal value of frequency and to obtain maximum displacement for sense direction. They show that there is dramatically growth of amplitude displacement of inertial mass along y direction when eigenfrequency is applied along x direction. Simulation results are presented for different inertial mass dimensions and spring length L_{arm}=500*10^{-3}m (fig.3). We observe, that geometrical dimensions leverage on sense displacement and moreover changes eigenfrequency. It can be observed, that geometrical dimensions have meaningful impact on characteristics in frequency domain. Spring dimensions change cause change both spring constant and total mass; inertial mass dimensions change – total mass only. Note, that large size of inertia mass (L_m=500·10^{-6}m) gives much better results of y displacement than in small size case (L_m=300·10^{-6}m). Concluding, simulation results point out that along with mass growth, resonant frequency drops and amplitude in resonant frequency meaningfully increases. For low frequencies, system is more sensitive on small frequency deviations, however for high frequencies, plot in frequency domain become more smooth and system better tolerates frequency fluctuations.

For various spring length (fig. 3) spring constant is calculated based on geometrical dimensions and eq. 4 for complex springs connection. Here, spring constant is next factor which leverages on y displacement results. It is especially visible also at resonant frequency. For spring length L=0,0005m there is the best result of y displacement (2.5·10^{-10}m) for frequency f=17040Hz. The worst results are for L_m=300·10^{-6}m (f=32000Hz, y=2,5·10^{-11}m).

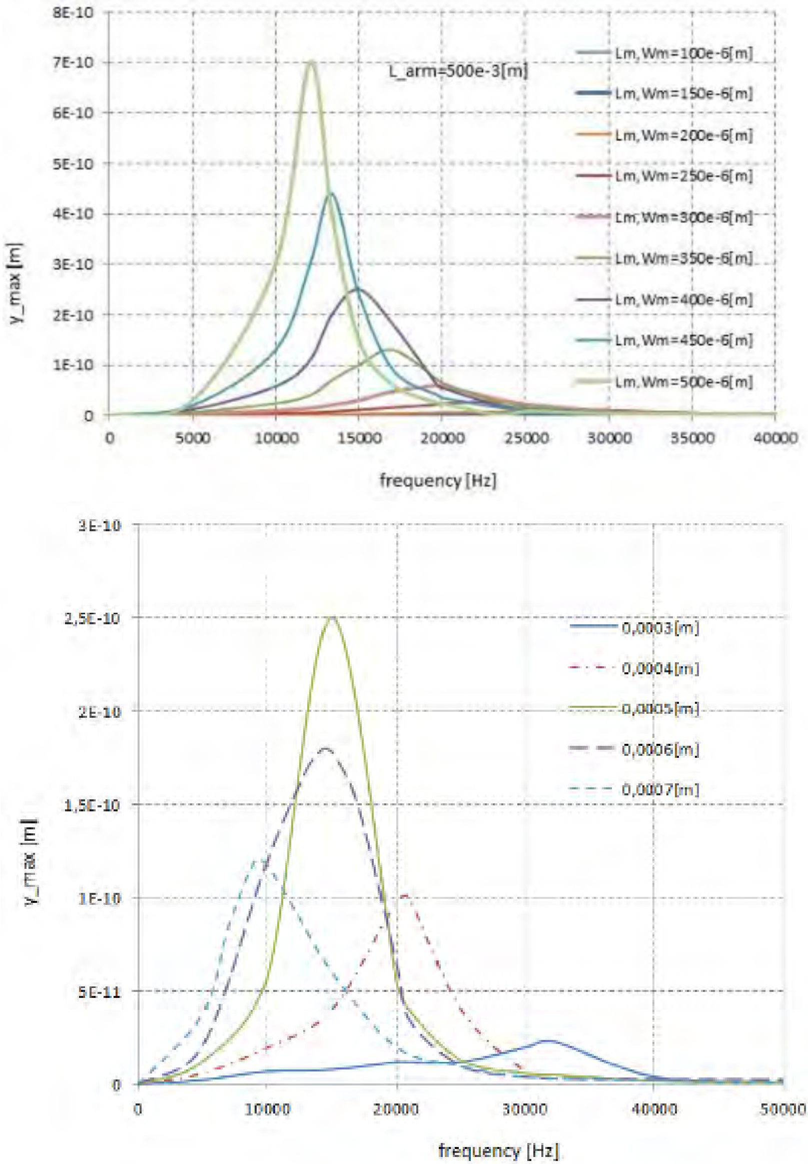

Fig.3 Displacement in *y* (sense) direction in frequency domain in dependency of inertia mass dimensions and spring length.

In fig. 4 there are presented maximum displacement in sense direction in dependency of spring length for specified 5 different frequencies. We observe that maximum displacement increases along with spring length and optimal frequency of actuator also increases; moreover here one can see that for given spring length there is maximum displacement threshold impossible to exceed. In fig. 4 we observe that for both drive and sense direction sense amplitude dependency on spring length are non-linear. Difference between amplitudes for both directions lessens along with spring length. Therefore, to get maximum sense displacement it is necessary to extend spring as much as possible.

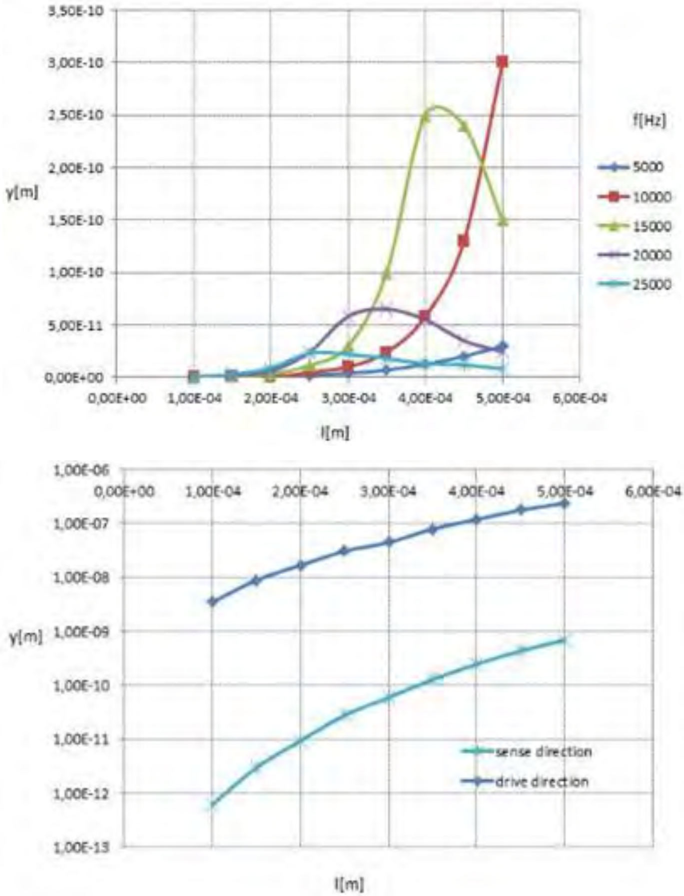

Fig.4 Displacement in y (sense) direction in frequency domain in dependency of inertia mass dimensions and spring length.

Next tests are considered response on dynamical changes angular velocity. Created electrical equivalent model of MEMS gyroscope allowed us simply substitute signal generator instead of constant value of angular velocity. Such generator is very useful for model testing in case of fast changes of angular velocity and to obtain information about response characteristics (*y* sense oscillations).

In fig. 5 there are presented entry angular velocity signals. In the first case angular velocity is 5 rad/s, next increases to 20 rad/s and then changes to -20 rad/s. We observe on fig. 6 that *y* amplitude changes slightly and for change rotation motion to opposite direction, time switching lasts longer (see plot between 5 and 6 ms). Note, that vibrations change their phase 180 degrees. Depend on point in time of angular motion change, some artefacts can appear – here they can be observed at 3ms where maximum of displacement is little bit higher than for previous vibrations. The best results are when angular velocity switching point is when amplitude in y is equal to 0. This fact should be particularly taken into consideration during processing amplitude output signal and transform its to other physical quantity because of falsified result.

In case 2 signal is similar but particular signal levels last shorten. Again, amplitudes of vibrations switches slightly, and there is enough time to switch and achieve targeted amplitude. Although signal is similar to this discussed in case 1 we see that artefacts do not appear. Here, we also observe that changes rotation direction causes changes vibration phase.

In case 3 we tested model of gyroscope with angular velocity increased after subsequent periods of times. In such case there is no switching phase and amplitude changes slightly in whole simulation time range.

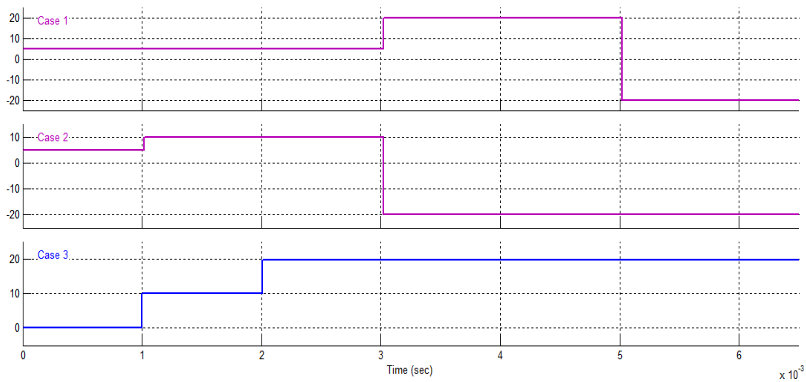

Fig.5 Input signal representing angular velocities for three cases.

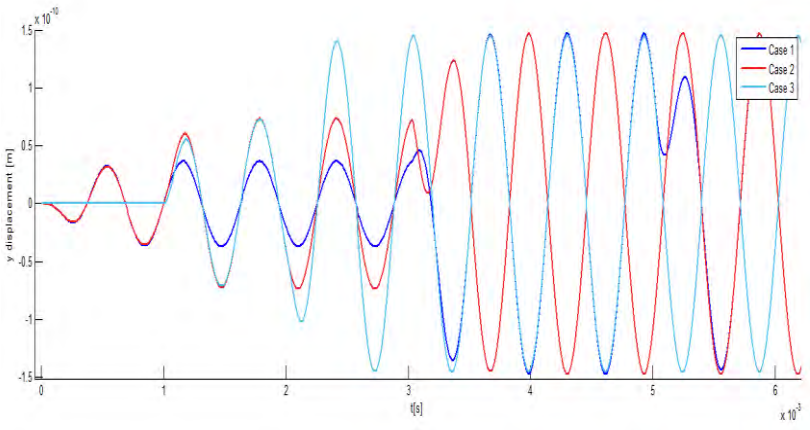

Fig.6 Input signal representing angular velocities for three cases.

CONCLUSIONS

In this paper electrical equivalent model of MEMS Gyroscope was created in Matlab/SIMULINK environment. Model is highly parametrizable regarding geometry details. The advantage is that it can be transferred to other software packages that allows to simulate electrical circuits. Another advantage of such model is that calculations takes less time than in case of Finite Element Analysis software, however we have to remember that presented model is simplified.

The sets of geometrical parameters were applied to model to compare results and get optimal gyroscope dimensions – inertial mass and spring length, which are crucial in case of searching for frequency to apply and based on this – quality factor (which is not considered in this paper) influence on damping behavior of the device.

Results of simulations of electrical model show strong dependency between drive and sense directions. However small amplitude along sense direction causes small capacitance changes (sensing element) what enforce to optimize geometry dimensions and apply appropriate resonant frequency.

REFERENCES

[1] Y. Zhang, X. Liu, W. Chen. "System-Level Modeling and Simulation of Force-Balance MEMS Accelerometers". *Journal of Semiconductors*, 5.Vol(29), 2008. pp. 917-922.

[2] G. L. Teodor, "The Matlab/Simulink modeling and numerical simulation of an analogue capacitive micro-accelerometer. Part 1: Closed loop", *MEMSTECH*, 2008. pp. 115-121.

[3] B. Homeijer, D. Lazaroff, D. Milligan, R. Alley, J. Wu, M. Szepesi, B. Bicknell, Z. Zhang, R. G. Walmsley, and P. G. Hartwell, "Hewlett packard's seismic grade MEMS accelerometer, " *in Proc. MEMS 2011*, pp. 585-588.
[4] B. E. Boser, R. T. Howe, Surface Micromachined Accelerometers, *IEEE J. Solid-State Circuits*, vol. SC-31, March 1996, pp. 366–375.
[5] Steven Nasiri. A Critical Review of MEMS Gyroscopes Technology and Commercialization Status. Inven Sense, 2003. http://blog.ncue.edu.tw/sys/lib/read_attach.php?id=17430.
[6] M. Jankowski, P. Zajac, Andrzej Napieralski, Fully differential read-out circuitry components for MEMS-based accelerometers, *IEEE Conference Proceedings of XIV-th International Conference Perspective technologies and methods in MEMS design*, Polyana, UKRAINE, 18-22 April 2018, pp. 86-90.
[7] C. Maj, A. Napieralski, "Mechanical simulation of 3-axis accelerometer using a single proof-mass", *2017 XIIIth International Conference on Perspective Technologies and Methods in MEMS Design (MEMSTECH)*, 2017, pp. 47–50.
[8] C. Maj, P. Amrozik, W. Zabierowski: "Meshing of symmetric structures in example of comb-drive accelerometer". *2018 XIV-th International Conference on Perspective Technologies and Methods in MEMS Design (MEMSTECH)*, 2018, pp: 151-154.
[9] Ramyar Rashed and H. Momeni, System Modeling of MEMS Gyroscopes, *Procedings of the 15th Midterranean Conference on Control & Automation*, July 27-29, 2007, Athens – Greece, https://ieeexplore.ieee.org/document/4433802/.