

SUSCEPTIBILITY OF LEAST SQUARES METHOD–BASED ALGORITHMS TO ERRORS RESULTING FROM THE NON-PERIODIC COMPONENT OF SHORT CIRCUIT CURRENT

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Неперіодичні складові струму короткого замикання можуть бути причиною похибок під час поточних вимірювань. Значення таких помилок, які отримують за допомогою різних алгоритмів, відрізняються між собою і залежать також від прийнятої моделі струму під час короткого замикання. Проаналізовано різні алгоритми, які використовують метод найменших квадратів, і здійснено їх порівняння стосовно точності оцінки критеріальних величин захистів від струмів короткого замикання. Показано, що найбільші похибки (до декількох десятків процентів) з'являються у разі використання моделі класичного сигналу. Модель електричної мережі на основі диференційного рівняння кола короткого замикання є резистивною для постійної складової струму, тоді як її подальше застосування обмежено оцінкою активного і реактивного опорів кола короткого замикання. Більше того, цей алгоритм є придатним для гармонічного спотворення, яке з'являється у вимірюваних сигналах. Запропоновано алгоритм, який застосовує розклад експоненційних функцій у ряди Тейлора. Цей алгоритм немає недоліків попередніх алгоритмів і є універсальним.

Non-periodic component of short circuit current can be a source of errors in the *on-line* measurements. Values of such errors generated by diverse algorithms is different and depends also on the assumed model of the fault current. In the paper, different algorithms using the least square method (*LSM*) have been analyzed and compared regarding accuracy of estimation of criterion values of the fault current based protections. It has been shown that the highest errors (up to some tens of percents, appear when the classic signal model is applied. Network model based on the differential equation of the fault loop is resistant to the direct-current component whilst the application of the latter is restricted to the estimation of the fault loop's resistance and reactance. Moreover, the algorithm is susceptible to harmonic distortions appearing in the measuring signals. An algorithm which implements the expansion of exponential function into the Taylor series has been proposed. The algorithm is free of drawbacks of precedent algorithms and shows to be the universal one.

Introduction and fundamental relationships. Modern automatic protections are expected to be more and more speedy and selective in operation. It refers mainly to the protections of HV objects important for the system and characterized by high fault currents with high probability of occurrence of direct current components with decay time constant of some tens to some hundreds milliseconds. These components can generate the transient errors of estimation of protection's criterion values [2]. As they are difficult to filter, the error correction algorithms are searched [3,5]. One of proposals presented in the work is an algorithm in which the potential function representing the direct current component in the signal model is expanded in the Taylor series [4].

Due to the type of transient processes accompanying the faults in the electric power system, there are numerous undesired and disturbing components in the measuring signals: harmonic, non-harmonic, steady and transient ones. Let us assume that a signal including K harmonics and direct-current component is put to the measuring unit's input. The signal can be analytically presented as:

$$y(t) = \sum_{m=1}^M Y_m \cos(m\omega_1 t - \varphi_m) + Y_0 \exp\left(-\frac{t}{T_p}\right) + \gamma(t) \quad (1)$$

where M – number of harmonics in consideration; ω_1 - fundamental pulsation; φ_m – phase angle of m^{th} harmonic of signal; T_p – non-periodic current component decay's time constant; $\gamma(t)$ – measuring error.

Second element of right side of (1) represents the component with Y_0 initial amplitude which decays at the decay time constant T_p . Regarding relatively fast decay, the component is not significant in the traditional measuring methods. However, it could result in important errors in the measuring circuits of fast electric power protections working according to the on-line algorithms.

Modern protection systems take decision to switch off the failed or risky object basing on the digital measurement of criterion values. Therefore, it is convenient to operate with discrete magnitudes. Signal (1) in the discrete form becomes:

$$\mathbf{y}(n) = \sum_{m=1}^M \left[Y_m \cos \left(m \frac{2\pi}{N} n - \varphi_m \right) \right] + Y_0 \exp \left(-\frac{T_i}{T_p} n \right). \quad (2)$$

In equation (2), n stands for discrete time whilst N is the number of samples in the fundamental period T_1 , T_i is the sampling period. Equation (2) is the signal model of the process under consideration.

To obtain high speed, selectivity and reliability, the measuring-decision algorithms have to meet rigid requirements concerning separation of components disturbing the measuring signals. Regarding high diversity of algorithms, it is not simple to systemize errors resulting from these disturbances. To compare algorithms to each other, their relative instantaneous values defined as

$$\delta x(n) = \frac{x_m(n) - x_1(n)}{x_1(n)} 100\% \quad (3)$$

can be assumed to be the errors measure, where $x_m(n)$ – instantaneous measured values of criterion magnitude under signal's disturbances, $x_1(n)$ – instantaneous precise values of criterion magnitude.

The error defined that way can not be used to estimate the dynamic properties of the algorithm. The mean error

$$\Delta X(n) = \frac{100}{X_1} \sqrt{\frac{1}{T_w} \sum_{k=1}^p [X(n-k) - X_1]^2} \% \quad (4)$$

is more suitable, where n – number of current sample; X – measured value of determined criterion magnitude; x_1, X_1 – precise value of determined criterion magnitude, T_w – measuring window (interval of analysis), p – number of samples in measuring window.

The conclusion from the formula (4) is that the mean error is a measure of mean deviations of measuring magnitude within the measuring window T_w , from the precise values of these magnitudes under steady conditions. Due to such averaging definition of error, the algorithm can be estimated regarding its susceptibility to the signal's variations with time; thus, it can be a measure of the algorithm's dynamics.

LSM– classic signal model. In the method, a true measuring model $y(t)$ is replaced with a model $z(t)$ which is a function of unknown parameters and defined coefficients; it can be written down in the discrete time domain in the form

$$z(n) = A(n)X. \quad (5)$$

For a model based on m variables, $A(n) = [a_{n1} \ a_{n2} \ \dots \ a_{nm}]$ is a matrix of known (assumed) model coefficients, $X = [X_1 \ X_2 \ X_m]$ – vector of searched parameters of the model.

Measured values of signal $y(n)$ differ from the model by the value of the measurement error $\gamma(n)$:

$$\mathbf{y}(n) = z(n) + \gamma(n). \quad (6)$$

When the expression (5) is placed instead $z(n)$ into (6), we receive

$$\mathbf{y}(n) = A(n)X + \gamma(n). \quad (7)$$

Implementing the least squares method, the unknown parameters of X model can be found by minimizing the square of absolute value of error vector $\gamma(n)$. This solution is described by the known relationship LSM:

$$X(n) = A^s Y(n). \quad (8)$$

and

$$A^g = (A^T A)^{-1} A^T \quad (9)$$

is a quasi-inverse matrix. If the matrix is only a function of model's coefficients (variables) known a priori, it can be computed off line and saved. The problem becomes more complex if the random components are already present in the signal, and their values are either unknown or approximate. The time constant T_p of the direct current component's decay in the fault current is a good example.

The non-periodic component of short circuit current is heavy to filter off due to continuous frequency spectrum, and the energy distribution within the spectrum depends on the decay time constant, i.e. is random as well as the time constant's value. Therefore, the practical value of algorithms of the measuring signal non-periodic current component elimination referred in the literature and assuming an a priori knowledge on the time constant of the component decay T_p is limited to the special cases. According to the statements of the algorithms' authors [1], any assumption of inappropriate value of the time constant may result in greater errors than those resulting from its neglecting in the measuring algorithm. It can be simply shown using the signal model (2).

Left side of model (2) represents a vector of successive samples of measured signal which, for the measuring window $T_w = N \cdot T_i$ (T_i – sampling period) equal to the fundamental component period T_i , becomes:

$$Y(n) = [y(n-N+1) \ y(n-N+2) \dots \ y(n)]^T \quad (10)$$

In turn, $A(n)$ in equation (7) is a matrix of model coefficients the n -th line of which (after having assumed a 3-variable signal model, i.e. model consisting of three components) can be written down as:

$$A(n) = \left[\sin\left(\frac{2\pi}{N} n\right) \cos\left(\frac{2\pi}{N} n\right) \exp\left(\frac{-T_i}{T_m} n\right) \right], \quad (11)$$

where T_m is the non-periodic current component decay time constant assumed in the model, and, in the general case, its value differs from the true time constant, i. e. $T_m \neq T_p$. $\gamma(n)$ is the vector of measuring errors.

The wanted vector of estimated signal parameters $X(n)$ can be described according to the known LSM relationship (8).

Third column of matrix (11) can be source estimation error if the non-periodic current component of unknown decay time constant T_p occurs in the measuring signal. In the work [1], there is the opinion that maximum error due to this component can reach 17 %. In fact, maximum errors due to the presence of direct current component in the measuring signal can be much higher as it is shown in Fig. 1.

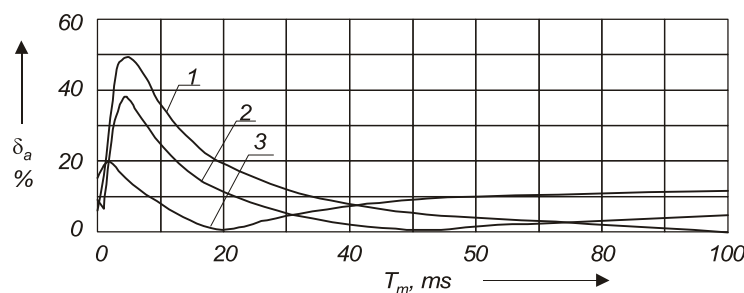


Fig. 1. Maximum errors $\delta_a\%$ of amplitude estimation using LSM for different values of direct current component decay time constant of the model (T_m) and for three different direct current component decay time constant in measuring T_p ;

1 – $T_p = 100$ ms, 2 – $T_p = 50$ ms, 3 – $T_p = 20$ ms

Maximum errors of amplitude estimation can reach some tens percent and arise when the time constant T_p arises. Presented characteristics of the error confirm once again that the estimation error will not occur if $T_m = T_p$. As the time constant value T_p depends on many random factors (fault location, fault type, transition resistance at the fault location, etc.), it is not but the theoretical case. However, the detailed conclusions with practical significance can be drawn from error characteristics; the optimum choice of the model's time constant value T_m

can be chosen. First, the maxima of the error curves always appear for $T_m < T_p$. Second, for time constant T_m assumed to be equal to 0 or ∞ (very large), the occurrence of the amplitude estimation error (caused by the direct current component in the measuring signal) identical in value and relatively low becomes highly probable. Referring to two conclusions as above, a third general conclusion can be drawn: the time constant of the model T_m should be chosen according to the requirement:

$$T_m \geq T_p \quad (12)$$

In practice, the direct current component decay time constant for signal model T_m should be assumed as the highest value of the measuring signal time constant T_p , that could appear under the most inconvenient fault conditions in the protective zone of the protection.

Circuit model. Let us assume the fault loop measuring voltage can be represented by a series of samples $u(n)$ described by the differential equation as below:

$$U(n) = Ri(n) + L\Delta i(n) + \Gamma(n) \quad (13)$$

where $\Delta i(n) = (i(n+1) - i(n-1))/2T_s$, and $\Gamma(n)$ is measuring error, whilst R and L are the model parameters we are searching for.

As it was a case with the signal parameters' estimation, equation (13) can be written down in the matrix form:

$$U(n) = A(n)X(n) + \Gamma(n) \quad (14)$$

In equation (14), the vector of samples of the measured signal is in the form like that in the equation (10):

$$U(n) = [\mathbf{u}(n - K+1) \ \mathbf{u}(n - K+2) \ \dots \ \mathbf{u}(n)]^T \quad (15)$$

Other expressions in matrix of the model variables $A(n)$ and of the vector of estimated parameters of model $X(n)$:

$$A(n) = [\mathbf{a}_1(n) \ \mathbf{a}_2(n)], \quad (16)$$

where $\mathbf{a}_1(n) = i(n)$, $\mathbf{a}_2(n) = \Delta i(n) = (i(n+1) - i(n-1))/2T_s$, and the vector of unknown parameters of the model is:

$$X(n) = [R \ L]^T \quad (17)$$

It can be computed from the relationship LSM presented above (8):

$$X(n) = A^g(n)U(n) \quad (18)$$

where quasi-inverse matrix $A^g(n)$ is given by equation (9).

In the estimation algorithm (18) based on the model of the fault circuit parameters (13), the non-periodic current component of fault current is effectively eliminated as shown in Fig.2. Waveforms in Fig. 2 are plotted for measuring window T_w equal to the fundamental period T_1 and sampling frequency $f_s = 1$ kHz.

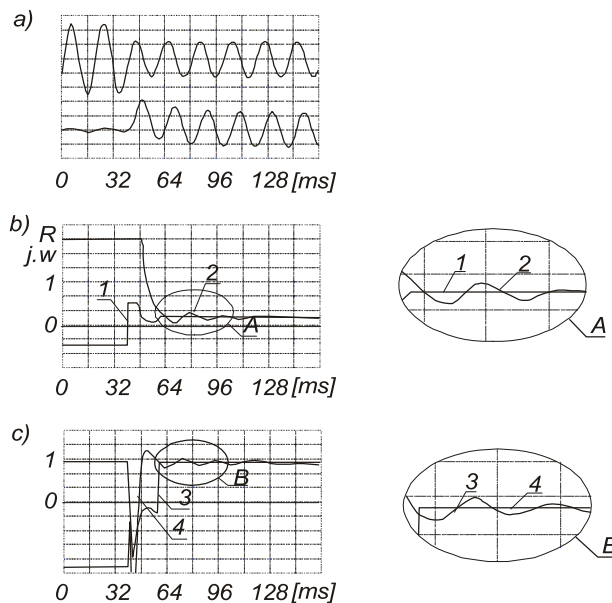


Fig. 2. Source signals (a), relative resistance values (b) and inductance (c) obtained using the LSM – based algorithm: 1, 3 – with circuit model, 2, 4 – with signal model

To compare properties of algorithms based on the *LSM* with signal model (2) and circuit model (13), the results of estimation of resistance and reactance obtained with both models are shown in Fig.2. The magnitudes have been dimensioned in relative units, taking a precise value of the fault loop impedance to be the basic unity. A good effectiveness of the network model in elimination of the impact of the direct current component of fault current is evidently shown.

However, a circuit model shows some restrictions. First of all, it works only for *R* and *L* estimations; moreover, the differential equations are formulated assuming linearity of the fault circuit parameters, i.e. lack of the higher harmonics in the source signals.

Model based on Taylor series. As it was stated above, the main and difficult to correct source of error of the algorithms of criterion magnitudes' estimation using fault current is the non-periodic current component with unknown time constant of decay T_p . It is widely being assumed that a good model of this current is a known expression:

$$i(t) = I \cos(\omega_1 t - \varphi) + I_0 \exp\left(-\frac{t}{T_p}\right). \quad (19)$$

The most right-hand component of the right side of the expression can be expanded in the Taylor series:

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}. \quad (20)$$

Number of sum's component shall be limited. In practice, regarding true conditions, if only three initial elements of series can be taken into account, the error will no exceed. Thus, we can write:

$$i(t) = I(\cos \omega t \cos \varphi + \sin \omega t \sin \varphi) + I_0 \left\{ 1 - At + 0,5A^2 t^2 - \left(\frac{1}{6} A^3 t^3\right) \right\} \quad (21)$$

where $A = 1/T_p$.

Equation (21) can be re-written in the form:

$$i(t) = I \cos \omega t \cos \varphi + I \sin \omega t \sin \varphi + I_0 - I_0 A t + 0,5 I_0 A^2 t^2 - I_0 \frac{1}{6} A^3 t^3. \quad (22)$$

As we can see, the equation as above is a linear algebraic equation of six unknowns: $I \cos \varphi, I \sin \varphi, I_0, I_0 A, I_0 A^2, I_0 A^3$. To simplify the formal notation, let's define the unknown quantities as follows:

$$I \cos \varphi = X_1, I \sin \varphi = X_2, I_0 = X_3, I_0 A = X_4, I_0 A^2 = X_5, I_0 A^3 = X_6. \quad (23)$$

Thus, equation (22) takes a discrete form:

$$i(n) = X_1 \cos n\Omega_1 + X_2 \sin n\Omega_1 + X_3 - X_4 n + 0,5 X_5 n^2 - \frac{1}{6} X_6 n^3. \quad (24)$$

To find these unknown values, at least six independent equations are needed. The best way is to define them for different instants. Thus, a set of the (K+1) equations takes a form:

$$\begin{aligned} i(n) &= X_1 \cos n\Omega_1 + X_2 \sin n\Omega_1 + X_3 - X_4 n + 0,5 X_5 n^2 - \frac{1}{6} X_6 n^3 + \gamma(n) \\ i(n-1) &= X_1 \cos(n-1)\Omega_1 + X_2 \sin(n-1)\Omega_1 + X_3 - X_4(n-1) + 0,5 X_5(n-1)^2 - \frac{1}{6} X_6(n-1)^3 + \gamma(n-1) \\ i(n-2) &= X_1 \cos(n-2)\Omega_1 + X_2 \sin(n-2)\Omega_1 + X_3 - X_4(n-2) + 0,5 X_5(n-2)^2 - \frac{1}{6} X_6(n-2)^3 + \gamma(n-2) \\ &\dots\dots\dots \\ i(n-K) &= X_1 \cos(n-K)\Omega_1 + X_2 \sin(n-K)\Omega_1 + X_3 - X_4(n-K) + 0,5 X_5(n-K)^2 - \frac{1}{6} X_6(n-K)^3 + \gamma(n-K) \end{aligned} \quad (25)$$

In the equation as above, $\Omega_1 = \omega_1 T_i$ is a relative pulsation in reference to sampling frequency $f_i = 1/T_i$ and γ is the measuring error.

Equation (25) can be written in a compact matrix form:

$$I(n) = A(n)X(n) + \gamma(n) \quad (26)$$

and vector of samples of the measured current signal is

$$I(n) = [i(n-K) \ i(n-K-1) \ \dots \ i(n)]. \quad (27)$$

Matrix of the model's known coefficients is

$$A(n) = [a_1(n) \ a_2(n) \ a_3(n) \ a_4(n) \ a_5(n) \ a_6(n)] \quad (28)$$

and its elements results directly from the equation set (25):

$$a_1 = \cos n\Omega_1, \ a_2 = \sin n\Omega_1, \ a_3 = 1, \ a_4 = -n, \ a_5 = 0,5n^2, \ a_6 = -\frac{1}{6}n^3. \quad (29)$$

Thus, all elements of matrix $A(n)$ are determined and can be calculated off line and saved.

Vector of model's unknown parameters is:

$$X(n) = [X_1 \ X_2 \ X_3 \ X_4 \ X_5 \ X_6]^T. \quad (30)$$

When the equation (30) is solved, we can find not only values of the fault current's orthogonal components (X_1, X_2) and its amplitude ($\sqrt{X_1^2 + X_2^2}$), but also the magnitudes inaccessible by other models, such as direct current component of fault current (X_3) and decay's time constant ($(T_p = \frac{X_3}{X_4} = \frac{X_4}{X_5} = \frac{X_5}{X_6})$). Simultaneously, with the signal model using the expansion of potential function into Taylor series, the inconveniences resulting from the presence of direct current component of unknown decay time constant and initial amplitude inside the measuring current signal can be avoided.

In Fig. 3, results of estimation of amplitude of both the direct component and alternate current component of fault current are presented along with the decay time constant of direct current component.

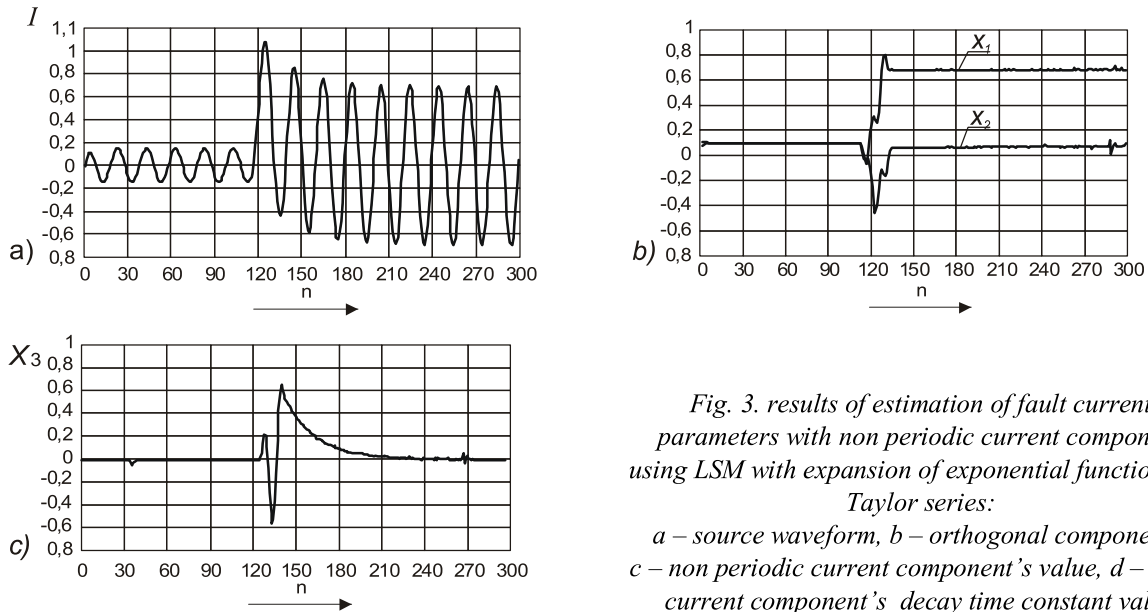


Fig. 3. results of estimation of fault current parameters with non periodic current component using LSM with expansion of exponential function into Taylor series:
a – source waveform, b – orthogonal components, c – non periodic current component's value, d – direct current component's decay time constant value

Fig. 3 shows that application of the signal model with expansion of its exponential component into Taylor series to estimate the fault current parameters using *LSM* makes the algorithm insensible completely to the direct current component. Moreover, due to such approach, estimation of magnitudes inaccessible with other methods, including random magnitudes like value and time constant of the fault current's non-periodic current component decay, becomes feasible.

Summary and final remarks. In the work, the least squares method (*LSM*) has been applied to estimate some criterion values based on fault current. Algorithms referring to three different models have been presented, i. e.:

- classic signal model in which the fault current is represented by three components: two orthogonal components and one exponential component representing non-periodic short circuit current component,
- circuit model in which the fault current loop is assumed to be represented by linear resistances and reactances connected in series; criterion magnitudes are computed from differential equations defined at different instants within measuring window,

- “non-classic” signal model which differs from the „classic” model by using expansion of exponential expression representing direct current component of fault current into Taylor series.

The computational complexity of all models is similar. Partial conclusions have been presented when each of individual algorithms has been discussed. To summarize, it can be stated that the third of the considered algorithms i.e. that with expansion of exponential function into Taylor series, has shown its supremacy over two others: it is highly universal and significantly insensitive to the direct current component of fault current. Regarding the paper’s volume, the analysis of errors resulting from the limitation of Taylor series elements number to three has not been reported. However, the reported figures give the graphical image of estimation and show that such limitation does not lead to significant errors.

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МОДЕЛЮВАННЯ НЕСИМЕТРИЧНИХ КОРОТКИХ ЗАМИКАНЬ НА НЕЙТРАЛЬ АСИНХРОНІЗОВАНОГО ГЕНЕРАТОРА

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Отримано диференційні рівняння асинхронізованого турбогенератора в режимі одно- та двофазного замикання на нейтраль для визначення характеристик струмів несиметричних режимів роботи.

Here we obtain differential equations of doubly fed wound induction generator in one- and two-phase fault on the neutral to determine the asymmetric mode.

Постановка проблеми. Застосування в енергосистемах асинхронізованих турбогенераторів (АСТГ) дозволяє ефективно вирішити проблему споживання надлишків реактивної потужності, що виникають в високовольтних мережах у години “провалів” графіка навантажень. Крім того, АСТГ позбавлені обмежень за стійкістю роботи в режимах споживання реактивної потужності, які властиві традиційним синхронним генераторам [1].

Проведені раніше дослідження [2] показують деяке зниження струмів трифазного короткого замикання в мережах з асинхронізованими генераторами, що додатково збільшує ефективність їх використання.

Аналіз останніх досліджень. Останнім часом ведуться роботи [2] з виявлення характеристик режимів АСТГ під час симетричного трифазного короткого замикання, однак особливості режимів