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Peculiarities of dynamics of the reservoir with a free–surface liquid on pendulum suspension with the moving suspension point

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A problem of dynamics of a reservoir of cylindrical shape, partially filled with liquid, on pendulum suspension with movable suspension point is under investigation. The problem is considered in nonlinear statement with the purpose of clarification of the effect of pendulum suspension on both frequency characteristics and the system behavior in the near-resonance zone. An analytical and numerical study shows that normal frequencies of oscillations have considerable changes for both quasi-rigid pendulum mode of motion and especially for the frequency of liquid sloshing modes. Numerical examples show that resonant properties of the system for below resonance, above resonance and near resonance modes are considerably different and the effect of amplitude modulation manifests strongly for all cases.

Keywords: *liquid oscillations, reservoir on pendulum suspension, near resonance modes of motion, amplitude modulation.*

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1. Introduction

The problem about dynamical behavior of the system "reservoir – liquid with a free surface" has both academic and engineering significance. This problem is much more complicated than the problem about a translational motion of such system because angular parameters of the carrying body motions appeared and, therefore, the Stokes-Zhukovsky vector potential of velocity should be supplementarily introduced into the description of the system kinematics. The study is done based on the previously developed method [1], which was developed and verified on a system of problems about free and constrained (pendulum suspension with the immovable suspension point). One of significant modification of the problem statement is connected with the behavior of the reservoir with liquid on pendulum suspension in the case when the foundation of this suspension performs the motion according to the prescribed law. Namely, this problem statement corresponds to earthquake, when inverse influence of the reservoir on the motion of the foundation is supposed to be absent due to considerable exceed of the foundation mass relative to the mass of the structure with liquid. In this case, it is necessary to consider that the system performs the prescribed translational motion together with the foundation, but the combined motion of liquid and angular motion of the reservoir. Such kind of problems was not considered before.

2. Object of study

Consider a rigid reservoir of the circular cylindrical shape. The reservoir of the radius R is partially filled with an ideal incompressible liquid with the depth H. We consider that at initial time instant the system is in a rest state. Therefore, according to the Lagrange theorem about vortex motion of an ideal liquid, its further motion will be vortex-free. Then, it is possible to describe the liquid motion by the velocity potential [1]

$$\boldsymbol{v} = \boldsymbol{\nabla}\left(\varphi + \dot{\boldsymbol{\varepsilon}} \cdot \boldsymbol{r} + \boldsymbol{\omega} \cdot \boldsymbol{\Omega}\right),$$

where the first addend φ corresponds to wavy notion of a liquid, the second addend corresponds to translational motion of liquid (here $\boldsymbol{\varepsilon}$ is the vector linear displacements of the reservoir, $\boldsymbol{r}$ is the radius vector of an arbitrary point of liquid), the third one corresponds to rotational motion of liquid (here ω is the reservoir angular velocity, $\boldsymbol{\Omega}$ is the Stokes-Shulovskiy vector potential).

According to the method, described in [1], we construct decomposition of a liquid free surface ξ with respect to normal modes of oscillations. The corresponding decompositions are constructed also for the potentials φ and $\boldsymbol{\Omega}$. Further solving the problem of construction of the mathematical model is based of the following premises.

1. Holding all kinematical boundary conditions.

a) Since we consider the decompositions of potentials with respect to normal modes of oscillations, these decompositions will be harmonic functions, which satisfy non-flowing conditions of the reservoir walls and the linear part of boundary conditions on a free surface of liquid.

b) By means of the Galerkin method we satisfy nonlinear kinematic conditions of a free surface of liquid analytically for arbitrary number of the considered normal modes of oscillations N. In this way actually we eliminate the coefficients of decompositions for all potentials of velocities into series. Thus, we pass to a free system, for which the number of variables coincides with the number of degrees of freedom.

2. Construction of the Lagrange function includes the kinetic and potential energies for the liquid and the reservoir in their translational and rotational motion (6 degrees of freedom) and wavy motion of a free surface of liquid (N degrees of freedom). According to the standard procedure we can pass to the system of ordinary differential equations with respect to six parameters of motion of the reservoir and N amplitude parameters of a free surface motion a_i.

Since we consider a particular case of the system motion, when the translational motion is in advance given, the system will have $N+3$ degrees of freedom and the parameters of translational motion of the system ε are supposed to be given.

In this case the resolving system of motion equation has the form

$$\begin{aligned}
&\sum_i \ddot{a}_i\left(\delta_{ir} + \sum_j a_j A^3_{rij} + \sum_{j,k} a_j a_k A^4_{rijk}\right) + \frac{1}{2\alpha^v_r}\sum_{s=1}^{3}\ddot{\alpha}_s\left[\sum_{p=1}^{3}\frac{\partial\omega_p}{\partial\dot{\alpha}_s}\left(E^{1*}_{pr} + \sum_i a_i E^{2*}_{pri} + \sum_{i,j} a_i a_j E^{3*}_{prij}\right)\right] \\
&\quad = -k_r\dot{a}_r - \frac{1}{\alpha^v_r}\ddot{\boldsymbol{\varepsilon}}\cdot\left(\boldsymbol{B}^1_r + \sum_i a_i\boldsymbol{B}^2_{ri} + \sum_{i,j} a_i a_j\boldsymbol{B}^3_{rij} + \sum_{i,j,k} a_i a_j a_k\boldsymbol{B}^4_{rijk}\right) \\
&\quad + \sum_{i,j}\dot{a}_i\dot{a}_j C^3_{ijr} + \sum_{i,j,k}\dot{a}_i\dot{a}_j a_k C^4_{ijkr} + \frac{1}{2\alpha^v_r}\sum_{p,s=1}^{3}\omega_p\omega_s\left(E^2_{psr} + 2\sum_i a_i E^3_{psir}\right) \\
&\quad + \frac{1}{2\alpha^v_r}\sum_{p=1}^{3}\omega_p\left[E_i\dot{a}_i\left(E^{2*}_{pir} - E^{2*}_{pri}\right) + 2\sum_{i,j}\dot{a}_i a_j\left(E^{3*}_{pijr} - E^{3*}_{prij}\right)\right] \\
&\quad + \frac{1}{2\alpha^v_r}\sum_{p=1}^{3}\omega^{(k)}_p\left(E^{1*}_{pr} + \sum_i a_i E^{2*}_{pri}\right) + \dot{\boldsymbol{\varepsilon}}\cdot\left(\sum_i \dot{a}_i\boldsymbol{D}^2_{ir} + \sum_{i,j}\dot{a}_i a_j\boldsymbol{D}^3_{ijr} + \sum_{i,j,k}\dot{a}_i a_j a_k\boldsymbol{D}^4_{ijkr}\right) \\
&\quad + \frac{1}{2\alpha^v_r}\dot{\boldsymbol{\varepsilon}}\sum_{p=1}^{3}\omega_p\left(\boldsymbol{F}^2_{pr} + 2\sum_i a_i\boldsymbol{F}^3_{pir} + 3\sum_{i,j} a_i a_j\boldsymbol{F}^4_{pijr}\right) \\
&\quad - \frac{\alpha^c_r}{\alpha^v_r}g(\sin\alpha_1\sin\alpha_3 - \cos\alpha_1\sin\alpha_2\cos\alpha_3) - \frac{\alpha^s_r}{\alpha^v_r}g(\sin\alpha_1\cos\alpha_3 + \cos\alpha_1\sin\alpha_2\sin\alpha_3)
\end{aligned}$$

$$-\frac{N_r}{\alpha_r^v} g\cos\alpha_1\cos\alpha_2-\frac{\sigma}{\rho\alpha_r^v}\left[\lambda\cos\theta_1+\varkappa_r^2 N_r a_r-\frac{1}{4}\sum_{i,j,k}\left(\delta^4_{ijkr}+\delta^4_{rijk}\right)a_i a_j a_k\right]; \quad (1)$$

$$\sum_i \ddot{a}_i\left[\sum_{p=1}^{3}\frac{\partial\omega_p}{\partial\dot{\alpha}_r}\left(\boldsymbol{E}^{1*}_{pi}+\sum_j a_j\boldsymbol{E}^{2*}_{pij}+\sum_{i,k}a_j a_k\boldsymbol{E}^{3*}_{pijk}\right)\right]$$
$$+\sum_{n=1}^{3}\ddot{\alpha}_n\left[2\sum_{p,s=1}^{3}\frac{\partial\omega_p}{\partial\dot{\alpha}_r}\frac{\partial\omega_s}{\partial\dot{\alpha}_n}\left(\frac{1}{\rho}I^{ps}_{\text{res}}+E^2_{ps}+\sum_i a_i E^2_{psi}+\sum_{i,j}a_i a_j E^3_{psij}\right)\right]$$
$$=-2\ddot{\varepsilon}\sum_{p=1}^{3}\frac{\partial\omega_p}{\partial\dot{\alpha}_r}\left(F^1_p+\sum_i a_i\boldsymbol{F}^2_{pi}+\sum_{i,j}a_i a_j\boldsymbol{F}^3_{pij}+\sum_{i,j,k}a_i a_j a_k\boldsymbol{F}^4_{pijk}\right)$$
$$+2\sum_{p,s=1}^{3}\left(\omega^*_{p,r}\omega_s+\omega^{(k)}_p\frac{\partial\omega_s}{\partial\dot{\alpha}_r}\right)\left(\frac{1}{\rho}I^{ps}_{\text{res}}+E^2_{ps}+\sum_i a_i E^2_{psi}+\sum_{i,j}a_i a_j E^3_{psij}\right)$$
$$+\sum_{p=1}^{3}\omega^*_{p,r}\left(\sum_i\dot{a}_i E^{1*}_{pi}+\sum_{ij}\dot{a}_i a_j E^{2*}_{pij}+\sum_{i,j,k}\dot{a}_i a_j a_k E^{3*}_{pijk}\right)+2\dot{\boldsymbol{\varepsilon}}\cdot\sum_{p=1}^{3}\omega^*_{p,r}\left(\boldsymbol{F}^1_p+\sum_i a_i\boldsymbol{F}^2_{pi}+\sum_{i,j}a_i a_j\boldsymbol{F}^3_{pij}\right)$$
$$-2\sum_{p,s=1}^{3}\frac{\partial\omega_p}{\partial\alpha_r}\left(\sum_i\dot{a}_i E^2_{psi}+2\sum_{i,j}\dot{a}_i a_j E^3_{psij}\right)-\sum_{p=1}^{3}\frac{\partial\omega_p}{\partial\dot{\alpha}_r}\left(\sum_{i,j}\dot{a}_i\dot{a}_j E^{2*}_{pij}+2\sum_{i,j,k}\dot{a}_i\dot{a}_j a_k E^{3*}_{pijk}\right)$$
$$-2\dot{\boldsymbol{\varepsilon}}\cdot\sum_{p=1}^{3}\frac{\partial\omega_p}{\partial\dot{\alpha}_r}\left(\sum_i\dot{a}_i\boldsymbol{F}^2_{pi}+2\sum_{i,j}\dot{a}_i a_j\boldsymbol{F}^3_{pij}\right)+2g\frac{\partial}{\partial\alpha_r}\left[\left(\cos\alpha_1\sin\alpha_2\cos\alpha_3-\sin\alpha_1\sin\alpha_3\right)\left(\sum_i a_i\alpha^c_i+Hl^c\right)\right.$$
$$\left.-\left(\cos\alpha_1\sin\alpha_2\sin\alpha_3+\sin\alpha_1\cos\alpha_3\right)\left(\sum_i a_i\alpha^s_i+Hl^s\right)\right]$$
$$+\frac{2g}{\rho}\left(M_r h_r+M_l h_l\right)\frac{\partial}{\partial\alpha_r}(\cos\alpha_1\cos\alpha_2)+\frac{2M_r}{\rho}. \quad (2)$$

The system of motion equations in the sloshing parameters a_i and the parameters of angular motion α_i is supplemented by dissipative forces according to [2]. Here in the equations we suppose the values ε_i to be variables of the finite order, and α_i, a_i are supposed to be small and they are taken into account up to the values of the third order of smallness. It is denoted in the system of motion equations (1)–(2) that δ_{ij} is the Kroneker symbol,

$$\omega^{(k)}_p=-\sum_{n=1}^{3}\dot{\alpha}_n\frac{\partial\omega_p}{\partial\alpha_n};\quad \omega^*_{p,k}=\frac{\partial\omega_p}{\partial\alpha_k}-\frac{d}{dt}\left(\frac{\partial\omega_p}{\partial\dot{\alpha}_k}\right),$$

ρ is the liquid density, g is the free falling acceleration, M_r and M_l are the masses of the reservoir and liquid, correspondingly, I^{ps}_{res} is the inertia moment of thee reservoir. All other coefficients are determined in the form quadratures from normal modes of oscillations of liquid over the undisturbed free surface of liquid [1].

Considerable simplification of the motion equations takes place when we consider the motion of the reservoir relative to the reference frame with the origin at the suspension point. In this case it is necessary to perform recalculation of the coefficients of the motion equations (1)–(2) according to the displacement l of the suspension point with respect to the center of a free surface of liquid according to the method of [1].

3. Normal frequencies for the system in combined motion

The problem of a study of the dynamics of reservoirs with liquid in the combined motion of its components is still investigated insufficiently. The main part of authors continue to consider that it is possible to investigate dynamics of this system as behavior of a system with given motion of the carrying body. This automatically fixes partial frequencies for the system reservoir – liquid as normal frequencies. However, the difference between partial frequencies and normal frequencies of the system in its combined motion is considerable, and it significantly depends on the way of restriction of mobility of the carrying body. The general scheme of the object of study is shown in Fig. 1.

Let us consider the numerical examples for the filling depth $H = R$, three suspensions lengths $l = R$, $l = 2R$ and $l = 8R$. Here we consider that $M_r = 0.1M_l$, tensor of inertia of the reservoir was determined according to [1].

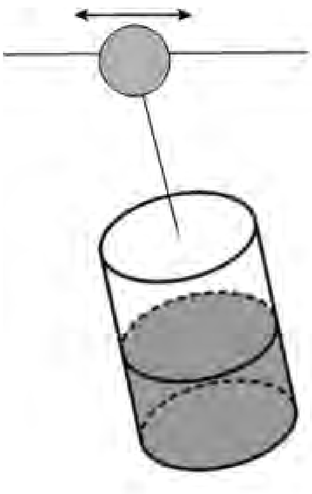

Fig. 1. General scheme of the object.

Let us study changes of frequencies, which take place in the case of considering the combined character of the system motion. Further numerical results are given for $R = 1\,\mathrm{m}$. For the suspension length $l = R$ the normal frequency of liquid motion with respect to the first normal mode is 6.6998 in the case of fixed suspension point and 16.6308 in the case of movable suspension point. At the same time, the partial frequency for the first normal mode is 4.1443. This amounts to 62% and 401% correspondingly. For the combined motion of pendulum with liquid in the case of fixed suspension point the normal frequency is 2.2431 and in the case of movable suspension point it is 2.2673. Comparison with partial frequency 2.4784 gives the following differences 9.5% and 8.5%.

In the case of the suspension length $l = 2R$, the normal frequencies for the liquid in the case of fixed and movable suspension point are 6.1082 and 12.4514 for the same partial frequency 4.1443. For $l = 8R$, the normal frequencies for liquid are correspondingly 5.5273 and 9.7510. The relative error decreases but still remain to be considerable. The same behavior is observed for frequencies of normal frequencies pendulum.

Variation of the system normal frequencies depending on the way of restriction of motion of the reservoir shows considerable differences between behavior of the system in its combined motion and in the case of the given motion of the reservoir. Therefore, our further study is completely based on the model of the combined motion of the system.

4. Near-resonant behavior of the reservoir with liquid on movable pendulum suspension

The study of dynamics of liquid in the near-resonant range of frequencies in the case of translational motion of the carrying body and in the case of the system motion on the pendulum suspension with the fixed suspension point showed general regularities of the system behavior. First of all, this is connected with evidence of the absence of the system tending to oscillations with steady amplitude. Namely, this mode of motion is predicted by a group of researches [3, 4]. This result is confirmed by several experiments [5–7]. This difference in theoretical results takes place because of the effect of modulation, which appeared due to including into the model of oscillations on normal frequencies of all normal modes, which is not done in [3, 4]. Therefore, we especially focus our attention on the manifestation of the effect of modulation.

Initially, the system is at rest and its motion is caused by the given law of motion of the suspension point $\varepsilon_x = A \sin pt$, where A is the amplitude of oscillations and p is the frequency of external excitation. The rest of the parameters were selected as in the example with determination of normal frequencies. A preliminary study of the system normal frequencies and the analysis of the application of the kinematic excitation shows that real excitation, which is applied not to the mass center of the system, finally results in simultaneous applying of translational and rotational excitations to the system with movable position of the mass center (due to liquid sloshing). Therefore, we initially study manifestation of the resonant properties of the system numerically on the frequency range $1-18$. The results of numerical study shows that the resonant frequency is $\omega_0 = 2.46$, which is slightly different from the predicted frequency for the system with fixed suspension point $\omega = 2.48$. Namely, this value of frequency should be taken into account, because for the given translational motion of the system the degree of freedom corresponding to the translational motion of the system is eliminated, therefore, the corresponding equation must be omitted too. This difference can be caused by the dependence of frequency on the amplitude, which is neglected within the framework of the linear theory of oscillations. Numerical analysis shows that the difference of the system behavior for these two frequencies is evident.

So, we split the frequency range into three zones, namely, the below-resonance zone (conventionally we accept $p = 0.75\omega_0$); near-resonance zone (conventionally we accept $p = \omega_0$) and the above-resonance zone (conventionally we accept $p = 1.25\omega_0$). to hit into the nonlinear range of oscillations we accept $A = 0.115$. The results of determination of the system behavior for variation in time of the liquid elevation on the tank wall and the inclination angle of the reservoir for the near-resonance zone are shown in Fig. 2 and Fig. 3, correspondingly.

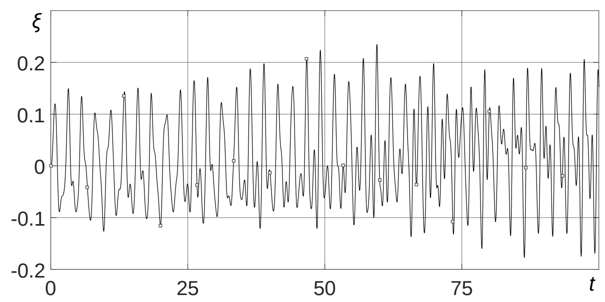

Fig. 2. Variation in time of the amplitude of wave on a free surface for the near-resonance mode.

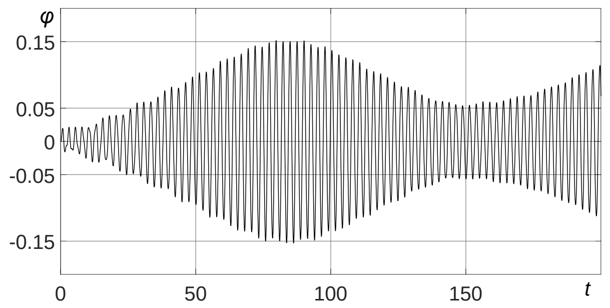

Fig. 3. Variation in time of the inclination angle of pendulum support for the near-resonance mode.

These graphs show the considerable manifestation of the modulation effect both for elevation of liquid on the reservoir wall and for inclination angle. The frequency of modulation for the inclination angle is very small, therefore we elongate the time interval to show the complete period of modulation. The modulation of liquid oscillations presents also, and it is evident that both drift of the mean value of oscillations and the effect of high-frequency modes of oscillations is manifested for this mode of motion.

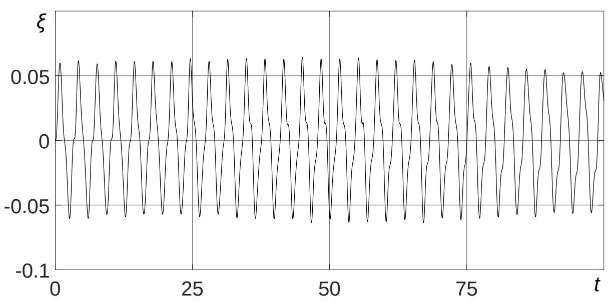

Fig. 4. Variation in time of the inclination angle of pendulum support for the below-resonance mode.

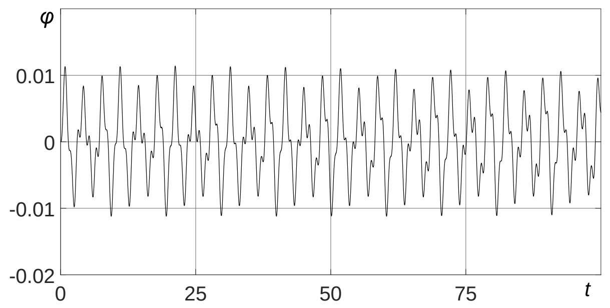

Fig. 5. Variation in time of the inclination angle of pendulum support for the below-resonance mode.

The system behavior for the below-resonance zone is shown in Fig. 4 and Fig. 5 for the same mechanical parameter of the system and the amplitude of excitation. In this case, the amplitude of liquid sloshing decreases considerably $(3-4$ times), but the amplitude of the inclination angle decreases in-significantly if we take into account its mean value on the period of modulation. The modulation effect is very weak. However, even in spite of development of oscillations in the linear range of amplitudes the weak variation of amplitude and manifestation of the high-frequency modes present.

In the case of the system excitation in the above-resonance range (Fig. 6 and Fig. 7) manifestation of the modulation increases and modulation frequency increases too. For the same amplitude of excitation, we obtain the amplitudes of excitations of a free surface of liquid even greater than in the near-resonance case, but the amplitudes of the inclination angle of the pendulum suspension reduce approximately twice. In the case of the suspension length $l = 2R$ the general picture of the system behavior does not change significantly, because according to the conclusion of [8] relative arrangement of frequencies is the same and it begins to change only after $l = 6.12R$.

If we compare the frequency of modulation according to the recommendations of the article [9], it is possible to state that the conclusion of this article is not true. Namely, the frequency of modulation for the modes of excitation with the resonance frequency multiplied by 0.75 and 1.25 normal differ multiply. The reason of this most probably is connected for nonlinear range of excitations with the property of dependence of the frequency on the amplitude, which is not taken into account in [9]. Therefore, the "mirror" systems gave considerably different characteristics of the modulation.

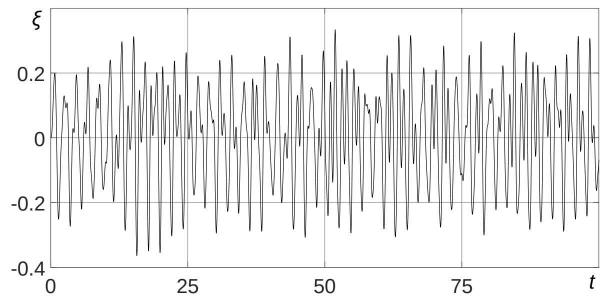

Fig. 6. Variation in time of the inclination angle of pendulum support for the above-resonance mode.

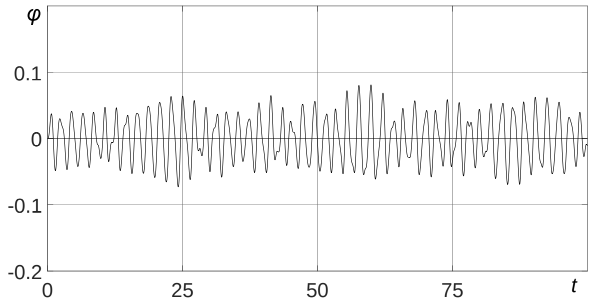

Fig. 7. Variation in time of the inclination angle of pendulum support for the above-resonance mode.

5. Conclusions

The problem about the near-resonance behavior of the system of the reservoir with a free surface liquid on pendulum suspension with movable suspension point, in which we consider the translation motion as given in advance and liquid oscillations with the angular motion of the reservoir is studied in the combined mode of motion. Such a statement of the problem is aimed at the research seismic processes. A distinctive peculiarity of the problem consists in different behavior of the system in the below-resonance, near resonance and above resonance modes. Strong effect of modulation is peculiar for all these modes of motion. Therefore, for all cases amplitudes of liquid oscillations and angular oscillations of the reservoir do not tend to the steady mode of motion, predicted by some publications.

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Особливості динаміки резервуара з рідиною з вільною поверхнею на маятниковому підвісі з рухомою точкою підвісу

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Розглянуто задачу динаміки резервуара циліндричної форми, частково заповненого рідиною, на маятниковому підвісі з рухомою точкою підвісу. Задачу розглядають у нелінійній постановці з метою визначення впливу маятникового підвісу на частотні характеристики і поведінку системи в білярезонансній зоні. Аналітично і чисельно досліджено, що власні частоти коливань суттєво змінюються як для квазітвердої маятникової форми руху, так і особливо для частоти коливань рідини. Чисельні приклади показали, що резонансні властивості системи для дорезонансного, зарезонансного і білярезонансного режимів суттєво відрізняються і для усіх випадків сильно проявляється ефект амплітудної модуляції.

Ключові слова: *коливання рідини, резервуар на маятниковому підвісі, білярезонансні режими руху, амплітудна модуляція.*

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