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MATHEMATICAL MODEL AND THE ANALYSIS OF A FIELD OF THE DISK IMPEDANCE ANTENNA WITH SPIRAL HETEROGENEIETY

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Abstract

In given paper the results of development of the mathematical model of the disk impedance antenna with spiral heterogeniety are presented. The model is presented by some class of the branched continual fractions.

***Keywords*:** disk modulated impedance antenna, mathematical model, impedance structures, recurrence formula, branched continual fractions.

1. INTRODUCTION

In paper [1] the mathematical model (MM) of the disk modulated impedance antenna (MIA) is constructed in the form of Fredholm integrated equation (IE) of the second sort with differential kernel.

In the given paper the mathematical model of the disk modulated impedance antenna (MIA) with spiral heterogeneity, is offered in the form of the branched continual fractions (BCF). Such fractions are asymptotic solutions in a spectral form of mentioned Fredholm integrated equation of the second sort for cases when the law of modulation of radial distribution of the superficial impedance is the result of addition of multiple one another of periodic sequences of rectangular functions (PSRF) [2].

The problem solution in the form of branched continual fractions differs that it is received in the closed form and consequently allows investigate a number of fundamental laws of formation of the spectrum of spatial harmonics of the field of the disk modulated impedance structures. Results of the given work can be used also for creation of elements of info-communicational systems on the basis of modulated nano-dimensional structures.

2. MATHEMATICAL MODEL

2.1. THE INTEGRATED EQUATION

The problem of the symmetric (concerning an axis) excitation of the disk MIA with spiral heterogeneity, entered in the flat screen (fig. 1), formulated in work [1], is led to the solution of the following IE:

$$I_\phi^M(s) + \frac{\hat{Z}_E(s)}{4}\int_{-L}^{L} H_0^{(2)}(|s-\eta|) I_\phi^M(\eta)d\eta = -2i\hat{Z}_E(s)H_\phi^{long}(s), \quad (1)$$

Where: $s = r/\lambda$; $L = R/\lambda$; λ- length of a wave; ($H_0^{(2)}$ - Hankel function). The equation (1) connects among themselves set laws of excitation of an impedance disk by a foreign source of a field $H_\phi^{long}(s)$ and modulation of the standardized superficial impedance $\hat{Z}_E(s)$ with the required law of distribution of magnetic currents $I_\phi^M(s)$.

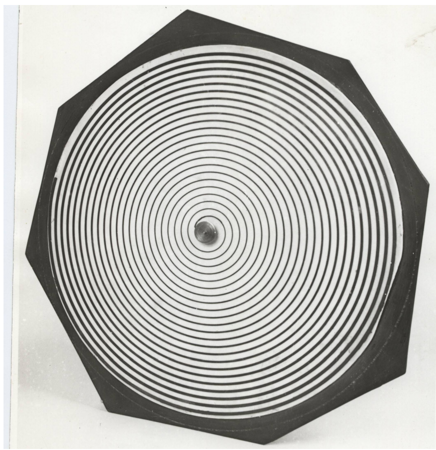


Fig.1. Disk modulated impedance antenna with spiral heterogeneity

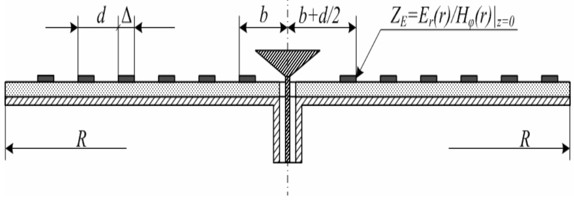


Fig.2. Radial section of the disk MIA

2.2. BRANCHED CONTINUAL FRACTIONS

For a case when in the equation (1) $L \to \infty$, and the law of modulation of the superficial impedance $\hat{Z}_E(s)$ is described by expression (2)

$$\hat{Z}_E(s) = \hat{Z}_0 + \sum_{i=1}^{N} \hat{Z}_i \sum_{n=-\infty}^{\infty} rect(\frac{s - n\hat{d}_i}{\hat{\Delta}}); \quad (2)$$

where: $\Delta << \lambda$; $\hat{d}_N = \prod_{i=1}^{N-1} p_i d_1$, $N = 1, 2, \ldots$;

the solution of the equation (1) concerning spectral density of the magnetic current

$$\tilde{I}_\phi^M(\hat{\chi}) = \frac{1}{2\pi}\int_{-\infty}^{\infty} I_\phi^M(s)e^{i\chi s}ds \quad (3)$$

will be described by the recurrence formula which in the unfold form represents BCF [2]:

$$\xi_N(\chi) \cong \xi_{N-1}(\chi) - \frac{A_N \sum_{n_N=-\infty}^{\infty} \xi_{N-1}(\chi - n_N T_N)C_{n_N}}{\prod_{m=0}^{N} D_{m,\Delta}(\chi)},$$

where:

$$D_{N,\Delta}(\chi) = 1 + A_N \sum_{n_N=-\infty}^{\infty} \frac{C_{n_N}}{\prod_{m=1}^{N} D_{m-1,\Delta}(\chi - n_N T_N)}; \quad (4)$$

$\xi_N(\hat{\chi}) = \tilde{I}_\phi^M(\hat{\chi})$; $A_N = \hat{Z}_{M_N}\Delta/d_N$;

$C_{n_N} = \sin c(n_N \pi\Delta / d_N)$; $T_N = 2\pi / d_N$; d_N – the period of spatial modulation of structures; N - number of the multiple PSRF imposed one each other; χ -the generalized wave number (spatial frequency). The recurrence formula (4) allows to define in the closed kind spectral density $\xi_N(\hat{\chi})$ N of multiply modulated structure through N-1 multiple. Function $\xi_0(\hat{\chi})$ is the solution of a problem symmetric (concerning an axis) excitation by a vertical electric dipole in length $\Delta\ell$ of a flat infinite impedance disk with homogeneous distribution of the superficial impedance. This function is base for construction under formulas (4) solutions of problems of the modulation of the superficial impedance by complex periodically non-uniform laws. To similarly work [2], can be shown, that function $\xi_0(\hat{\chi})$ has a following

view: $\xi_0(\chi) = 2\Phi_0(\chi) / D_{0,\Delta}(\chi)$, (5)

where: $D_{0,\Delta}(\chi) = \sqrt{\hat{\chi}^2 - 1} - Z_0$; (6)

$\Phi_0(\hat{\chi}) = \hat{\chi}^2 A$; $A = (8\pi i)^{-1} I_0 \Delta\ell$. (7)

2.3. Modulation of impedance by PSRF

As an example let take mathematical model of impedance disk modulated by one PSRF.

$$\hat{Z}_E(s) = \hat{Z}_0 + \hat{Z}_1 \sum_{n=-\infty}^{\infty} rect(\frac{s - n\hat{d}_1}{\hat{\Delta}}); \quad (8)$$

Such PSRF is presented on fig. 3:

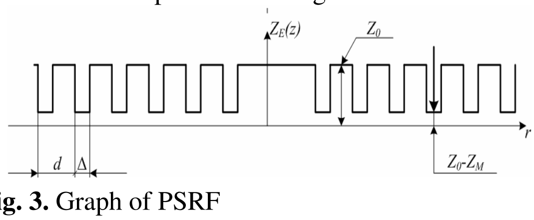


Fig. 3. Graph of PSRF

Mathematical model of the antenna:

$$\xi_1(\chi) = \Phi_0(\chi)\phi_0(\chi)\phi_{1,\Delta}(\chi); \quad (9)$$

where: $\varphi_0(\hat{\chi}) = 1/(\sqrt{\hat{\chi}^2 - 1} - \hat{Z}_0)$; (10)

$$\phi_{1,\Delta}(\hat{\chi}) = 1/(1 + \hat{Z}_{M_1}\frac{\Delta}{d_1}\sum_{n_1=-\infty}^{n_1=\infty}\frac{\Phi_0(\hat{\chi} - n_1 T_1)\sin c(n_1\pi\Delta/d_1)}{\sqrt{(\hat{\chi} - n_1\lambda/d_1)^2 - 1} - \hat{Z}_0})$$

Where:

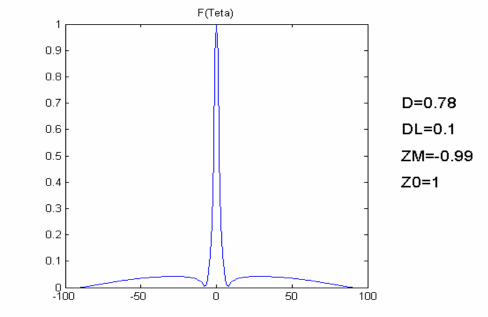
$T_1 = \lambda / d_1$; $\Phi_0(\hat{\chi} - n_1 T_1) = A(\hat{\chi} - n_1 T_1)^2$. (11)

For the construction of the radiation patterns in the field of "visible corners" ($-90^0 \le \theta^0 \le 90^0$) it is considered, that

$F(\hat{\chi}) = (\sqrt{\hat{\chi}^2 - 1})\xi_1(\hat{\chi})$; $\hat{\chi} = \sin\theta^0$; θ^0 – a corner which is counted from a normal to a surface of structure.

3. Results of calculations

On Fig. 4 the radiation patterns of the disk MIA are resulted:



θ^0 (degrees)

Fig. 4. Radiation pattern of the disk MIA

4. Conclusion

The mathematical model in the form of branched continual fractions does not consider the waves reflected from the ends of the structure. However such decision is valuable for the research of fundamental properties of the disk modulated impedance structures. It transparent connects among themselves constructive parameters of the antennas with parameters of a spectrum of the spatial harmonics. Applied value of such researches in the long term increases for development of new info - communication systems on the base of nano-dimensional structures.

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