MATHEMATICAL MODELS OF THE RADIATING AND WAVEGUIDE STRUCTURES WITH N-FOLD PERIODICITY

Hoblyk V. V.

Departament "Electronic equipment of information computer networks"
Lviv Polytechnic National University, Lviv, Ukraine,
E-mail: viktor_hoblyk@mail.ru

Abstract

In present article the results of development of the mathematical models of the wide class of radiating and waveguiding structures of divisible periodicity of constructive parameters are represented.

Keywords: mathematical model, branched continual fractions, radiating and waveguiding structures.

1. INTRODUCTION

Researching problems of electrodynamic, optical and info-communicative properties of structures with multiple periodicity of constructive parameters become more and more actual in connection with development of info-communicative systems, antennas technics, development of ways of processing, transfer and storage of information based on nano-sized structures and nanotechnology. The role of mathematical and radio-physical modelling considerably grows in research of these problems.

In the present work the review of results of developing the mathematical models of a wide class of periodically heterogeneous radiating and waveguiding structures with N-fold periodicity of change of their constructive parameters on the basis of a method offered by A.F.Chaplin in work [1] is resulted. This method found in the closed form the definite solution of the problem of electrodynamic excitation of an infinite impedance plane, surface impedance of which is modulated by two imposed multiple periodic sequences of δ - functions.

In works [2-12] A.F.Chaplin’s method is extended to the solution of problems of electrodynamic excitation of a wide class of periodically heterogeneous structures, which are modulated by any number of N-fold periodic sequences of impulse functions.

Solutions of problems of electrodynamic excitation of circular and elliptic impedance cylinders modulated by periodic sequences of impulse functions (a squared shape and Gaus functions) are found. Problems of electrodynamic excitation of periodically heterogeneous dielectric plate and the circular dielectric cylinder with modulation of complex dielectric permeability by multiple periodic sequences of impulse functions are solved. Solutions of a problems of electrodynamic excitation of a flat wave guide, a surface impedance of which walls is modulated by multiple periodic sequences of rectangular functions are obtained.

Mathematical models of the linear antenna lattices the loading of which changes under the periodic laws formed as a result of imposing of multiple periodic impulse functions are constructed.

Wide range of particular problems is generalised in a form of recurrence formula. It allows to construct by corresponding algorithm mathematical models of structures with difficult laws of modulation of constructive parameters on the basis of the previous more simple solutions of a problem.

It is found out that the mathematical models developed for the description of electrodynamic properties of radiating and waveguiding structures with N-fold periodicity belongs to a separate class of branched continual fractions, theory of which is based in works [13,14].

2. MATHEMATICAL MODEL

The spectral density $\xi_N(\chi)$ of field’s spatial distribution of one-dimensional modulated antenna arrays, impedance and dielectric structures can be presented by the next recurrence formula, which in a unfold form looks like branched continual fraction:

$$\xi_N(\chi) \cong \xi_{N-1}(\chi) - \frac{A_N \sum_{n_N=-\infty}^{\infty} \xi_{N-1}(\chi - n_N T_N) C_{n_N}}{\prod_{m=0}^{N} D_{m,\Delta}(\chi)}$$

where: (1)

$$D_{N,\Delta}(\chi) = 1 + A_N \sum_{n_N=-\infty}^{\infty} \frac{C_{n_N}}{\prod_{m=1}^{N} D_{m-1,\Delta}(\chi - n_N T_N)}$$

A_N - coefficients, which describe parameters of modulation of the antenna arrays, the impedance and dielectric structures: amplitude of modulation of load resistance, period and linear sizes of heterogeneities;

C_{n_N} - coefficients, which describe form of curve and the parameters of structures can be modulated by means of the curve. The structures' parameters can be modulated by the functions, represented as the superimposed multiple periodic sequences of rectangular, triangular gauss and other impulse functions, also $\delta-$ functions.

$T_N = 2\pi / d_N$; d_N – period of spatial modulated structure;

N- number of the functions' superimposed multiple periodic sequences;

χ – spatial frequency. For construction of structure's directional diagram in a field "visible angles" the quantity $\chi = k \sin\theta^0 -$ should be put instead of χ in the formula (1), where is

k- wave - number of free space, θ^0 – angle, counted from normal till aperture of structure

The function $D_{0,\Delta}(\chi)$ in the formula (1) has next look:

$$D_{0,\Delta}(\chi) = Z_0(a,\omega,\varepsilon_a) - P_0(\chi) B_0(a,\chi) \; ; \qquad (2)$$

$Z_0(a,\omega,\varepsilon_a)$ – constant component of structure's surface impedance, defined by it geometrical and radiophysical parameters.

$$P_0(\chi) = -i\sqrt{\chi^2 - k^2} \; ; \qquad (3)$$

$B_0(a,\chi)$ – function, which describe transverse geometry of structure;

$$\xi_0(\chi) = 2\Phi(\chi) / D_{0,\Delta}(\chi) \qquad (4)$$

solution of the problem under condition that spatial modulation of structure is absent;

$\Phi(\chi) -$ spectral destiny of primary field.

Concrete form of branched continual fraction's components (1) for computation of the scattered field of modulated antenna arrays, impedance and dielectric structures are represented in the works [5-12].

4. Conclusion

We have shown usage of the branched continual fractions in the problems of the theory of the modulated antenna arrays, the periodic impedance and dielectric structures. The branched continual fractions are also effective for the description of thin effects of photons' interaction with subarrays, which constructed from nanodimensional elements.

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